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# Integrator Design for High-Frequency Active Filter Applications

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**Abstract**—The combined effects of deviation from ideal in integrator magnitude and phase on the performance of integrator-based active filters is investigated. The limitations of using the integrator  $Q$ -factor as a measure of integrator performance are discussed. Conditions for minimizing the operational amplifier gain bandwidth product effects on integrator-based active filters are established which are dependent only

upon the characteristics of the integrators. Inverting and noninverting integrators for use in active filters which are independent of first-order and second-order operational amplifier time constant effects and require no amplifier matching are introduced. These new integrators are compared with existing actively compensated configurations directly and in a filter structure. Experimental results are presented which confirm the theoretical development.

## I. INTRODUCTION

**I**NTEGRATORS find widespread use in the design of active filters. This popularity is a result of the well-known fact that it is possible to obtain low passive sensitiv-

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ities, low component spread, convenient tuning, multiple output characteristics (e.g., low-pass, bandpass, high-pass, and any desired combination thereof), and simple design procedures by employing ideal integrators as the active portion of the filter. The important class of filters that employ integrators (possibly lossy) include the "biquad" of Thomas [1], the "state variable" filter of Kerwin, Huelsman, and Newcomb [2], the "resonator" of Tow [3], the "leapfrog" of Girling and Good [4], the Akerberg-Mossberg configuration [5], Bruton's multiple amplifier configurations [6], [7], the "two integrator loop" of Girling and Good [8], etc.

The observed response of most active filters employing internally compensated operational amplifiers that are designed to operate at high frequencies or at a high  $Q$  changes significantly if the operational amplifiers used in the design are replaced with devices of the same type but with slightly different characteristics. These changes in the response of the active filter are a result of the dependence of the response on the characteristics of the operational amplifiers themselves, in particular, on the high-frequency rolloff of the operational amplifier gain. This dependence, in turn, is strongly dependent upon the particular circuit topology used to realize a particular filter response.

Filters employing integrators are particularly affected by the high-frequency rolloff of the operational amplifiers [9], [10] and consequently are limited to low-frequency applications. This has spurred considerable interest throughout the past ten years in the design of integrators that perform well at high frequencies [11]–[15]. Although improvements in integrator performance and consequently improvements in filter response have been made, the useful operating range of the integrator-based filters is still significantly lower than that of many other active  $RC$  filter designs [16]–[19].

Probably the main reason that the high-frequency performance of integrator type filters is so poor today is because of the fact that the criterion which has been almost exclusively used for the design of integrators is the *integrator  $Q$ -factor* (discussed later) introduced by Girling and Good [20] in 1969. Although the integrator  $Q$ -factor is useful, its success in predicting the performance of active filters is limited for two reasons:

- 1) the integrator  $Q$ -factor is based only upon the integrator phase (i.e., it contains no integrator magnitude information), and
- 2) the integrator  $Q$ -factor is a characteristic of the integrator rather than a characteristic of the filter employing the integrator.

Several integrators which offer reduced op amp dependence are discussed here that have circuit complexity comparable to the state of the art designs but which offer significant improvements in the high-frequency performance of filters employing these devices. Details of the designs of these integrators follow which are based directly

upon the filter performance rather than upon integrator performance.

## II. INTEGRATOR DESIGN

An integrator is characterized by a voltage transfer function of the form

$$I(j\omega) = Ie^{j\theta} \quad (1)$$

where ideally  $\theta = -\pi/2$  and  $I = \pm \omega_0/\omega$  for some constant  $\omega_0$ . For notational convenience assume an active filter employs only two integrator blocks with gains given by  $I_1(j\omega)$  and  $I_2(j\omega)$ . Extension to higher order structures is straightforward.

### A. Effects of Integrators on Filter Performance

To study the effects of the integrators on the filter performance, consider the transfer function of the filter which may be written functionally as

$$T(j\omega) = F(I_1, I_2, \theta_1, \theta_2) \quad (2)$$

where  $I_1, I_2, \theta_1,$  and  $\theta_2$  are, respectively, the magnitude and phase of the two integrators. It follows from a MacLaurin series expansion of  $F$  in the four variables  $I_1, I_2, \theta_1,$  and  $\theta_2$  that the relative change in  $T(j\omega)$  due to deviations of the integrators from ideal can be approximated by

$$\frac{\Delta T(j\omega)}{T(j\omega)} = (S_{I_1}^F) \frac{\Delta I_1}{I_1} + (S_{I_2}^F) \frac{\Delta I_2}{I_2} + (S_{\theta_1}^F) \frac{\Delta \theta_1}{\theta_1} + (S_{\theta_2}^F) \frac{\Delta \theta_2}{\theta_2} \quad (3)$$

where the standard sensitivity expressions are evaluated at the ideal values of the integrator magnitudes and phases.

A good integrator design should force the sum of the magnitudes (or magnitudes squared) of all four terms on the right hand side of (3) to be as small as possible. The four sensitivity expressions are dependent upon the *filter* topology and unaffected by the integrator gains. Good integrator designs thus should force the integrator dependent terms,  $(\Delta I_1)/I_1, (\Delta I_2)/I_2, (\Delta \theta_1)/\theta_1, (\Delta \theta_2)/\theta_2,$  to be small. The relative importance that should be associated with each of these ratios is dependent upon the magnitude of the respective sensitivity expressions that multiply these terms which are in turn dependent upon both  $F$  and the specific topology used to realize  $F$ . It is our contention that these sensitivities are often of comparable magnitude indicating it is important to consider both the integrator's magnitude and phase response when designing integrators. We will establish this quantitatively for one specific, popular second-order biquad. The examples that follow involving higher order cascaded and leapfrog structures further emphasize the importance of considering the integrator magnitude response along with the phase response.

Consider the state-variable bandpass structure of Fig. 1. The voltage transfer function for this structure is given by

$$T(j) = F(I_1, I_2, \theta_1, \theta_2) = \frac{-I_1 e^{j\theta_1}}{1 + K_1 I_1 e^{j\theta_1} + K_2 I_1 I_2 e^{j(\theta_1 + \theta_2)}} \quad (4)$$

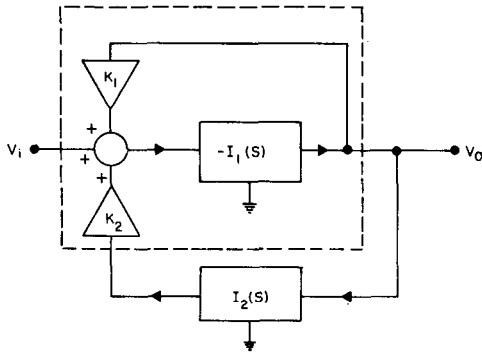


Fig. 1. Second-order integrator-based bandpass filter.

where ideally

$$\begin{aligned} K_1 &= 1/Q \\ K_2 &= 1 \\ I_1 &= I_2 = \frac{\omega_0}{\omega} \\ \theta_1 &= \theta_2 = -\pi/2 \end{aligned} \quad (5)$$

so that the ideal filter gain is given by

$$T_I(j\omega) = \frac{\omega \omega_0 j}{\omega^2 - \omega_0^2 - j\omega \frac{\omega_0}{Q}} = \frac{s \omega_0}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2} \Bigg|_{s=j\omega} \quad (6)$$

It follows from (3), (4), and (5) that for this filter

$$\begin{aligned} \frac{\Delta T(j\omega)}{T(j\omega)} &= \left( \frac{\omega^2}{\omega^2 - \omega_0^2 - j \frac{\omega \omega_0}{Q}} \right) \\ &\cdot \left( \frac{\Delta I_1}{I_1} + \frac{\Delta I_2}{I_2} \frac{\omega_0^2}{\omega^2} + \frac{\Delta \theta_1}{\theta_1} \pi/2 + \frac{\Delta \theta_2}{\theta_2} \pi/2 \right). \end{aligned} \quad (7)$$

For this structure it follows that the magnitude and phase effects of the integrator are of comparable importance and that little is to be gained by increased improvement in either the magnitude or phase response without simultaneous improvement of the other.

One interesting observation is to be made from (7). In addition to minimizing the integrator magnitude and phase deviations, improvements in this filter can be obtained if the signs of  $\Delta \theta_1$  and  $\Delta I_1$  are opposite to those of  $\Delta \theta_2$  and  $\Delta I_2$ .

#### A. Limitations of Integrator $Q$ -Factor

The inadequacy of the integrator  $Q$ -factor itself as a figure of merit for evaluating integrator performance can be demonstrated at this point. Since the integrator gain given in (1) is a complex quantity it may be alternately expressed as

$$I(j\omega) = \frac{1}{R(\omega) + jX(\omega)} \quad (8)$$

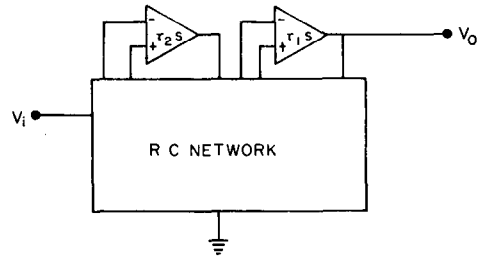


Fig. 2. General two op amp integrator.

where ideally  $R(\omega = 0)$  and  $X(\omega) = \omega/\omega_0$ . The integrator  $Q$ -factor is defined [20] to be

$$Q_I = \frac{X(\omega)}{R(\omega)}. \quad (9)$$

It now follows from (1) that

$$Q_I = \tan(-\theta). \quad (10)$$

Since the integrator  $Q$ -factor is dependent only upon  $\theta$ , it follows from (7) that it is of limited use in predicting integrator performance. The state of the art integrators [12]–[15] seek to maximize this factor alone.

#### B. Minimization of $\Delta I/I$ and $\Delta \theta/\theta$

The effects of the parameters of the operational amplifiers on the performance of the integrators will now be investigated. Assume the op amps are ideal except for the frequency dependent gain

$$A(s) = \frac{1}{\tau s} \quad (11)$$

where the op amp time constant  $\tau$  is the reciprocal of the gain-bandwidth product and is ideally 0.

It follows from the truncation after first-order terms of a MacLaurin expansion of the integrator magnitude and phase expressions that for an integrator employing two operational amplifiers

$$\begin{aligned} \Delta I &\approx \frac{\partial I}{\partial \tau_1} \tau_1 + \frac{\partial I}{\partial \tau_2} \tau_2 \\ \Delta \theta &\approx \frac{\partial \theta}{\partial \tau_1} \tau_1 + \frac{\partial \theta}{\partial \tau_2} \tau_2 \end{aligned} \quad (12)$$

where the partial derivatives are evaluated at the ideal values of the op amp time constants,  $\tau_1 = \tau_2 = 0$ . It thus suffices to minimize the four partial derivatives in (12). Emphasis here will be placed upon actually forcing these terms to vanish.

A general integrator employing two op amps is shown in Fig. 2. It can be readily shown with a standard sensitivity analysis that all four partials in (12) will vanish independent of operational amplifier matching provided  $I(s)$  is expressible in the form

$$I(s) = \frac{I_0(1 + \theta_1 \tau_1 s)}{s(1 + \theta_1 \tau_1 s) + \theta_{12} \tau_1 \tau_2 s^2 D_{RC}} \quad (13)$$

where  $\theta_1$  and  $\theta_{12}$  are constants and  $D_{RC}$  is a polynomial in  $s$

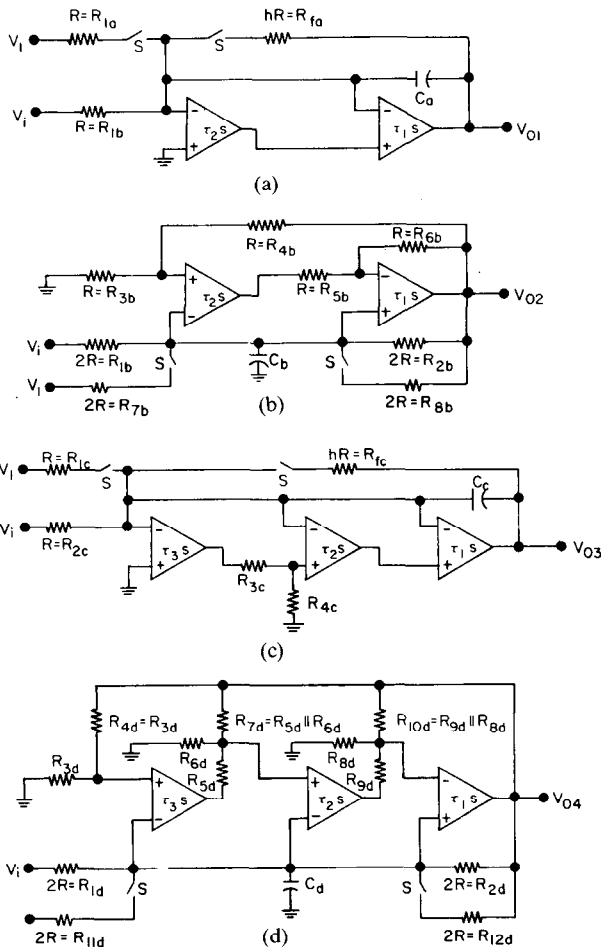


Fig. 3. High-frequency inverting and noninverting integrators. (a) Two op amp inverting. (b) Two op amp noninverting. (c) Three op amp inverting. (d) Three op amp noninverting.

dependent only upon passive parameters.

When extended to integrators employing three op amps, it can be shown that both the first and second derivatives in the corresponding MacLaurin expansion simultaneously vanish provided \$I(s)\$ is expressible in the form

$$I(s) = \frac{I_0(1 + \theta_1\tau_1s + \theta_2\tau_2s + \theta_3\tau_3s + \theta_{12}\tau_1\tau_2s^2 + \theta_{13}\tau_1\tau_3s^2 + \theta_{23}\tau_2\tau_3s^2)}{[s(1 + \theta_1\tau_1s + \theta_2\tau_2s + \theta_3\tau_3s + \theta_{12}\tau_1\tau_2s^2 + \theta_{13}\tau_1\tau_3s^2 + \theta_{23}\tau_2\tau_3s^2) + \theta_{123}\tau_1\tau_2\tau_3s^3D_{RC}]} \quad (14)$$

where the \$\theta\$'s are constants and \$D\_{RC}\$ is again a polynomial in \$s\$ dependent only upon passive components.

C. New Integrator Structures

Four integrators,<sup>1</sup> two inverting and two noninverting, are shown in Fig. 3. With all switches open the gain of

<sup>1</sup>The integrators of Fig. 3(a) and Fig. 3(b) were presented at the IEEE International Symposium on Circuits and Systems, (Houston, TX), April 1980 and have been subsequently examined by S. Natarajan [21].

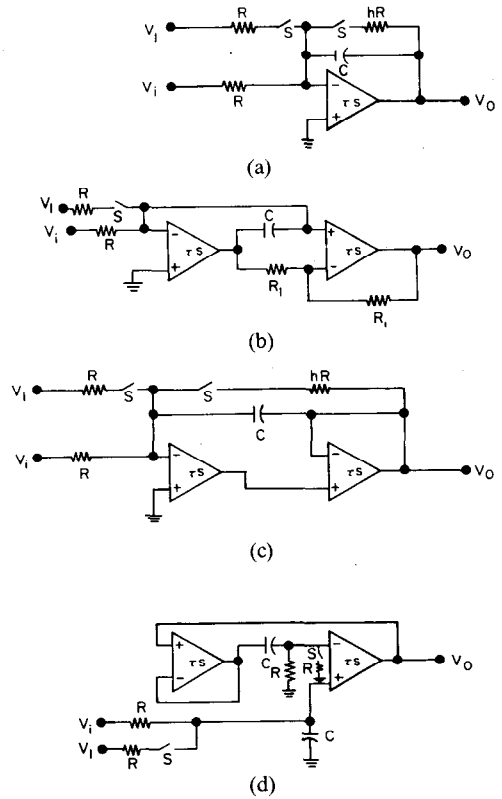


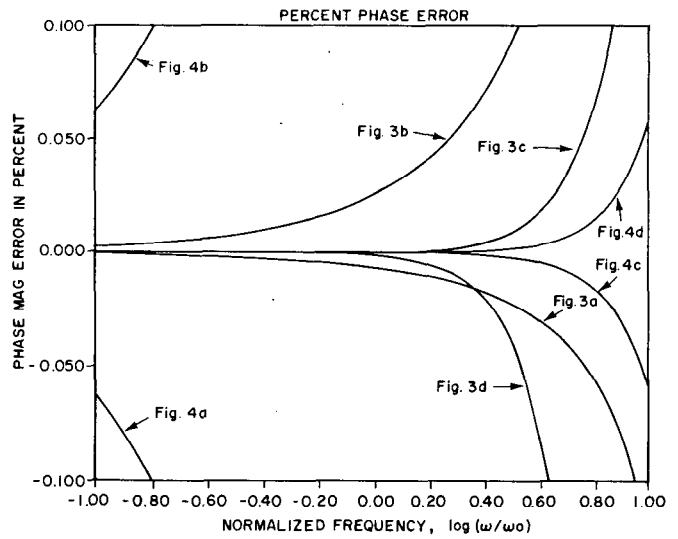
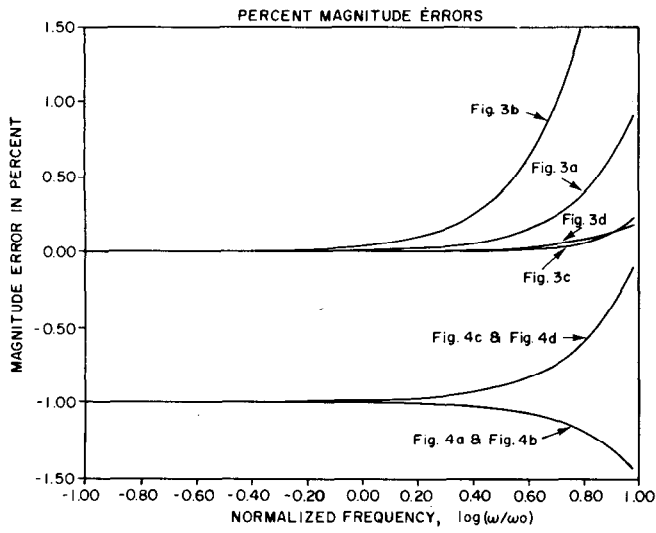
Fig. 4. Existing integrator structures, \$\omega\_0 = 1/RC\$. (a) Popular inverting. (b) Modified Miller inverting. (c) High-Q inverting. (d) Compensated balanced time constant noninverting [15].

these integrators are given, respectively, by

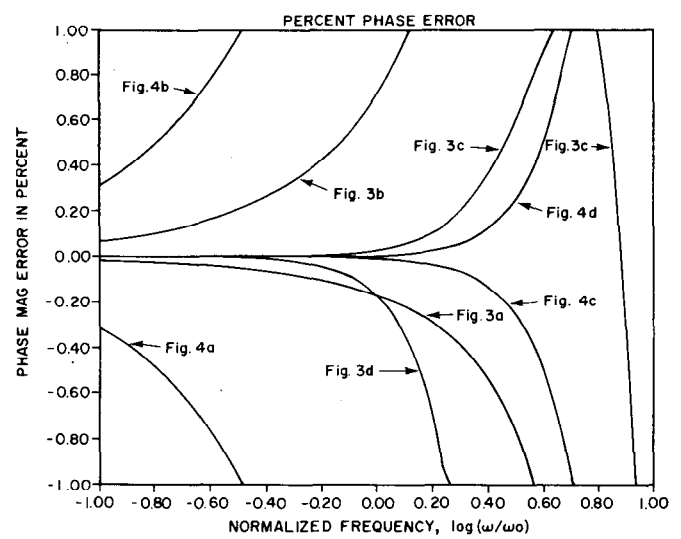
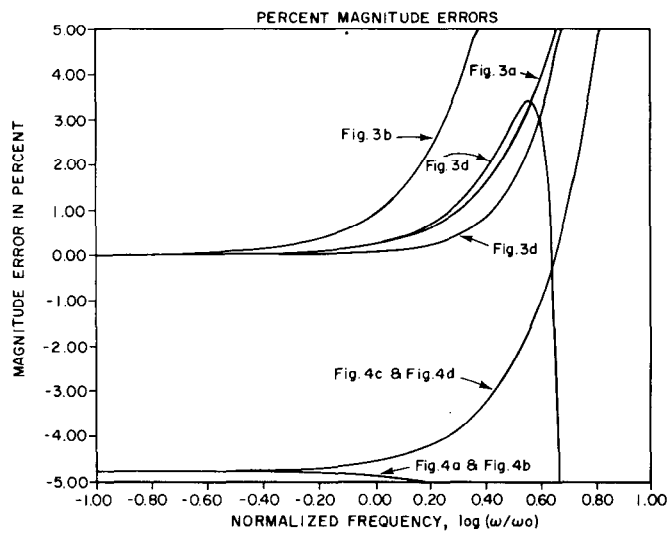
$$\begin{aligned} \frac{V_{O1}}{V_i} &= \frac{-(1 + \tau_{2n}s_n)}{s_n(1 + \tau_{2n}s_n) + \tau_{1n}\tau_{2n}s_n^2(1 + s_n)} \\ \frac{V_{O2}}{V_i} &= \frac{1 + 2\tau_{2n}s_n}{s_n(1 + 2\tau_{2n}s_n) + 4\tau_{1n}\tau_{2n}s_n^2(1 + s_n)} \\ \frac{V_{O3}}{V_i} &= \frac{-(\nu + \tau_{3n}s_n + \tau_{2n}\tau_{3n}s_n^2)}{s_n(\nu + \tau_{3n}s_n + \tau_{2n}\tau_{3n}s_n^2) + \tau_{1n}\tau_{2n}\tau_{3n}s_n^2(1 + s_n)} \\ \frac{V_{O4}}{V_i} &= \frac{\nu_1\nu_2 + \nu_1\tau_{3n}s_n + \tau_{2n}\tau_{3n}s_n^2}{s_n(\nu_1\nu_2 + \nu_1\tau_{3n}s_n + \tau_{2n}\tau_{3n}s_n^2) + 2\tau_{1n}\tau_{2n}\tau_{3n}s_n^3(1 + s_n)} \end{aligned} \quad (15)$$

where \$\omega\_0 = 1/RC\$, \$s\_n = s/\omega\_0\$, and \$\nu\$, \$\nu\_1\$, and \$\nu\_2\$ satisfy the equations

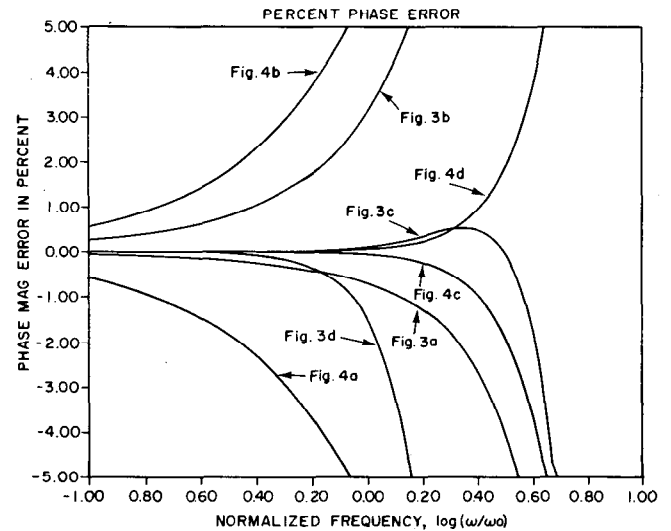
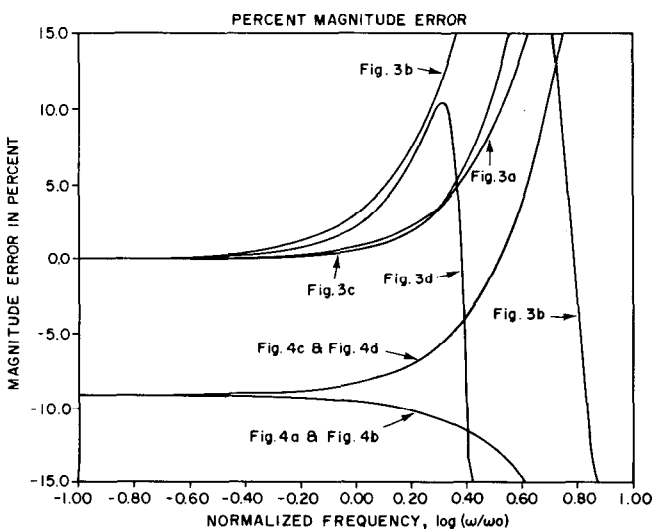
$$\begin{aligned} \nu &= \frac{R_{4c}}{R_{3c} + R_{4c}} \\ \nu_1 &= \frac{\frac{R_{8d}}{2} \parallel R_{9d}}{\frac{R_{8d}}{2} \parallel R_{9d} + R_{9d}} \\ \nu_2 &= \frac{\frac{R_{6d}}{2} \parallel R_5}{\frac{R_{6d}}{2} \parallel R_5 + R_{5d} + R_{5d}} \end{aligned} \quad (16)$$



(a)



(b)



(c)

Fig. 5. Comparison of integrator performance. (a)  $\tau_n = 0.01$ . (b)  $\tau_n = 0.05$ . (c)  $\tau_n = 0.10$ .

The parameters  $\nu$ ,  $\nu_1$ , and  $\nu_2$  are chosen to force the parasitic poles introduced by the op amps to be in the left half-plane. Methods of determining those parameters is discussed elsewhere [22]. Note that the gain of the first two is of the functional form of (13) necessary to eliminate first-order effects whereas the second two are of the functional form of (14) necessary to eliminate both first-order and second-order terms in the MacLaurin expansions for the integrator magnitude and phase. It follows from (3) that the corresponding operational amplifier effects vanish in any active filter employing these integrators. A comparison of the magnitude and phase of these integrators (for  $\nu = 0.2$  in Fig. 3(c) and  $\nu_1 = 0.5$ ,  $\nu_2 = 0.2$  in Fig. 3(d) and all switches open in Fig. 3 and Fig. 4) with that of the popular configurations of Fig. 4 is made in Fig. 5 for normalized time constants  $\tau_n = \omega_0/GB$ , of 0.01, 0.05, and 0.10. Note that in these comparisons the configurations of Fig. 3 exhibit significantly superior performance in the magnitude response in the region of interest around  $\omega/\omega_0 = 1$  whereas the phase response is comparable to that attainable with the high- $Q$  configurations of Fig. 4(c) and Fig. 4(d). If all switches in Fig. 3 are closed, the derivatives discussed earlier still vanish. These switches allow for additional excitations and/or loss to be introduced in these integrators while still retaining the desirable performance characteristics. Most practical filter applications require these capabilities to avoid the need for additional summing amplifiers.

### III. INTEGRATOR-BASED FILTER PERFORMANCE

The performance of three filter structures employing the new integrator is compared with the same structures employing the integrators of Fig. 4 in this section.

#### A. Second-Order Biquad

If the filter structure of Fig. 1 is modified by replacing the dashed-portion by a single summing inverting integrator and using a noninverting structure for  $I_2$  it is a well known fact that a reduction in components is possible. With the switches closed for the inverting configurations of Fig. 3 and Fig. 4 one obtains the required summing action to obtain a second-order bandpass response where for convenience all capacitors are equal valued and  $h$  is chosen to equal the desired pole  $Q$ .

Fig. 6 shows a comparison of the frequency response of the second-order bandpass filter of Fig. 1 for four different integrator pairs with values of  $\tau_n = \omega_0/GB = 0.01, 0.05,$  and  $0.10$  where  $\omega_0$  is the ideal center frequency. The op amps have been assumed matched for convenience. The pairing of the integrators is listed in Table I. The first pair uses the popular inverting and noninverting integrators. The second was chosen because the integrators have been reported to have high integrator  $Q$ -factors. The third involves the two integrators of Fig. 3 which force first-order op amp effects to vanish. The final pair uses the integrators

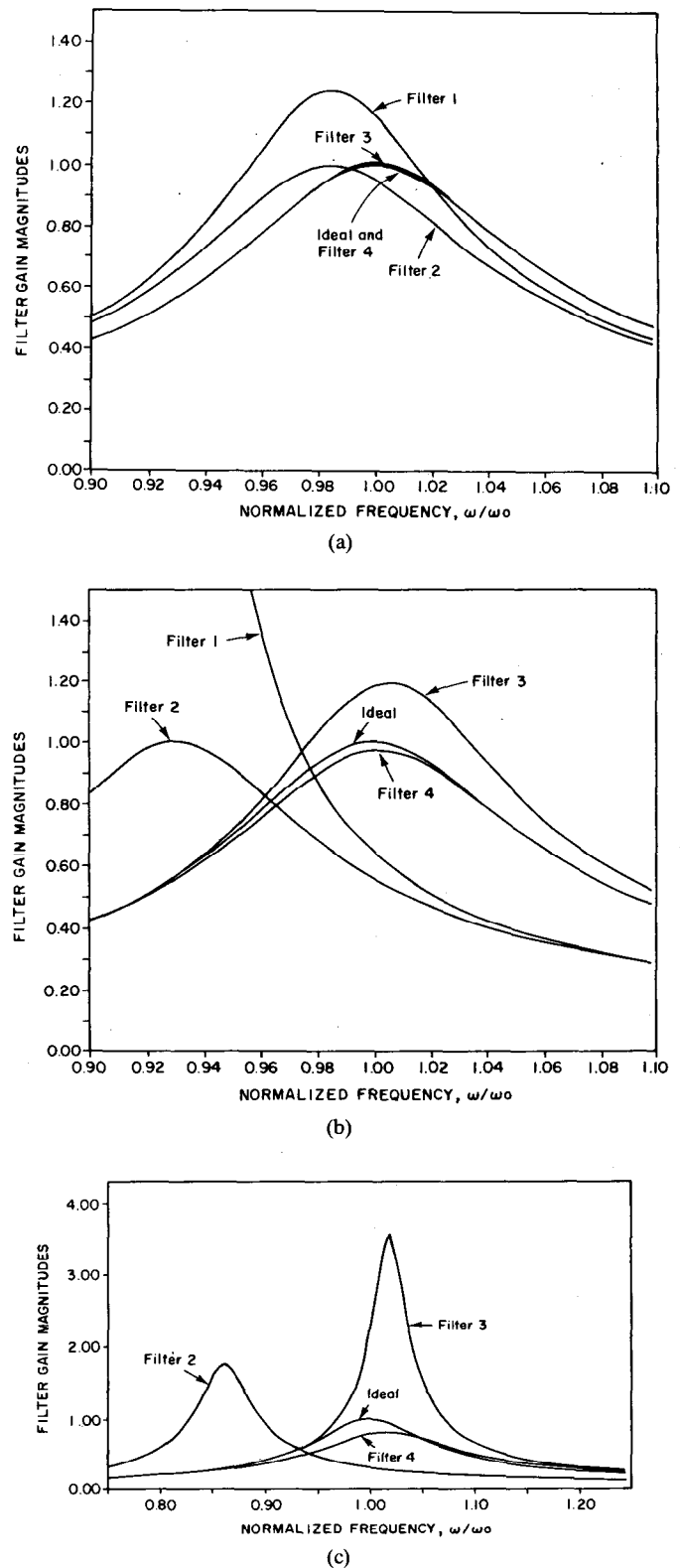


Fig. 6. Comparative frequency response of bandpass filters. (a)  $\tau_n = 0.01$ . (b)  $\tau_n = 0.05$ . (c)  $\tau_n = 0.10$ .

of Fig. 3 which force both first-order and second-order effects to vanish. The results need no interpretation.

The improvements in filter performance are comparable

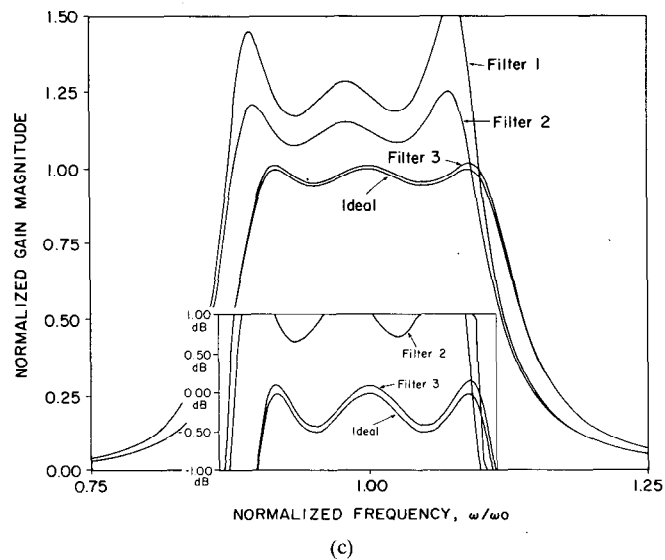
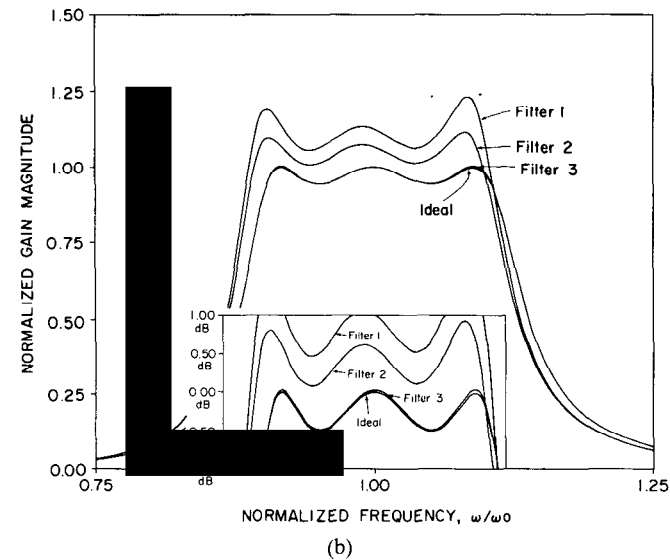
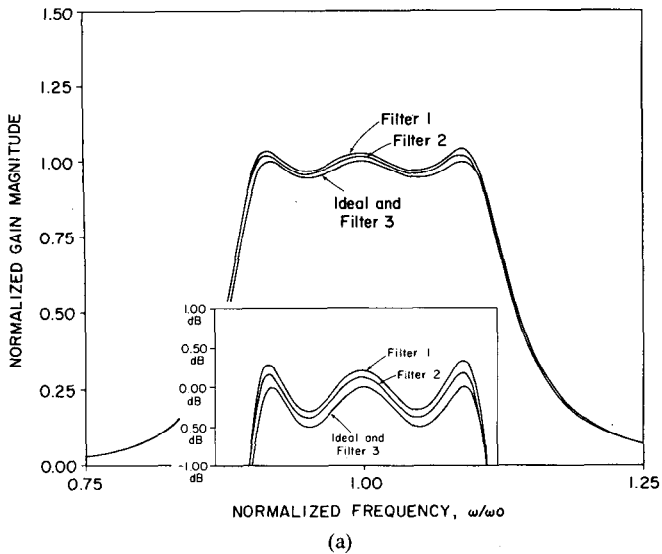


Fig. 7. Comparative frequency response of a sixth-order cascaded-biquad bandpass filter. (a)  $\tau_n = 0.001$ . (b)  $\tau_n = 0.025$ . (c)  $\tau_n = 0.05$ .

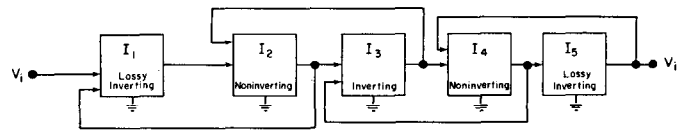


Fig. 8. Leapfrog structure for fifth-order low-pass filter.

TABLE I  
INTEGRATOR SELECTION FOR FILTER ANALYSIS

	$I_1$	$I_2$
Filter 1	Fig. 4(a)	Fig. 4(b)
Filter 2	Fig. 4(c)	Fig. 4(d)
Filter 3	Fig. 3(a)	Fig. 3(b)
Filter 4	Fig. 3(c), $v = .2$	Fig. 3(d), $v_1 = .5, v_2 = .2$

to those previously reported by Geiger and Budak [16], [17] with the direct zero active sensitivity designs. This is not surprising since it can be shown that any filter employing the new integrators is actually in the large class of zero active sensitivity active filters.

**B. Sixth-Order Bandpass Filter Using Cascaded Biquads**

A more demanding application involving cascaded biquads is considered here. To obtain quantitative comparisons, a sixth-order bandpass Chebyshev filter realized by cascading three of the biquads previously discussed that has 0.5-dB passband ripple and a 0.5-dB bandwidth of 20 percent of the center frequency was designed. Using the same pairing of integrators as listed in Table I, the filter response shown in Fig. 7 was obtained where the frequency axis has been normalized by the center frequency  $\omega_0$ . The advantages offered by the integrators of Fig. 3 are quite obvious. It should be noted that even in the low frequency case,  $\tau_n = 0.001$  (corresponding to a center frequency of 1 kHz with 1-MHz op amps), there is more than 0.2-dB passband error for both filter 1 and filter 2. Filter 4 is not included in this comparison since the deviation from ideal in these comparisons is negligible as can be concluded from the previous example.

**C. Fifth-Order Low-Pass Leapfrog Filter**

A fifth-order leapfrog low-pass structure employing five integrators is shown in Fig. 8 where  $I_1$  and  $I_5$  are lossy inverting summing integrators,  $I_3$  is a lossless summing inverting integrator, and  $I_2$  and  $I_4$  are noninverting summing integrators. All inputs for all integrators are equally weighted. Additional details about leapfrog designs are readily available [23].

For comparison purposes a low-pass Chebyshev filter with 0.5-dB passband filter and a 0.5-dB cutoff frequency

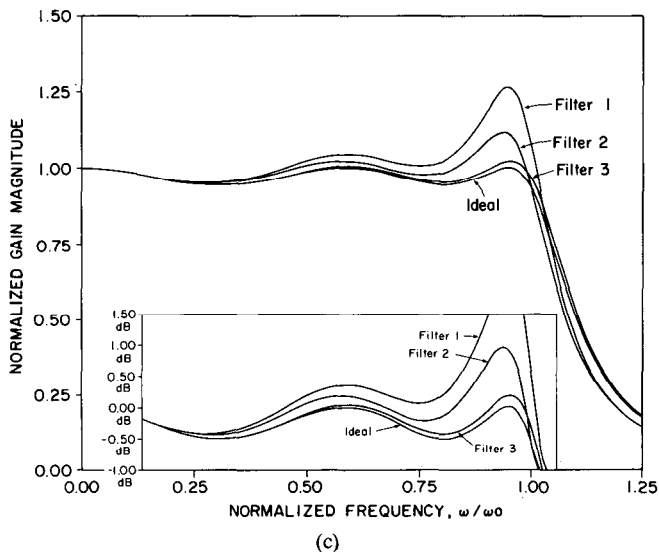
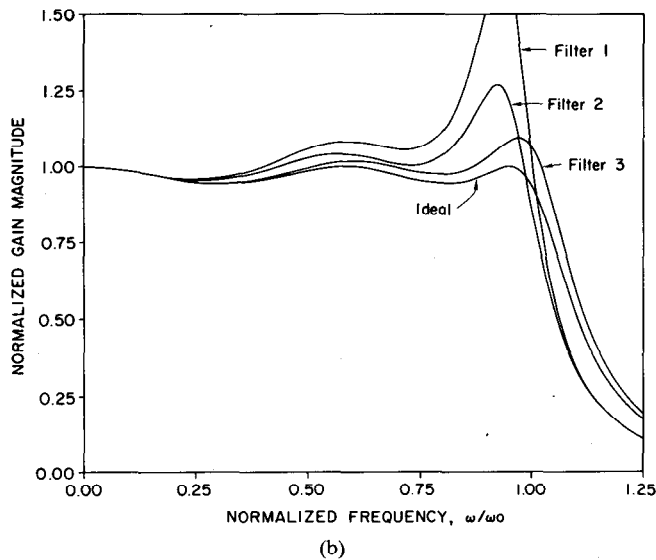
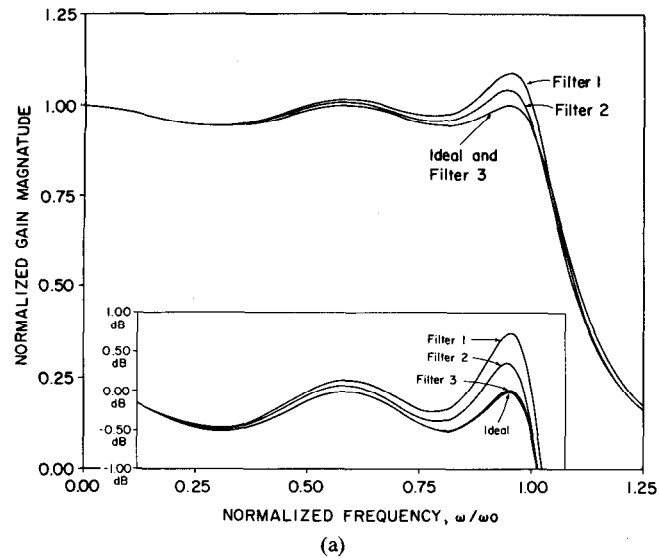


Fig. 9. Comparative frequency response of a fifth-order leapfrog structure. (a)  $\tau_n = 0.01$ . (b)  $\tau_n = 0.025$ . (c)  $\tau_n = 0.05$ .

TABLE II  
RESULTS OF EXPERIMENTAL EVALUATION

Experimental Results						
	$f_{o,exp}$	$f_{o,th}$	% error in $f_o$	$Q_{exp}$	$Q_{th}$	% error in $Q$
Filter 3	26.336KHz	26.277KHz	+0.22%	11.098	10.29	+7.8%
Filter 4	26.292KHz	26.277KHz	+0.01%	9.88	10.29	-4.0%
Component Values						
$R_{1a}=597.8\Omega$	$R_{1b}=1.195K$	$R_{1c}=597.8\Omega$	$R_{1d}=1.195K$	$R_{8d}=\infty$		
$R_{1b}=597.1\Omega$	$R_{2b}=1.195K$	$R_{2c}=597.1\Omega$	$R_{2d}=1.195K$	$R_{9d}=5.012K$		
$R_{fa}=6.152K$	$R_{3b}=5.015K$	$R_{3c}=3.858K$	$R_{3d}=5.047K$	$R_{10d}=5.014K$		
$C_a=10.132nf$	$R_{4b}=5.011K$	$R_{4c}=962.4\Omega$	$R_{4d}=5.047K$	$C_d=10.132nf$		
	$R_{5b}=5.018K$	$R_{fc}=6.152K$	$R_{5d}=5.029K$			
	$R_{6b}=5.016K$	$C_c=10.132nf$	$R_{6d}=3.250K$			
	$C_b=10.132nf$		$R_{7d}=1.975K$			

of  $\omega_0$  was designed. If all operational amplifiers are ideal, this requires that

$$\begin{aligned}
 I_1 &= -\frac{1}{1.7058} \frac{\omega_0}{s + \frac{1}{1.7058} \omega_0} \\
 I_2 &= \frac{1}{1.2296} \frac{\omega_0}{s} \\
 I_3 &= -\frac{1}{2.5408} \frac{\omega_0}{s} \\
 I_4 &= \frac{1}{1.2296} \frac{\omega_0}{s} \\
 I_5 &= -\frac{1}{1.7058} \frac{\omega_0}{s + \frac{1}{1.7058} \omega_0}
 \end{aligned} \quad (17)$$

In Fig. 9 the ideal response of this filter is again compared with that of using the integrators of Fig. 3 and Fig. 4. The same pairings as denoted in Table I for  $\tau_n = 0.01$ , 0.025, and 0.05 were used. Again, the performance improvements are significant.

#### D. Experimental Results

The second-order bandpass circuits discussed previously using filters 3 and 4 of Table I designed for  $Q = 10$  and



$f_0 = 25$  kHz were constructed using 741 type op amps with gain bandwidth products that were measured to be between 710K rad/s and 840K rad/s. The ideal op amp theoretical and experimental center frequency and 3-dB bandwidth are compared in Table II. Even closer agreement follows if these experimental results are compared with those predicted in Fig. 6 when extrapolated to the  $\tau_n = 0.03$  curves. Component values which were measured to within 0.1 percent are also listed.

These filters were observed to have an unstable mode of operation instigated by power-supply start up similar to that often reported in the literature. Since the nature of this instability, which may not even exist with some other types of op amps, is strongly dependent upon the particular op amp used and since it can be readily and economically detected and removed, details will not be presented here.

#### IV. CONCLUSIONS

It has been shown that the integrator  $Q$ -factor alone is of limited use for predicting integrator performance. Four integrators which were designed to directly improve filter performance have been discussed. The magnitude and phase of the integrators are simultaneously optimized. Two of these are comparable in circuit complexity to the high- $Q$  configurations but offer advantages in filter performance. The other two offer even further performance improvements at the expense of increased circuit complexity.

The new integrators have been compared in a second-order bandpass filter structure, in a cascaded biquad circuit, and in a leapfrog design with the popular and high- $Q$  configurations. The improvements in performance are significant as verified by both a theoretical and experimental evaluation.

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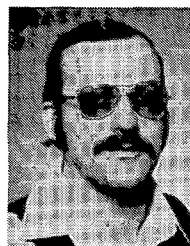


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