Biquadratic SC Filters with Small GB Effects

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***Abstract*—The effects of proper switch phasing in SC filters to yield low gain–bandwidth (GB) product dependence is discussed. A figure of merit for comparing GB effects in SC networks based upon the topology dependent effective GB matrices has been introduced. Motivated by this consideration, two general biquadratic switched-capacitor filter structures are presented. These circuits are stray-insensitive, have small ω_0 and Q sensitivities, and are easy to apply. A significant feature of these biquads is their reduced dependence on the operational amplifier GB product. Filter center frequencies up to the 50-kHz range are feasible for op amps with typical values of $GB/2\pi = 10^6$ Hz, making these circuits attractive for high-frequency applications.**

The advantages of the proposed structures are demonstrated by an example. Results enabling the designer to determine the ω_0 and Q deviations for different values of GB are presented. A comparison of theoretical and experimental results shows good agreement.

I. Introduction

SWITCHED-CAPACITOR (SC) filters constructed with MOS capacitors, MOS switches and MOS operational amplifiers (op amps) have been recognized as a practical method for realizing precision monolithic filters.

A number of SC filters have been reported which realize a general or semigeneral biquadratic transfer function [1]–[6]. These topologies were presented without taking into consideration the operational amplifier (op amp) gain–bandwidth (GB) product effects on the performance of the filters. Several authors have recently presented systematic analysis procedures for determining the influence of GB on the performance of SC active filters [7]–[10]. In many SC filters several different phasing schemes can ideally be used without affecting the overall transfer function. However, in practice the different switch phasing often leads to significant differences in filter performance. This is more acute for small ratios of GB/f_s, where f_s is the sampling frequency. This fact was first reported in [8] and confirmed in [11].

In this paper, we attempt to generalize and formalize the principles for designing SC filters with low GB dependence. Two biquadratic SC filter topologies are proposed. It is demonstrated that the GB effects are small in these structures. The structures employ two op amps and at most 10 capacitors. They have favorable capacitor values, are immune to stray capacitances, and have low passive sensitivities.

Finally, experimental results are presented and compared with the theoretically predicted performance. These show close agreement.

II. Effect of Switch Clock Arrangements on GB Product Dependence in SC Filters

Insight into the low GB effects in SC structures can be attained by reviewing the analysis of SC filters in which the op amp is modeled with a single dominant pole. A brief review of the analysis procedure is given in the Appendix; details can be found in the literature [7]–[10]. From the Appendix (equation (6)) we see that the response of a single op amp SC circuit due to step inputs applied at time $t = t_1$ is an exponential ramp of the type

$$v_o(t) = v_o(t_1)e^{-(t-t_1)\widehat{GB}} + v_{od}\left[1 - e^{-(t-t_1)\widehat{GB}}\right] \quad (1)$$

where $\widehat{GB}$ is *the effective gain–bandwidth product* and equal to aGB, a is a topology-dependent voltage divider and v_{od} is the desired output which is dependent upon the topology, the inputs, and the initial charges on the capacitors. For the common case where the noninverting terminal of the op amp is grounded and the circuit contains no nodes other than those connected directly to an input or a terminal of the op amp it can be shown that the value of a for each clock phase is obtained from the expression

$$a = \frac{\sum C_f}{\sum C_i} \quad (2)$$

where the C_f sum is over all feedback capacitors (connected directly between the op amp output and the inverting input terminal) and the C_i sum is over all capacitors connected to the negative terminal of the op amp. Note that a is bounded between 0 and 1.

It can be readily seen from (1) that for single op amp SC filters, the step response is another step if the op amp is ideal (i.e., $Z_i = \infty$, $Z_o = 0$, and $GB = \infty$). It can be further argued from (1) that for single op amp circuits the aGB product determines the rise time of the response and thus that both a and GB should be maximized. GB is fixed once the op amp has been specified. The parameter, a, which is determined by the circuit topology can be controlled by the circuit designer.

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In the case that the circuit contains a single capacitor and that capacitor is connected between the output and inverting input terminal to the op amp, the parameter a assumes its upper bound of unity and $\widehat{\mathrm{GB}} = \mathrm{GB}$.

The situation is somewhat more involved in the multiple op amp case. As in the single op amp case, it can again be shown that the step response (at each op amp output) of multiple op amp SC filters is also a step provided all op amps are ideal. It can thus be argued that one should strive to force all op amp outputs to be as similar to a step function as possible to obtain a reduction in GB dependence. It has been shown in [10] that the response to any phase-constant excitation of an m-op amp SC filter during any phase defined by $t_1 \leqslant t < t_2$ is given by

$$v(t) = e^{-GA(t-t_1)}M_1 + M_2 \tag{3}$$

$$e^{\mathrm{GB}_{11}\Gamma A(t-t_1)} = \begin{bmatrix} e^{\mathrm{GB}_{11}a_{11}(t-t_1)} & 0 \\ \left(\dfrac{\gamma_{22}a_{21}}{a_{11}-\gamma_{22}a_{22}}\right)\left(e^{\mathrm{GB}_{11}a_{11}(t-t_1)} - e^{\mathrm{GB}_{11}\gamma_{22}a_{22}(t-t_1)}\right) & e^{\mathrm{GB}_{11}\gamma_{22}a_{22}(t-t_1)} \end{bmatrix} \tag{6}$$

where $v(t)$ is the m-dimensional vector of op amp output voltages, G is the m-dimensional diagonal matrix with $g_{ii} = \mathrm{GB}_i$, $i = 1,\cdots,m$, A is a time independent $m \times m$ matrix determined by the circuit topology and the $m \times 1$ vectors, M_1 and M_2, are topology dependent but independent of t and GB.

From (3) it can be concluded that all GB dependence during any phase is determined by A and M_1 since it is assumed that the GB of the op amps is fixed. One way to minimize GB dependence is to make the effective GB matrices "large" and real during *each* phase (an m-phase SC network will have m effective GB matrices which are typically all distinct). In the single op amp case, the effective GB matrix becomes the scalar a defined in (2). As in the single op amp case, it can be shown that all entries of the effective GB matrix can be obtained by inspection of the circuit following a simple algorithm similar to that used to obtain (2).

The duration of all time-domain transients is determined by the real part of the eigenvalues of the effective GB matrix. To reduce GB dependence, one would thus like to have the minimum of the real parts of all eigenvalues for all phases as large as possible. *The minimum of the real part of all eigenvalues can thus serve as a figure of merit for comparing SC filters with respect to GB* for the case of widely separated distinct eigenvalues. Since all GB-related time constants of the filter are determined by the topology dependent A matrices, we will term these matrices of dimensionless entries the *effective GB matrices*.

In the case that the eigenvalues are not distinct, are not widely separated, or are comparable in magnitude for two circuits under comparison one must consider the off-diagonal elements of the effective GB matrix as well. A more detailed investigation of GB effects in the parametrically tractable two op amp triangular effective GB matrix case follows. To gain additional insight into the effects of GB matching, the matrix G in (3) will be expressed in terms of the gain bandwidth product of the first op amp, GB_{11}, as

$$G = \mathrm{GB}_{11}\Gamma \tag{4}$$

where Γ is a diagonal matrix referred to here as the GB matching matrix which in the two op amp case is expressed as

$$\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & \gamma_{22} \end{bmatrix}. \tag{5}$$

It thus follows from (3) that *all* GB effects are determined by the exponentiation of the matrix $\mathrm{GB}_{11}\Gamma A(t-t_1)$ which becomes in the two op amp case for unequal $(a_{11} \neq \gamma_{22}a_{22})$ and equal $(a_{11} = \gamma_{22}a_{22})$ eigenvalues, respectively,

and

$$e^{\mathrm{GB}_{11}\Gamma A(t-t_1)} = \begin{bmatrix} e^{\mathrm{GB}_{11}a_{11}(t-t_1)} & 0 \\ \gamma_{22}a_{21}\mathrm{GB}_{11}(t-t_1)e^{\mathrm{GB}_{11}a_{11}(t-t_1)} & e^{\mathrm{GB}_{11}a_{11}(t-t_1)} \end{bmatrix}. \tag{7}$$

From these matrices the effects of the off-diagonal element a_{21} on the time-domain response can be seen; thus in addition to large diagonal elements in the effective GB matrix, one should also strive to have small off diagonal elements.

It is particularly important to investigate the effects of the off-diagonal elements in the case of equal or near equal eigenvalues. Since it is desirable to have all transients reduced to negligible levels at the end of a clock phase, the relative importance of the off-diagonal element, a_{21}, can be obtained by comparing the coefficient $\gamma_{22}a_{21}\mathrm{GB}_{11}(t-t_1)$ in (7) to unity at the end of the clock phase. In the popular symmetric two-phase clock case, the term $t - t_1$ will be equal to $T/2$ at the end of any clock phase. The ratio of this coefficient to unity is thus given by

$$\rho = \frac{\gamma_{22}a_{21}}{2}\mathrm{GB}_{11}T. \tag{8}$$

From this expression it can be seen that unless a_{21} is very small, the effects of GB can be significantly more severe in SC structures which have matched or nearly matched eigenvalues.

Two integrator loops have been used extensively in the design of biquadratic SC filters. Since it has been argued that one should strive to obtain op amp outputs that are as similar to a step function as possible to reduce GB effects, it can be further argued that all inputs to each integrator in the loop should be as similar to a step function as possible since any delayed response in an input will cause a corresponding delayed response in the output. Actual step inputs to each integrator during each phase are possible by combining two forward integrators, one of the noninverting and the other of the inverting type [1], however, this combination typically leads to high Q sensitivity structures. Therefore, a loop with an inverting backward integrator and a noninverting forward integrator is commonly preferred [3], [5], [6], [12]. In the next section, two possible combinations of this loop are presented.

III. Minimum GB Effect SC Topologies

The design of SC filters with reduced GB effects will be addressed in this section. In order to illustrate the design approach, we consider the basic cells shown in Fig. 1. A noninverter integrator is shown in Fig. 1(a). Assuming the input is constant during each clock phase (see Appendix), the transfer function of the circuit is

$$H_n^{ee}(z)=\frac{V_o^e(z)}{V_i^e(z)}=\frac{\alpha_n z^{-1}}{1-z^{-1}}=\alpha_n\hat{z}^{-1} \tag{9}$$

where

$$\hat{z}^{-1}=z^{-1}/(1-z^{-1}).$$

An inverting backward integrator is shown in Fig. 1(b). Its transfer function is given by

$$H_i^{ee}(z)=\frac{V_o^e(z)}{V_i^e(z)}=\frac{-\alpha_i}{1-z^{-1}}$$

$$=-\alpha_i\frac{1-z^{-1}+z^{-1}}{1-z^{-1}}=-\alpha_i(1+\hat{z}^{-1}). \tag{10}$$

If the input to the integrators changes at an intermediate point during one clock period, (9) and (10) along with the flow graph notation of Figs. 1 and 2, remain valid provided inputs and outputs of the integrators are sampled during the same clock phase. If input and output are sampled during different phases (ϕ_1 and ϕ_2) then an additional 1/2 period delay occurs. This will introduce $z^{1/2}$ terms in (9) and (10). A capacitive amplifier is shown in Fig. 1(c). The input–output relationship is simply

$$V_o^e=-\alpha_s V_1^e. \tag{11}$$

The signal flow graph for the topology of the first biquad is shown in Fig. 2(a). Assuming the input and output are sampled during the same clock phase, (e.g., the even clock phase) it follows from Mason's rule and the definition of $\hat{z}^{-1}$ that the transfer function of *Biquad 1* is

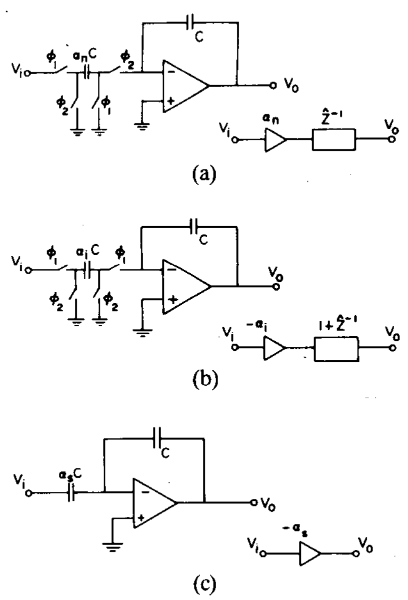

Fig. 1. Basic cells. (a) Noninverting integrator. (b) Inverting integrator. (c) Inverting amplifier.

K, B_1, B_2, r, and θ can be obtained. It can be readily shown that the α's in (12) can be chosen to realize any biquadratic transfer function [5].

A SC implementation of Biquad 1 is shown in Fig. 2(b). Note that this structure is insensitive to parasitic capacitance associated with all capacitors.

A simplified investigation of the sensitivities of the resonant frequency ω_0 and the selectivity Q of the pole pair defined by (12) follows. ω_0 and Q are defined [1] from the impulse invariant transformation of the general second-order s-domain polynomial $s^2+s\omega_0/Q+\omega_0^2$. For the simplified analysis the following approximations [5] which relate ω_0 and Q to θ and r are made. These approximations are good for high sampling frequency f_s and high Q conditions

$$\theta\cong 2\pi\frac{f_0}{f_s}=\omega_0 T_s \tag{14}$$

$$r=1-\frac{\theta}{2Q}. \tag{15}$$

By truncating the Maclaurin series expansion for $\cos\theta$

$$H^{ee}(z)=\frac{V_{o_2}^e(z)}{V_i^e(z)}=-\frac{\alpha_5'}{1+\alpha_8}\frac{z^2-z[\alpha_5+\alpha_5'-\alpha_2(\alpha_1+\alpha_4)]/\alpha_5'+[\alpha_5-\alpha_2\alpha_4]/\alpha_5'}{z^2-z[2+\alpha_8+\alpha_9-\alpha_2\alpha_7]/(1+\alpha_8)+(1+\alpha_9)/(1+\alpha_8)}. \tag{12}$$

$H(z)$ of (12) has the general form

$$H^{ee}(z)=\frac{V_{o_2}^e(z)}{V_i^e(z)}=-K\frac{z^2+B_1z+B_2}{z^2-2r\cos\theta z+r^2} \tag{13}$$

where the locations of any pair of complex-conjugate poles are at $re^{\pm j\theta}$ and B_1 and B_2 determine the type [5] of filter. From (12) and (13) the relationship between the α's and

after the second-order terms, we obtain for small θ

$$2r\cos\theta\cong 2\left(1-\frac{\theta}{2Q}\right)\left(1-\frac{\theta^2}{2}\right)\cong 2-\frac{\theta}{Q}-\theta^2. \tag{16}$$

Comparing (14)–(16) with the denominator of (12), it can be shown that

$$Q=\frac{\sqrt{(\alpha_2\alpha_7)(1+\alpha_8)}}{\alpha_8-\alpha_9} \tag{17}$$

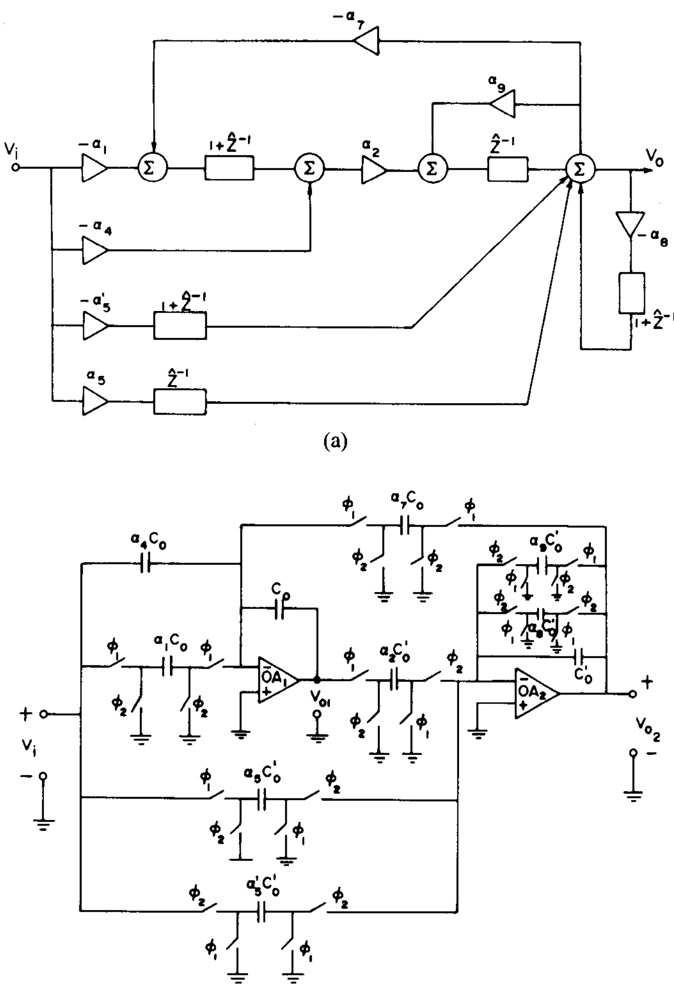

Fig. 2. Generation of Biquad 1. a) Flow diagram. b) SC implementation.

and

$$f_0 = \frac{1}{2\pi T_s}\sqrt{\frac{\alpha_2\alpha_7}{1+\alpha_8}}. \tag{18}$$

The sensitivities of f_0 and Q with respect to the different α's are shown in Table I.

Tradeoffs between sensitivity and total capacitor area can be made when $\alpha_9 \neq 0$ [13]. It can be seen that for $\alpha_9 = 0$ the passive sensitivities are small and comparable to those of other published biquads.

Biquad 2, which was obtained from a second flow graph representation of the biquadratic transfer function, is shown in Fig. 3. Observe that topologically the main difference between Biquad 1 and Biquad 2 is in the switch phase arrangements and the deletion of $\alpha_4 C_0$ in Biquad 2.

The ideal transfer function of the Biquad 2 is given by

$$H^{ee}(z) = \frac{V_{o2}^e(z)}{V_i^e(z)} = -\frac{\alpha_5'}{1+\alpha_8}\cdot\frac{z^2 - z(\alpha_5' + \alpha_5 - \alpha_1\alpha_2)/\alpha_5' + \alpha_5/\alpha_5'}{z^2 - z\dfrac{2+\alpha_8+\alpha_9-\alpha_2\alpha_7}{1+\alpha_8} + \dfrac{1+\alpha_9}{1+\alpha_8}}. \tag{19}$$

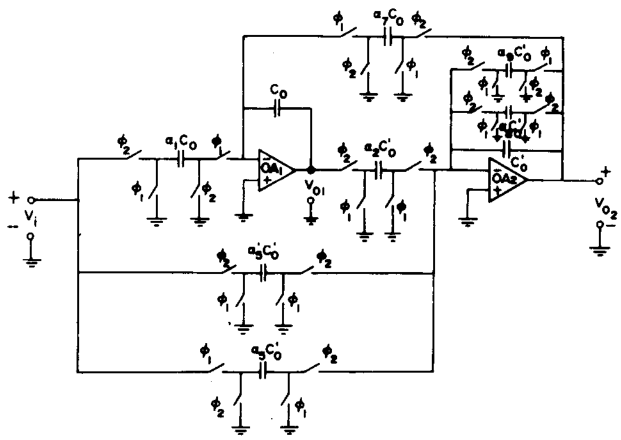

Fig. 3. SC Implementation of Biquad 2.

TABLE I
SENSITIVITIES OF f_0 AND Q WITH RESPECT TO CAPACITOR RATIOS α_i

	α_i Values			
	α_2	α_7	α_8	α_9
$S^{f_0}_{\alpha_i}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}\frac{\alpha_8}{1+\alpha_8}$	0
$S^{Q}_{\alpha_i}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}\frac{\alpha_8}{1+\alpha_8} - \frac{\alpha_8}{\alpha_8-\alpha_9}$	$\frac{\alpha_9}{\alpha_8-\alpha_9}$

The ideal design equations for the poles are the same as for Biquad 1. The effects of GB on these networks will now be considered by investigating the effective GB matrices. Since both networks are two-phase, there are two A matrices for each network. It thus remains to determine the eigenvalues of each of these four matrices. Prior to undertaking this routine task a couple of useful observations applicable to these circuits as well as may others which have appeared in the literature will be made. Details of the justifications are left to the readers.

Observation 1: If an SC network with m op amps is unilateral (with output voltage ordering $v_1, \cdots, v_m$) during a given clock phase and if $v(t)$ in (3) is defined by

$$v(t) = \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix} \tag{20}$$

then the A matrix in (3) is lower triangular[1] and the eigenvalues are the diagonal entries of the A matrix.

Observation 2: If during a given phase all capacitors which are connected to an op amp terminal have one plate connected to an op amp output node and another to an op amp node, then the entries of the A matrix during this

[1] "Unilateral" is defined in the Appendix. If the ordering of the entries in the $v(t)$ vector is reversed, the A matrix becomes upper triangular.

TABLE II
EFFECTIVE GB MATRICES FOR BIQUADS 1 AND 2

PHASE	BIQUAD 1	BIQUAD 2
ϕ_1	$\begin{bmatrix} \frac{1}{1+\alpha_1+\alpha_4+\alpha_7} & 0 \\ \frac{\alpha_7}{1+\alpha_1+\alpha_4+\alpha_7} & 1 \end{bmatrix}$	$\begin{bmatrix} \frac{1}{1+\alpha_1+\alpha_7} & 0 \\ 0 & 1 \end{bmatrix}$
ϕ_2	$\begin{bmatrix} \frac{1}{1+\alpha_4} & 0 \\ 0 & \frac{1+\alpha_8}{1+\alpha_2+\alpha_5+\alpha_5'+\alpha_8+\alpha_9} \end{bmatrix}$	$\begin{bmatrix} 1 & \frac{\alpha_2}{1+\alpha_8+\alpha_9+\alpha_2+\alpha_5+\alpha_5'} \\ 0 & \frac{1+\alpha_8}{1+\alpha_2+\alpha_5+\alpha_5'+\alpha_8+\alpha_9} \end{bmatrix}$

phase are given by

$$a_{l_m} = \frac{\sum C_{fi} + \sum C_{fn}}{\sum C_{ii} - \sum C_{ni}} \tag{21}$$

where the C_{fi} sum is over all *f*eedback capacitors from the output of op amp l to the *i*nverting input of op amp m, the C_{fn} sum is over all *f*eedback capacitors from the output of op amp l to the *n*oninverting input of op amp m, the C_{ii} sum is over all capacitors connected to the *i*nverting *i*nput of op amp m and the C_{in} sum is over all capacitors connected to the *n*oninverting *i*nput of op amp m.

If the conditions necessary for Observation 2 are satisfied the effective GB matrix can be obtained by inspection. These conditions are satisfied by most of the better stray insensitive SC structures which have appeared in the literature. If they are not satisfied, the notational complexity required to describe obtaining the effective GB matrix by inspection is cumbersome.

From these observations, it follows that the effective GB matrices for Biquad 1 and Biquad 2 are as shown in Table II.

IV. DESIGN EXAMPLE

This example allows us to compare both biquads introduced in the previous section. The analysis methods used to obtain the transfer functions is the same as presented before [10]. For comparative purposes the design specifications are the same as used by Fleischer and Laker [5]; specifically, a bandpass structure with center frequency/sampling frequency equals 1.633 kHz/8 kHz, a quality factor of $Q = 16$ and a peak gain of 10 dB at the center frequency. The resulting transfer function of the type[2] BPOO [5] is given by

$$H^{ee}(z) = \frac{0.1953(z-1)z}{z^2 - 0.5455z + 0.9229}. \tag{22}$$

[2]The notation [5] LP_{ij}, denotes a transfer function where the number of $(1+z^{-1})$ factors is i and the number of z^{-1} terms is j in the numerator.

TABLE III
BANDPASS REALIZATION WITH $\alpha_9 = 0$

Capacitor	Initial Design	Dynamic range Adjusted	Final Design
C_0	1.0	1.249	1.0
C_0'	1.0	1.0	11.970
$\alpha_2 C_0'$	1.0	1.249	14.953
$\alpha_5' C_0'$	0.212	0.212	2.533
$\alpha_7 C_0$	1.492	1.493	1.195
$\alpha_8 C_0'$	0.084	0.084	1.0
C_T	(Total capacitance in terms of unit capacitance, Cu)		32.651

TABLE IV
NUMERICAL VALUES OF THE EFFECTIVE GB MATRICES FOR BIQUADS 1 AND 2 CONSIDERED IN THE EXAMPLE

PHASE	BIQUAD 1	BIQUAD 2
ϕ_1	$\begin{bmatrix} 0.456 & 0 \\ 0.544 & 1 \end{bmatrix}$	$\begin{bmatrix} 0.456 & 0 \\ 0 & 1 \end{bmatrix}$
ϕ_2	$\begin{bmatrix} 1 & 0 \\ 0 & 0.426 \end{bmatrix}$	$\begin{bmatrix} 1 & 0.491 \\ 0 & 0.426 \end{bmatrix}$

TABLE V
SENSITIVITIES OF BANDPASS BIQUADS, $\alpha_9 = 0$

	α_i Values			
	α_2	α_7	α_8	α_9
$S^{f_0}_{\alpha_i}$	-0.585	-0.585	0.023	0.0
$S^{Q}_{\alpha_i}$	-0.585	-0.585	0.985	0.0

The design parameters, which are ideally the same for both biquads, are shown in Table III. The entries of the effective GB matrices for these designs as defined in Table II are given in Table IV. Since the eigenvalues of Biquad 2 are identical to those of Biquad 1 and since the eigenvalues for each circuit are far from matched, it can be concluded that the GB related performance of these circuits is comparable. The performance of Biquad 2 is actually a little better due to a modest reduction in the off-diagonal terms of the effective GB matrices. The passive sensitivities for both biquads, which are also the same, are given in Table V.

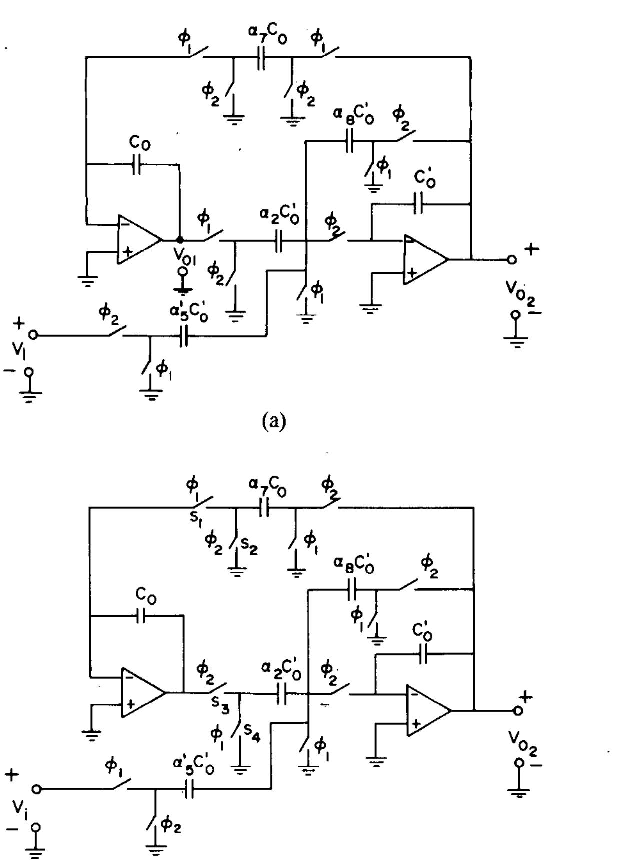

Fig. 4. SC Bandpass minimum switch configurations. (a) Biquad 1. (b) Biquad 2.

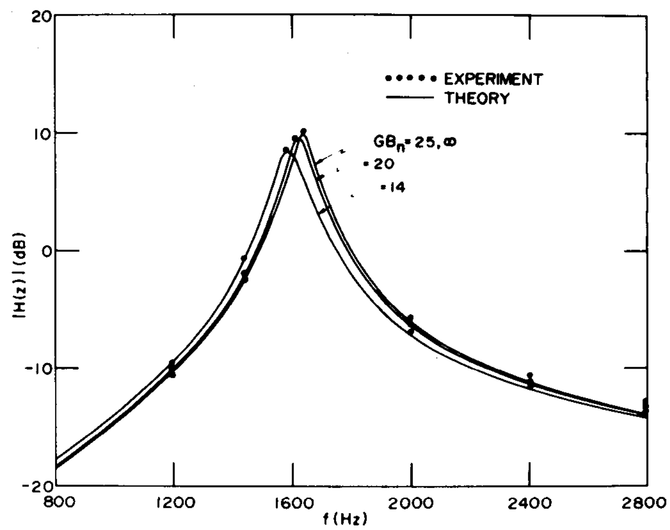

Fig. 5. Theoretical and experimental frequency response of Biquad 1.

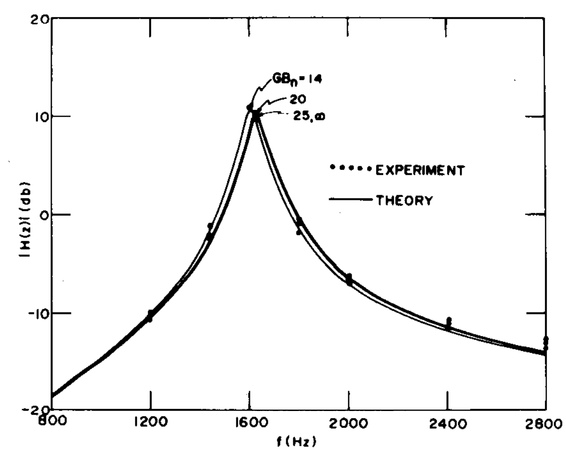

Fig. 6. Theoretical and experimental frequency response of Biquad 2.

More practical reduced (minimum switch) versions of these biquads (with $\alpha_1 = \alpha_5 = 0$) are shown in Fig. 4. The experimental[3] and theoretical results for Biquad 1 which show GB effects are shown in Fig. 5. They show excellent agreement.

The corresponding results for Biquad 2 are shown in Fig. 6. These theoretical and experimental results are also in good agreement.

In order to better appreciate the characteristics of the two biquads and make comparisons with other structures, a series of z-plane root locus plots as a function of $GB_n = GB/\omega_0$ are presented. Fig. 7 shows the root locus for both biquads assuming identical GB's for the op amps. In the sample plot a fixed grid of ω_0 and Q variations is included. This allows the designer to determine the Q and ω_0 variations for certain value of GB_n. It can be seen from this figure that the op amp effects on both ω_0 and Q for Biquad 2 are less severe than for Biquad 1. Also included is a plot of the pole locus of a third filter designed to realize the same transfer function using the configuration of Martin ([12, fig. 7]) which is also a special case of the Fleischer–Laker Biquad ([5, fig. 1]). The topology for the Martin–Fleischer–Laker implementation differs from the bandpass version of Biquad 2 shown in Fig. 4(a) only in the phasing of the four switches (component values are identical to listed in Table III), S_1–S_4, and consequently in the entries of the effective GB matrix. The corresponding effective GB matrix for the Martin–Fleischer–Laker Biquad is shown in Table VI. From Table VI it can be seen that the eigenvalues of this circuit during phase ϕ_2 are very close and thus the off-diagonal entry will play a major role. For example, if $GB_n = 18$, the off-diagonal entry in (6) is nearly ten times as large as the main diagonal entries. Since the minimum of the eigenvalues for the new biquads are comparable to those of the Martin–Fleischer–Laker circuit, it can be concluded that the latter circuit has more serious GB limitations due to the large off-diagonal term in the effective GB matrix. This fact is verified in the pole locus plot of Fig. 7. Since the three circuits compared in Fig. 7 differ only in switch phasing, they all have the same total capacitor area. It is easy to verify that *these* differences in phasing do not affect any of the passive sensitivity expressions.

[3]The op amps employed were of the μA741 type and the switches HI-201.

Fig. 8 shows the root locus for Biquad 1 with mismatched op amps. The corresponding root locus for Biquad 2 is shown in Fig. 9. It is interesting to note from these root-locus plots that the value of GB_2 is more critical

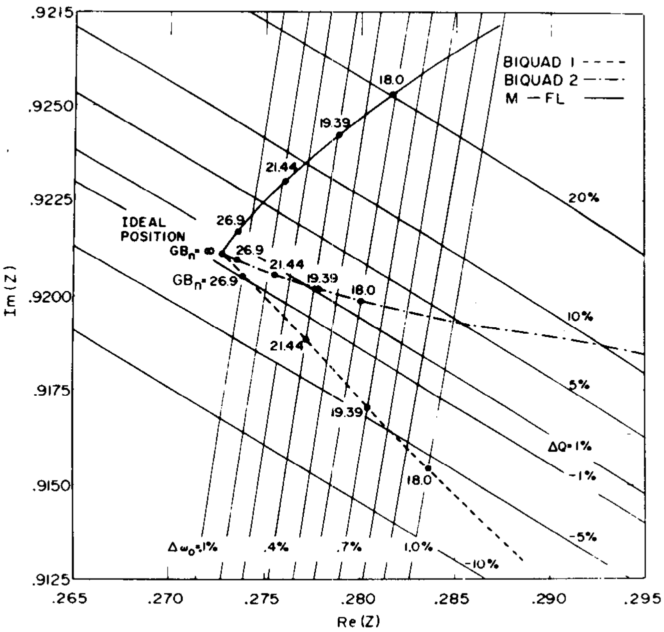

Fig. 7. Root locus of Biquad 1, 2, and M-FL for the case of identical GB's.

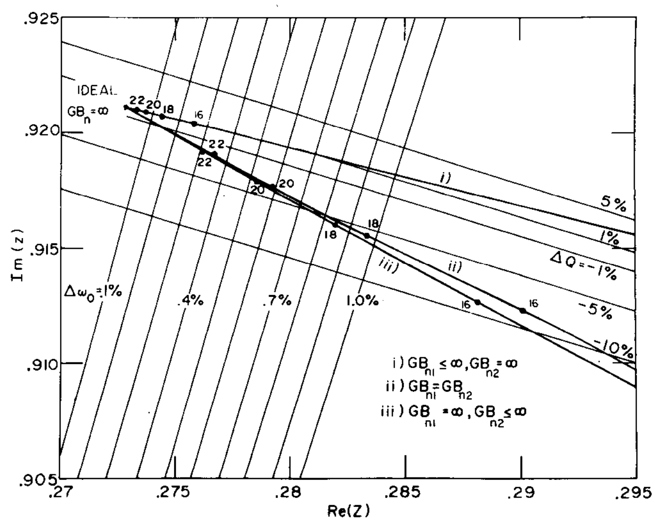

Fig. 8. Root locus of Biquad 1 for different values of GB_n's. $GB_i = GB_{ni}$, $i = 1,2$.

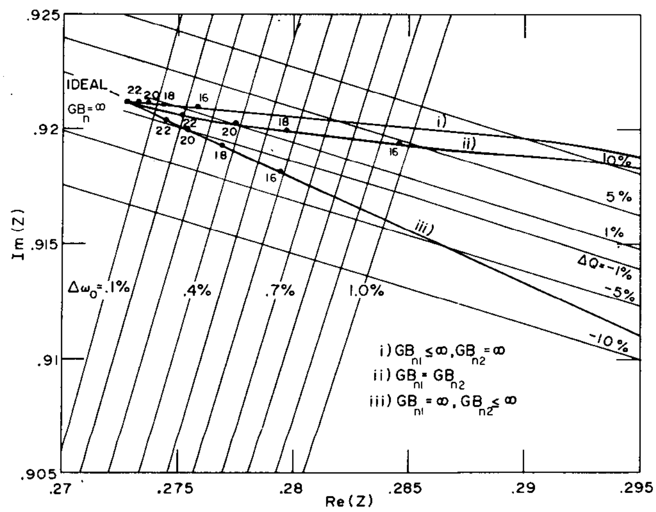

Fig. 9. Root locus of Biquad 2 for different values of GB_n's. $GB_i = GB_{ni}$, $i = 1,2$.

TABLE VI
NUMERICAL VALUES OF THE EFFECTIVE GB MATRICES FOR THE MARTIN–FLEISCHER–LAKER BIQUAD

Phase	
ϕ_1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
ϕ_2	$\begin{bmatrix} .456 & 0 \\ .544 & .426 \end{bmatrix}$

than GB_1. This can be explained from (6) and Table IV by observing that the minimum of the eigenvalues occurs during phase ϕ_2 for both biquads and that it is this eigenvalue that determines the exponential decay associated with GB_2. The designer should consider this fact when specifying/designing the op amps.

V. CONCLUSIONS

General design guidelines to select appropriate switching phasing in obtaining low GB dependance SC filter have been presented. The SC effective GB matrices, which can generally be obtained by inspection, have been shown to be useful in predicting GB effects. Two general biquadratic SC topologies with reduced GB effects have been reported. These circuits were shown to have favorable passive sensitivities and total capacitances. The theoretical and experimental results are in good agreement.

The analysis for both biquads allows the designer to predict the ω_0 and Q deviations due to the GB's of the op amps.

APPENDIX
BRIEF REVIEW OF THE ANALYSIS OF SC FILTERS WITH OP AMPS MODELED BY ONE DOMINANT POLE

The basic ideas on the analysis of SC filters with op amps modeled by one dominant pole, ω_1, are reviewed. First, the analysis assumptions are:

i) ϕ_1 and ϕ_2 are complementary nonoverlapping clocks with 50-percent duty cycles and period T. They will be *arbitrarily* defined with ϕ_2 high for $(n-1)T < t \leqslant (n-1/2)T$ and ϕ_1 high for $(n-1/2)T < t \leqslant nT$.

ii) $v_i(t)$ is constant during clock phases and defined by

$$v_i(t) = v_i(nT-T), \qquad \text{for } nT-T \leqslant t < nT. \quad (1)$$

iii) The op amp output, $v_0(t)$, is continuous for all time.

iv) The op amp is characterized by the popular single-pole model

$$A(s) = V_o(s)/V_a(s) = \mathrm{GB}/s$$

which can be represented in the time domain as

$$\frac{dv_o(t)}{dt} + \mathrm{GB}v_a(t) = 0 \quad (2)$$

where $v_a(t)$ is the differential input voltage to the op amp.

If the op amp is used in a SC filter, $v_a(t)$ can during any clock phase be expressed as a function of $v_o(t), v_i(t)$, capacitor voltages due to initial conditions and some other signals coming from the outputs of other op amps in the filter structure. The expression for $v_a(t)$ becomes

$$v_a(t) = av_o(t) + B(t) + b \quad (3)$$

where a is a topology dependent constant, $B(t)$ denotes the effects on $v_a(t)$ of all other op amp output voltages, and b depends on the circuit topology and the excitation $v_i(t)$. Thus (2) can be expressed as

$$\frac{dv_o(t)}{dt} + \widehat{\mathrm{GB}}v_o(t) = (B(t)+b)\mathrm{GB} \quad (4)$$

where $\widehat{\mathrm{GB}}$ is the effective gain–bandwidth product and equal to aGB. In the case that $B(t)$ is independent of $v_o(t)$, a general solution of (4) valid when t and t_1 are both in the same clock half period can be written as

$$v_o(t) = e^{-\widehat{\mathrm{GB}}t}\widehat{\mathrm{GB}}\int_{t_1}^{t} e^{\widehat{\mathrm{GB}}t}[B(t)+b]\,dt + v_o(t_1)e^{-\widehat{\mathrm{GB}}(t-t_1)}. \quad (5)$$

Consider the special case

$$B(t) = 0$$
$$v_i(t) = v_i(t_1). \quad (6)$$

In this case b is a constant and (5) reduces to

$$v_o(t) = v_o(t_1)e^{-(t-t_1)\widehat{\mathrm{GB}}} + b\left[1 - e^{-(t-t_1)\widehat{\mathrm{GB}}}\right]. \quad (7)$$

It is observed from (7) that the step response of a single op amp SC subcircuit is an exponential ramp. Since as t goes to infinity, the response must approach the desired value, v_{od}, we know that $b = v_{od}$.

Popular higher order SC filters typically contain cascaded strings of integrators. It is often the case that during a given phase the input signals propagate unilaterally[4] through the cascade. In that case, the output of the first stage, v_{01}, will assume the functional form given by

$$v_{01}(t) = (v_{01}(t_1) - b_1)e^{-\widehat{\mathrm{GB}}_1(t-t_1)} + b_1 \quad (8)$$

[4] A cascade of m op amp circuits is termed unilateral if $\exists$ an ordering of the op amps, 0A1, $\cdots$, 0Am such that V_k is independent of V_l $\forall k > l$ where V_i, $i \in \{1,\cdots,m\}$ represents the output voltage of ith op amp.

where $\widehat{\mathrm{GB}}_1 = a_{11}\mathrm{GB}_1$. This serves as the input to the next stage. In this case, b in (5) is again a constant and $B(t)$ can be expressed as $hv_{01}(t)$ where h is a topology dependent constant. It follows from (5) that the output of the second stage is of the form

$$v_{02}(t) = \frac{h\widehat{\mathrm{GB}}_2}{\widehat{\mathrm{GB}}_2 - \widehat{\mathrm{GB}}_1}e^{-\widehat{\mathrm{GB}}_1(t-t_1)}(v_{01}(t_1) - b_1) + e^{-\widehat{\mathrm{GB}}_2(t-t_1)} \cdot \left[v_{02}(t_1) - \frac{\widehat{\mathrm{GB}}_2 h}{\widehat{\mathrm{GB}}_2 - \widehat{\mathrm{GB}}_1}(v_{01}(t_1) - b_1) - hb_1 - b_2\right] + hb_1 + b_2 \quad (9)$$

where $\widehat{\mathrm{GB}}_2 = a_{22}\mathrm{GB}_2$.

This response is a sum of constants and decaying exponentials. If this is used as the input to the next stage, the output would again be a sum of constants and decaying exponentials. The same conclusions follows for all subsequent outputs in the unilateral cascade.

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Transactions Briefs

Optimal Design of State-Space Digital Filters by Simultaneous Minimization of Sensitivity and Roundoff Noise

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Abstract —**The relation between a newly defined sensitivity measure and the noise power gain for scaled filters is given. By means of this relation it is shown that the necessary and sufficient condition to minimize the noise power gain is to minimize the sensitivity measure for unscaled filters. It is shown that the minimization of this sensitivity measure can be carried out without taking into account the dynamic range constraint. A procedure for the simultaneous minimization of the sensitivity measure and the noise power gain is also given.**

I. INTRODUCTION

In the design of digital filters the problem of minimizing the roundoff noise under the dynamic range constraint has been studied and the necessary and sufficient conditions have been given [1], [2]. On the other hand, using the well-known equivalent transformation of state-variables a minimization procedure has been developed [3]. In the second-order case some optimal forms for the state-matrices have been proposed [2], [4].

From practical considerations it is equally important to synthesize filters with low-coefficient sensitivities. Hence, it has always been the interest of designers to simultaneously minimize sensitivity and roundoff noise under the dynamic range constraint. However, so far such a minimization procedure has not been found. The main aim of this paper is, therefore, to give a general optimization procedure to minimize the sensitivity and the roundoff noise simultaneously. To this end, a relation between a newly defined sensitivity measure and the well-known noise power gain [5] under the dynamic range constraint has been obtained.

II. SENSITIVITY AND ROUNDOFF NOISE MEASURES

Consider the discrete system transfer function

$$H(z)=\frac{\sum_{i=0}^{n} a_i z^{-i}}{1+\sum_{i=1}^{n} b_i z^{-i}}$$

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