Parasitic Pole Approximation Techniques for Active Filter Design

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Abstract—**A method of approximating the parasitic poles of active filters employing one, two, or three operational amplifier(s) is presented. These approximations are good for both a single-pole and two-pole model of the operational amplifier and are useful for determining filter stability. The approximation expressions are sufficiently simple that they may be included in the active filter design process.**

Stability criteria for active filters employing two-pole operational amplifiers are stated in terms of the parasitic pole approximations obtained using the simpler single-pole model of the operational amplifier. The necessity of including the more accurate two-pole model of an operational amplifier in some applications is discussed. Detailed comparisons of the actual and approximate parasitic poles in two examples are made.

I. Introduction

IT IS A well-known fact that the frequency response of an operational amplifier (op amp) affects the performance of active filters employing these devices. The frequency dependent gain of the op amp causes both a perturbation in the desired poles of the active filter from their nominal position and the introduction of parasitic poles in the active transfer function. It has also been observed by many filter designers that the performance is affected to the extent that some active filters that look promising when designed assuming the op amp's are ideal either perform poorly in the laboratory when actual op amp's are used or are actually unstable. This degradation in performance and/or possible instability can often be attributed primarily to the movement of the desired poles and zeros from their ideal position [1]–[3].

Much attention has been devoted to the design of active filters which are less sensitive to the parameters of the op amp's [4]–[8]. This has often been achieved by incorporating an accurate model of the op amp itself [7], [8] into the design process. The desired poles in these designs exhibit a reduced dependence upon the parameters of the op amp's.

Some designs in which the desired poles are very nearly located at the ideal position are, however, experimentally observed to be unstable. This is often a result of the fact that the *parasitic poles* lie in the right half of the s-plane. Such instabilities are often characterized in the laboratory by a high frequency approximately sinusoidal oscillation which may vary in amplitude from a few millivolts to several volts depending upon the circuit topology and particular op amp used.

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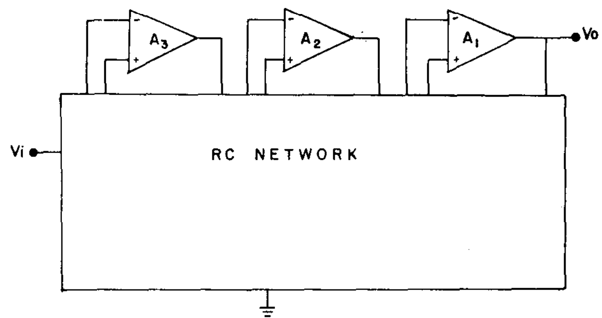

Fig. 1. General three op amp active filter.

This paper is concerned with obtaining approximate expressions for the *location of the parasitic transfer function poles* of active filters employing one, two, and three operational amplifiers. The expressions derived herein approximate the parasitic poles for both single-pole and two-pole models of the op amp's. By incorporating stability considerations obtainable from these parasitic pole approximations into the design process itself, some promising configurations that have in the past been observed to be unstable may be modified slightly to obtain a practical usable filter.

The magnitude of the parasitic pole approximation problem can be appreciated by realizing that an active filter employing three op amp's, each modeled by a two-pole model, which is designed to realize a second-order transfer function actually has an eighth-order denominator polynomial that must be factored to obtain the two desired poles and the six parasitic poles.

Although the frequency response of the op amp is usually (and often quite successfully) approximated by a single-pole transfer function, it is shown by example that a two-pole model may be more useful for determining stability. In this example a filter is given which has all poles in the left half-plane when a single-pole model of the op amp is used but which has a pair of parasitic poles in the right half-plane when a two-pole model is employed. The predicted instability is in agreement with observed laboratory results.

II. Pole-Defining Equation for Active Filters

The most general active filter employing three op amp's is shown in Fig. 1. The op amp's are all assumed to have infinite input impedance, infinite common-mode rejection ratio and zero-output impedance. The transfer function of

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TABLE I
POLE-DEFINING EQUATIONS FOR ACTIVE FILTERS EMPLOYING UP TO THREE OP AMP'S

#OP AMPs	#Poles in Model	Equation No.	Pole-Defining Equations
1	1	(i)	$D_{1,1}(s,\tau) = D_o(s) + \tau s D_p(s) = 0$
1	2	(ii)	$D_{1,2}(s,\tau) = D_o(s) + \tau s(1+\theta\tau s)D_p(s) = 0$
2	1	(iii)	$D_{2,1}(s,\tau) = D_o(s) + \tau s P_1(s) + \tau^2 s^2 D_p(s) = 0$
2	2	(iv)	$D_{2,2}(s,\tau) = D_o(s) + \tau s(1+\theta\tau s)P_1(s) + \tau^2 s^2(1+\theta\tau s)^2 D_p(s) = 0$
3	1	(v)	$D_{3,1}(s,\tau) = D_o(s) + \tau s P_1(s) + \tau^2 s^2 P_2(s) + \tau^3 s^3 D_p(s) = 0$
3	2	(vi)	$D_{3,2}(s,\tau) = D_o(s) + \tau s(1+\theta\tau s)P_1(s) + \tau^2 s^2(1+\theta\tau s)^2 P_2(s) + \tau^3 s^3(1+\theta\tau s)^3 D_p(s) = 0$

this filter may be expressed as [7]

$$\frac{V_0}{V_i} = \frac{N_0 + \frac{N_2}{A_2} + \frac{N_3}{A_3} + \frac{N_{23}}{A_2 A_3}}{D_0 + \frac{D_1}{A_1} + \frac{D_2}{A_2} + \frac{D_3}{A_3} + \frac{D_{12}}{A_1 A_2} + \frac{D_{13}}{A_1 A_3} + \frac{D_{23}}{A_2 A_3} + \frac{D_p}{A_1 A_2 A_3}} \tag{1}$$

where all N's and D's are polynomials in s dependent only upon the passive RC network and A_1, A_2, and A_3 are gains of the op amp's and are ideally infinite. D_0 is the desired denominator polynomial and D_p is the characteristic polynomial of the passive RC network.

At this point, for the convenience of reduced notational complexity, the assumption of identical op amp's will be made. Thus for $A_1 = A_2 = A_3 = A$, the pole-defining equation (the denominator of (1)) may be expressed as

$$D_3(s,A) = D_0 + \frac{P_1(s)}{A} + \frac{P_2(s)}{A^2} + \frac{D_p(s)}{A^3} = 0 \tag{2}$$

where $P_1(s) = D_1 + D_2 + D_3$ and $P_2(s) = D_{12} + D_{13} + D_{23}$.

The following development can readily be modified to handle the more general nonidentical op amp situation except where otherwise noted by considering the actual denominator of (1) instead of the simplified expression of (2) as the pole-defining equation.

The pole-defining equations in the two identical op amp and single op amp cases may similarly be expressed, respectively, as

$$D_2(s,A) = D_0(s) + \frac{P_1(s)}{A} + \frac{D_p(s)}{A^2} = 0 \tag{3}$$

$$D_1(s,A) = D_0(s) + \frac{D_p(s)}{A} = 0. \tag{4}$$

The gain function of the single-pole model of the op amp is assumed to be of the form [1]

$$A(s) = \frac{1}{\tau s} \tag{5}$$

where τ is the reciprocal of the gain-bandwidth product of the op amp. The gain function of the two-pole model of the op amp is assumed to be of the form

$$A(s) = \frac{1}{\tau s(1+\theta\tau s)} \tag{6}$$

where τ is again the reciprocal of the gain-bandwidth product and θ is a parameter of the op amp that determines the location of the second pole. When $\theta = 0$, the two-pole model reduces to the single-pole model.

The pole-defining equations (2)–(4) when the single-pole model and two-pole model of the op amp is employed are listed in Table I.

III. APPROXIMATE ROOT EXPRESSIONS

If the op amp's are assumed to be ideal, then $\tau = 0$ and the pole-defining equations listed in Table I all reduce to $D_0(s) = 0$. The roots of $D_0(s)$ are referred to as the "desired roots." In practice, however, τ is small but not zero. As a consequence, there will be a small movement of the desired roots as well as the existence of additional roots which are, in general, strongly dependent upon τ. These additional roots are termed "parasitic roots." In most *good* designs, the movement of the desired roots from their ideal position due to the nonzero op amp time constants is small. Approximations of the desired roots and the associ-

TABLE II
APPROXIMATE POLE-DEFINING EQUATIONS

# Of AMPs	# Poles in Model	Equation No.	Approximate Pole-Defining Equations
1	1	(i′)	$D_{1,1}(s,\tau) = D_o(s) + \tau s D_p(s) \simeq \left(\sum_{i=0}^{n_o} d_{oi}s^i\right)\left(\sum_{i=0}^{n_p+1-n_o} a_i\tau^i s^i\right) = 0$
2	1	(iii′)	$D_{2,1}(s,\tau) = D_o(s) + \tau s P_1(s) + \tau^2 s^2 D_p(s) \simeq \left(\sum_{i=0}^{n_o} d_{oi}s^i\right)\left(\sum_{i=0}^{n_p+2-n_o} a_i\tau^i s^i\right) = 0$
3	1	(v′)	$D_{3,1}(s,\tau) = D_o(s) + \tau s P_1(s) + \tau^2 s^2 P_2(s) + \tau^3 s^3 D_p(s) \simeq \left(\sum_{i=0}^{n_o} d_{oi}s^i\right)\left(\sum_{i=0}^{n_p+3-n_o} a_i\tau^i s^i\right) = 0$

ated stability problem can be found in the literature [1]–[5], [7], [8], [10], [11]. The emphasis here is placed upon approximating the parasitic roots.

Approximate Root Equations with Single-Pole Model of the Op Amp

Let the degree of the passive RC network be n_p. Then the polynomials $D_0(s)$, $P_1(s)$, $P_2(s)$, and $D_p(s)$ in the pole-defining equations of Table I are of degrees n_0, n_1, n_2, and n_p, respectively, where n_0, n_1, and n_2 are less than or equal to n_p and where the highest order coefficient of D_0 is assumed to be equal to unity.

The following assumption will now be made.

Assumption 1: Assume τ is so small that $D_0(s)$ is a factor of the pole-defining equations (i), (iii), and (v) of Table I.

Although $D_0(s)$ is in general not a factor of the pole-defining equations, the actual location of the desired poles are very near to the roots of $D_0(s)$ for small values of τ since the roots of the pole defining equation are analytic functions of τ [12]. With this assumption, the pole-defining equations (i), (iii), and (v) of Table I can be approximated by equations (i′), (iii′) and (v′) of Table II, where

$$D_0(s) = \sum_{i=0}^{n_0} d_{0i}s^i.$$

A reasonable choice of the "a" coefficients in (i′) can be made by equating the coefficients of the highest $n_p - n_0 + 2$ powers of s on both sides of "$\simeq$" in (i′). Equating these coefficients results in a set of linear equations in the "a" variables which can readily be solved. The "a" coefficients in (iii′) and (v′) are determined in a similar manner. If τ is sufficiently small, it is often the case that each of these linear equations is essentially dependent upon only a single "a" variable making the solution very simple.

With τ small, the equations of Table II of the form

$$\sum_{i=0}^{n_p - n_0 + j} a_i(\tau s)^i = 0, \qquad j = 1,2,3 \tag{7}$$

are the equations that can be solved to approximate the parasitic roots when the single-pole model of the op amp is used.

For existing active filters, it is most often the case that $n_0 = n_p$ so that the order of the equation that determines the parasitic roots is equal to the number of op amp's in the circuit. If one or two op amp's are used, the expressions for the parasitic roots are particularly simple.

Several alternative methods for obtaining the parasitic roots are discussed by Heinlein and Holmes [3, p. 216–226]. These methods give results similar to those presented here.

Approximate Root Expressions for Two-Pole Model of the Op Amp's

Two methods of obtaining the approximate parasitic roots when a two-pole model of the op amp is employed will be considered.

The first directly parallels the single-pole model development provided equations (ii), (iv), and (vi) of Table I are used in place of (i), (iii), and (v) in the preceding derivation. If the number of op amp's is very large, the order of the equation corresponding to (7) is rather high.

The second method, which is generally more tractable, extracts the two-pole model parasitic roots directly from those obtained by applying the much simpler single-pole model. This method is not, however, readily extendable to the nonidentical op amp situation.

The following development of the second method is dependent upon the assumption:

Assumption 2: $\lim_{\tau \to 0} a_i \neq 0$ for all "a" coefficients of (7).

This assumption roughly says that the "a" coefficients are independent of τ for small τ. This assumption is satisfied in most stable existing filters.

With this assumption and for small values of τ it follows that (7) may be written in factored form as

$$\prod_{i=1}^{n_p-n_0+j} (\tau s - p_i) = 0, \qquad j=1,2,3 \tag{8}$$

where for all i, p_i is not a function of τ. The parasitic poles obtained with the single-pole model are thus located at $s = p_i/\tau$, $i = 1, \cdots, n_p - n_0 + j$.

At this point, observe that the use of the two-pole model of the op amp of (6) actually involves making the transformation

$$\tau s \to \tau s(1+\theta\tau s) \tag{9}$$

in the pole-defining equations (i), (iii), and (v) of Table I. Since the τs term appears only in the second part of the product in the pole-defining equations of Table II, it follows that a factored form of the pole-defining equation for active filters using the two-pole approximation of the op amp's can readily be obtained by replacing τs by $\tau s(1+\theta\tau s)$ in (8). The parasitic poles can thus be determined from a knowledge of the single-pole parasitic poles from the expression

$$\prod_{i=1}^{n_p-n_0+j} (\tau s[1+\theta\tau s] - p_i) = 0, \qquad j=1,2,3. \tag{10}$$

The parasitic poles obtained from this expression are thus located at

$$s = -\frac{1}{\tau(2\theta)}\left(1 \pm \sqrt{1+4\theta p_i}\right). \tag{11}$$

This equation can be used to approximate all parasitic poles with a two-pole model of the op amp once the parasitic poles with the single-pole model have been either determined or approximated.

In the case that p_i is a complex number,

$$p_i = -\alpha + j\beta \tag{12}$$

p_i^* is also a root of (10). In this case it can readily be shown that the four second-order model roots obtainable from the first-order-model roots p_i and p_i^* are given by the expression

$$s = -\frac{1}{\tau(2\theta)}\left[\begin{array}{c} 1 \pm \sqrt{\dfrac{1-4\theta\alpha}{2} + \dfrac{\sqrt{1-8\theta\alpha+16\theta^2\alpha^2+16\theta^2\beta^2}}{2}} \\ \pm j\sqrt{\dfrac{4\theta\alpha-1}{2} + \dfrac{\sqrt{1-8\theta\alpha+16\theta^2\alpha^2+16\theta^2\beta^2}}{2}} \end{array}\right]. \tag{13}$$

IV. Stability Considerations

Since it is assumed in these approximations that τ is sufficiently small that the desired roots are essentially at the ideal locations, the filter stability will be determined by the parasitic pole locations. In the single-pole model case the parasitic roots themselves if solved for or the Routh–Hurwitz criterion can be used to determine stability. If a two-pole model of an op amp is necessary the stability question can be answered based solely upon the location of the more readily determined single-pole model parasitic roots provided the op amp's are identical. This is formalized in the following claim.

Claim: If τ is sufficiently small so that (7) is a good approximation of the pole-defining equation of the parasitic roots for the single-pole model of the op amp, then the filter will be stable if a two-pole model of the op amp is employed when the following equation is satisfied for each single-pole model parasitic root $p_i = -\alpha + j\beta$.

$$\theta\beta^2 < \alpha. \tag{14}$$

This claim follows from (13) with some routine algebraic manipulations by requiring that the real part of all complex-conjugate poles be negative.

The approximations presented thus far and ultimately the stability criterion have relied upon the fact that τ is sufficiently small. At this point the statement should be made that if a circuit is stable with sufficiently small values of τ, then it will often be stable for values of τ in which the approximations of Table II can not be justified provided the op amp's are identical. This can be established by showing that the parasitic pole movement as a function of increasing τ for parasitic poles with a nonzero imaginary part is approximately in a constant Q manner towards the origin of the s-plane. This fact will now be derived for the single-pole model case directly from Assumptions 1 and 2.

Assume τ is sufficiently small that the approximate parasitic pole-defining equation (7) is justified and assume that the "a" coefficients in (7) are not dependent upon τ. Let p_k be any parasitic root. Thus (7) may be written in the form

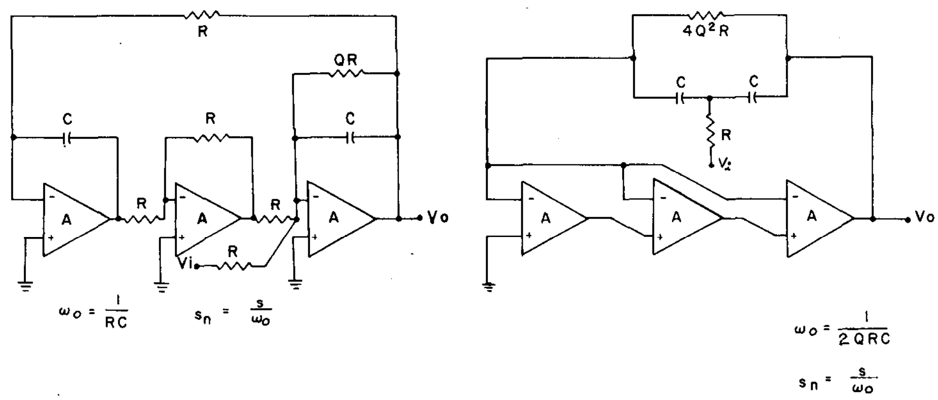
$$\sum_{i=0}^{n_p-n_0+1} a_i(\tau p_k)^i = 0. \tag{15}$$

Since the "a" coefficients are independent of τ, it follows that

$$\frac{\partial p_k}{\partial \tau} = \frac{-p_k}{\tau}. \tag{16}$$

Thus the change in p_k from its value at $\tau = \tau_1$ where τ_1 is sufficiently small that (7) is justifiable to its position $\tau_2 = \tau_1 + \Delta\tau$ may be approximated by

$$\Delta p_k \simeq \left.\frac{\partial p_k}{\partial \tau}\right|_{\tau=\tau_1} \cdot \Delta\tau = \frac{-p_k}{\tau_1}\Delta\tau. \tag{17}$$



$$\frac{V_o}{V_i} = \frac{s_n + \frac{1}{A}(3s_n+1) + \frac{1}{A^2}(2s_n+2)}{s_n^2 + \frac{s_n}{Q} + 1 + \frac{1}{A}\left(4s_n^2 + \left[3+\frac{4}{Q}\right]s_n + \frac{1}{Q}\right) + \frac{1}{A^2}\left(5s_n^2 + \left(9+\frac{5}{Q}\right)s_n + 2 + \frac{3}{Q}\right) + \frac{1}{A^3}\left(2s_n^2 + \left(6+\frac{2}{Q}\right)s_n + 4 + \frac{2}{Q}\right)}$$

$$\frac{V_o}{V_i} = \frac{2Qs_n}{s_n^2 + \frac{s_n}{Q} + 1 + \frac{1}{A}\left(s_n^2 + \frac{s_n}{Q} + 1\right) + \frac{1}{A^2}\left(s_n^2 + \frac{s_n}{Q} + 1\right) + \frac{1}{A^3}\left(s_n^2 + s_n\left[2Q + \frac{1}{Q}\right] + 1\right)}$$

Fig. 2. Biquadratic bandpass filters.

This shows that the movement as a function of τ of parasitic poles with a nonzero imaginary part is towards the origin in a *constant* Q manner as claimed above.

The above derivation extends readily in the case that the two-pole model of the op amp is employed.

It can be concluded that filter stability related to the position of the parasitic poles with either a single-pole or two-pole model of the op amp can be approximated by investigating the position of the parasitic poles obtained by considering the op amp's to have a single-pole model with τ sufficiently small so that the approximations of Table II are justifiable.

The assumption of identical op amp's is necessary to obtain reasonably simple expressions for the approximate parasitic poles in the general single-pole model case and is crucial for determining the parasitic poles with a two-pole model of the op amp directly from those obtained with the single-pole model via (11). It is, however, reasonable to expect that a filter which is predicted to be unstable when identical op amp's are used will not be useful if unmatched op amp's are employed. It is also reasonable to expect that the approximations listed above will be quite good as long as the parameters of the op amp's do not differ greatly.

No mention has been made to this point about how small τ should be to make the parasitic-pole approximations previously derived. The answer to this question depends in part upon the required accuracy of the approximations. Based upon a few examples it appears that the approximations derived above will probably be quite good if for a particular value of τ the real and imaginary parts of the approximate *desired-pole* locations obtainable from the references previously cited differ by at most a few percent from their ideal $\tau = 0$ value.

V. Example

The locations of the poles of the two second-order bandpass active filters of Fig. 2 will now be approximated and a comparison made of the approximate pole locations with the actual computer generated poles. The first circuit is a form of the popular state-variable filter [11]. The second is of interest because it is an example of a promising filter that appears stable when a single-pole model of the op amp is employed but which is actually unstable because of a right half-plane parasitic pole that appears when the typical second pole of an op amp such as the 741 is included in the analysis.

An analysis will now be made of the approximate pole locations for the state-variable filter of Fig. 2(a). The analysis of the remaining configuration is straightforward, hence only the results will be stated.

When a single-pole model of the op amp is employed, the pole-defining equation for the circuit of Fig. 2(a) is

$$\begin{aligned} D_{31}(s_n, A) &= s_n + \frac{s_n}{Q} + 1 + \tau_n s_n\left(4s_n^2 + s_n\left(3 + \frac{4}{Q}\right) + \frac{1}{Q}\right) \\ &\quad + \tau_n^2 s_n^2\left(5s_n^2 + s_n\left[9 + \frac{5}{Q}\right] + 2 + \frac{3}{Q}\right) \\ &\quad + 2\tau_n^3 s_n^3\left(s_n^2 + s_n\left[3 + \frac{1}{Q}\right] + 2 + \frac{1}{Q}\right) \\ &\simeq \left(s_n^2 + \frac{s_n}{Q} + 1\right)\left[a_3(\tau_n s_n)^3 + a_2(\tau_n s_n)^2 \right. \\ &\quad \left. + a_1(\tau_n s_n) + a_0\right]. \end{aligned} \tag{18}$$

The normalizations $s_n = s/\omega_0$ and $\tau_n = \tau\omega_0$ have been made to eliminate the ω_0 dependence from these equations thus making the analysis simpler. By equating the coefficients

TABLE III
COMPARISON OF APPROXIMATE AND EXACT PARASITIC POLE LOCATIONS FOR A SINGLE-POLE MODEL OF THE OP AMP

ROOT LOCATIONS SINGLE-POLE MODEL OF OP AMP		
τ_n	EXACT	APPROXIMATE
	STATE VARIABLE FILTER OF FIGURE 2a	
.001	-4.79071E-02 ± j0.997347E+00 *	-4.79500E-02 ± j.998788E+00
	-1.00018E+03	-1.00000E+03
	-1.00291E+03	-1.00000E+03
	-5.00009E+02	-5.00000E+02
.01	-2.97293E-02 ± j.984257E+00 *	-2.95000E-02 ± j.990700E+00
	-1.02562E+02	-1.00000E+02
	-1.00403E+02	-1.00000E+02
	-5.00753E+01	-5.00000E+01
	ZERO SENSITIVITY CIRCUIT OF FIGURE 2b	
.001	-5.00000E-02 ± j.998756E+00 *	-5.000000E-02 ± j.999687E+00
	-4.94932E-01 ± j.994975E+03	0.0 ± j1.0E+03
	-1.01010E+03	-1.0E+03
.01	-5.00019E-02 ± j.998760E+00 *	-5.000000E-02 ± j.999687E+00
	-4.47319E+00 ± j.947864E+02	0.0 ± j1.0E+02
	-1.11054E+02	-1.0E+02

*Indicates desired root

of the four highest powers of s the "a" coefficients of (7), subject to the assumption $\tau_n \ll 1$, are readily found to be

$$\left.\begin{aligned} a_3 &\simeq 2 \\ a_2 &\simeq 5 \\ a_1 &\simeq 4 \\ a_0 &\simeq 1 \end{aligned}\right\}. \tag{19}$$

The single-pole model parasitic pole-defining equation (7) is thus

$$2(\tau_n s_n)^3 + 5(\tau_n s_n)^2 + 4(\tau_n s_n) + 1 = 0. \tag{20}$$

The roots of this cubic polynomial are

$$\left.\begin{aligned} s_n &= -1.0/\tau_n \\ s_n &= -1.0/\tau_n \\ s_n &= -0.5/\tau_n \end{aligned}\right\}. \tag{21}$$

These approximate parasitic roots as well as the single-pole model approximate parasitic roots for the circuit of Fig. 2(b) are listed in Table III for $\tau_n = 0.001$ and 0.01 and for $Q = 10$. If the gain-bandwidth product of the op amp's are $2\pi \times 10^6$ rad/s, a typical value for the 741, then the $\tau_n = 0.001$ and 0.01 cases correspond to bandpass filters centered at 1 kHz and 10 kHz, respectively. The computer generated poles are also listed in the table. A comparison of the parasitic poles of the two circuits of Fig. 2 with the computer generated poles for $\tau_n = 0.001$ shows that the approximations are excellent. The approximations are quite good even when $\tau_n = 0.01$. The approximate desired roots [2, p. 390] are also listed in the table.

TABLE IV
COMPARISON OF APPROXIMATE AND EXACT PARASITIC POLE LOCATIONS FOR A TWO-POLE MODEL OF THE OP AMP

ROOT LOCATIONS TWO-POLE MODEL OF OP AMP		
τ_n	EXACT	APPROXIMATE
	STATE VARIABLE FILTER OF FIGURE 2a	
.001	-4.79065E-02 ± j.997348E+00 *	-4.79500E-02 ± j.998788E+00
	-1.00020E+03 ± j.998815E+03	-1.00000E+03 ± j1.00000E+03
	-1.00130E+03 ± j1.00268E+03	-1.00000E+03 ± j1.00000E+03
	-1.00001E+03 ± j1.96572E+00	-1.00000E+03
		-1.00000E+03
.01	-2.96584E-02 ± j.984355E+00 *	-2.95000E-02 ± j.990700E+00
	-1.00181E+02 ± j1.00171E+02	-1.00000E+02 ± j1.00000E+02
	-1.01300E+02 ± j1.01312E+02	-1.00000E+02 ± j1.00000E+02
	-1.00040E+02 ± j1.93346E+00	-1.00000E+02
		-1.00000E+02
	ZERO SENSITIVITY CIRCUIT OF FIGURE 2b	
.001	-5.00000E-02 ± j.998750E+00 *	-5.00000E-02 ± j.999687E.00
	-1.00505E+03 ± j1.00498E+03	-1.00000E+03 ± j1.00000E+03
	-2.27222E+03 ± j.799113E+03	-2.27202E+03 ± j.786161E+03
	+2.67270E+03 ± j.783093E+03	+2.72020E+02 ± j.786151E+03
.01	-5.00021E-02 ± j.998760E+00 *	-5.00000E-02 ± j.999687E+00
	-1.05488E+02 ± j1.04701E+02	-1.00000E+03 ± j.999687E+00
	-2.22741E+02 ± j.806809E+02	-2.27202E+02 ± j.786151E+02
	+2.29011E+01 ± j.754664E+02	+2.72020E+01 ± j.786151E+02

* Indicates desired root

Using (11), the parasitic roots for the circuit of Fig. 2(a) when a two-pole model of the op amp with $\theta = 0.5$ (a typical value for the 741) is used can be readily obtained from (21) to obtain the six approximate parasitic roots

$$\left.\begin{aligned} s_n &= (-1.0 \pm j1.0)/\tau_n \\ s_n &= (-1.0 + j1.0)/\tau_n \\ s_n &= -1.0/\tau_n \\ s_n &= -1.0/\tau_n \end{aligned}\right\}. \tag{22}$$

These approximate parasitic roots as well as those for the circuit of Fig. 2(b) are listed in Table IV for $\tau_n = 0.001$ and 0.01 and $Q = 10$ along with the computer generated roots. The approximate desired roots [2, p. 168] are also included. The exact and approximate parasitic roots for the circuits of Fig. 2 are likewise seen to be in close agreement when the two-pole model of the op amp is employed.

The necessity of including the second pole of the op amp in the analysis of some active filters is borne out by the circuit of Fig. 2(b). It can be seen from Table III that if the simpler but less accurate single-pole model of the op amp is used an exact computer analysis indicates the filter is stable since all poles have a negative real part. By including the typical second-pole of the op amp in the analysis, the approximate and exact root locations listed in Table IV indicate that the filter is actually unstable because of a right half-plane parasitic pole. This instability is in agreement with experimental results. The instability could, of course, have been predicted by considering the single-pole model of the op amp and using (14).

It is interesting to note that the desired pole locations for this circuit are essentially unchanged from the values obtained by assuming the op amp's to be ideal. This circuit is thus an example of an active filter in which it appears that the performance in the region of interest is essentially independent of the nonideal parameters of the op amp but which is actually unstable due to the presence of a right half-plane pole caused by the second-poles of the op amp's.

VI. Conclusion

A method of approximating the parasitic roots of active filters employing multiple op amp's has been presented. These approximations are good for either a single-pole or two-pole model of the op amp when the op amp time constant τ is sufficiently small. These approximation techniques involve solving low-order polynomials rather than the higher order polynomials necessary to obtain the exact roots. Of particular importance is the fact that for two op amp's or less a closed-form expression for the parasitic poles in terms of the components of the circuit is usually possible for either a single-pole or two-pole model of the op amp. Such a closed-form expression should be useful to the filter designer in the design process itself.

It has also been shown that since the parasitic pole movement is in a *constant Q* manner as a function of increasing τ, stability criterion can be approximated for values of τ at which the pole approximations can not be justified based upon the approximations when τ is sufficiently small.

An example has been given of a circuit which appears to perform very well when a single-pole model of the op amp is used but which is actually unstable when a typical second pole is included in the model of the op amp. The importance of considering the second pole of the op amp during the design process is borne out by this example.

Finally, comparisons of the readily obtained parasitic pole approximations are made with the actual computer generated poles for two circuits. These comparisons show that the approximations are quite good.

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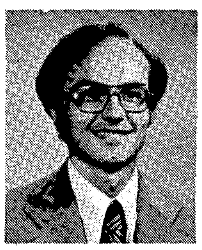

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