It is interesting to note that, unlike in the scalar cases, in the multivariable systems we can directly utilize a_i only in (20) for the left matrix fraction decomposition of a strictly positive real function, that $u(z)$, $x(z)$, and $v(z)$ that in (17)–(19) are functions of f_i and t_{ij} and that $f_i \neq a_i$. It indicates that to construct a strictly positive real function we have to solve for T^{-1}, the solution of the Lyapunov equation (3). The applications of construction of $r(z)$, for given A and B in (2) are in the model reference adaptive system, when A is known from the autoregressive method. The strict positive realness of $r(z)$ of the model adaptive reference adaptive system implies the asymptotic hyperstability, and, hence, the convergence of the scheme [3].

Numerical Example

Consider (1)–(4) with $m = 2$ and $n = 3$

$$a_0 = \begin{bmatrix} 0.3 & -0.06 \\ 0 & 0.24 \end{bmatrix} \qquad a_1 = \begin{bmatrix} -0.36 & 0 \\ -0.06 & -0.42 \end{bmatrix}$$

$$a_2 = \begin{bmatrix} 0.15 & -0.3 \\ 0.3 & 0.18 \end{bmatrix}.$$

The eigenvalues of the matrix A in (2) are $-0.9012 \pm j0.1697$. $0.3951 \pm j0.4869$, and $0.3411 \pm j0.3184$. We compute T and T^{-1} using Cyber 170 under NOS BE1. We applied the algorithms in [13], [14]. Then

$$T = \begin{bmatrix} 2.08888 & -.04174 & -1.08808 & -.38794 & 1.15365 & .39044 \\ & 2.15849 & .47335 & -1.21488 & -.40700 & 1.27230 \\ & & 2.08888 & -.04174 & -1.08808 & -.38794 \\ & & & 2.15849 & .47335 & -1.21488 \\ & \text{Symmetric} & & & 2.08888 & -.04174 \\ & & & & & 2.15849 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} .88980 & .03694 & .24368 & .28560 & -.42173 & .01365 \\ & .95977 & -.31779 & .29573 & -.07523 & -.46454 \\ & & .87152 & -.00683 & .26503 & .30116 \\ & & & .90178 & -.30253 & .27449 \\ & \text{Symmetric} & & & .90390 & .01547 \\ & & & & & .94356 \end{bmatrix}$$

$$f_0 = \begin{bmatrix} 0.30743 & -0.06447 \\ -0.00454 & 0.23515 \end{bmatrix}$$

$$f_1 = \begin{bmatrix} -0.38441 & -0.00485 \\ -0.07030 & -0.39374 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 0.14441 & -0.30689 \\ 0.28790 & 0.18558 \end{bmatrix}.$$

Note that $f_i \neq a_i$.

The strictly positive real function $r(z)$ follows immediately from (16)–(20).

Conclusion

We have shown the inverse of the solution of the Lyapunov equation in equations (1)–(2) transforms A into F in (4) by a similarity transformation. We also have shown that the first (last) m columns of T^{-1} in (3) yield the left (right) matrix fraction decomposition of a strictly positive real function. The known results in the scalar cases follow as particular cases.

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Tradeoffs Between Passive Sensitivity, Output Voltage Swing, and Total Capacitance in Biquadratic SC Filters

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Abstract—**Design guidelines for biquadratic SC filters taking into consideration passive sensitivity, output voltage swing, and total capacitance area are described. In this paper, we discuss the tradeoffs between these three parameters. A popular SC topology with additional positive feedback is used to illustrate the tradeoffs involved in the design of SC filters.**

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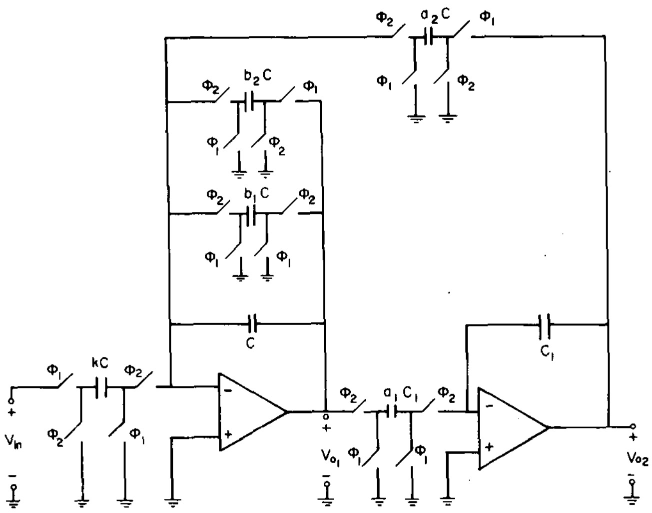

Fig. 1. A popular switched-capacitor filter.

I. Introduction

A number of SC filters have been published which realize general biquadratic transfer functions [1]–[4]. In particular, SC topologies that have partial positive feedback are capable of significantly reducing the total capacitance [5], [6]. However, attempts to minimize total capacitance in a filter structures are often accompanied by an increase in Q sensitivity and/or a reduction in the corresponding output voltage swing of the op amps. Conversely, when the designer only concentrates on minimizing the sensitivities large capacitor areas typically result.

In this paper, we show the tradeoffs between total capacitance and Q sensitivities for a certain output voltage swing of the op amps. Using information presented here the designer can specify the maximum Q sensitivity value permitted and obtain the minimum total capacitance, C_T, under certain op amp output voltage swing conditions, or vice versa, namely, the designer can specify the op amp output voltage swing, C_T, and determine the resultant Q sensitivity.

II. Second-Order Filter with Positive Feedback

Fig. 1 shows a popular biquadratic SC filter structure which includes a local positive feedback loop [5], [6]. This filter, without the positive feedback capacitor b_2C, reduces to the popular F-circuit reported by Fleisher and Laker [3]. We focus attention here on the positive feedback and the effects of b_2C on the pole frequency and pole-quality factor, Q, sensitivities.

III. Design and Pole Equations

The loop equation defining the pole locations for the circuit of Fig. 1 is given by

$$D(z) = 1 - z^{-1}(2 + b_1 + b_2 - a_1a_2)/(1+b_1) + z^{-2}(1+b_2)/(1+b_1). \quad (1)$$

From this equation we can identify

$$r^2 = \frac{1+b_2}{1+b_1} \quad (2)$$

and

$$2r\cos\theta = \frac{2+b_1+b_2-a_1a_2}{1+b_1} \quad (3)$$

where r and θ correspond to the pole radius and angle, respectively.

Assuming r, and θ are given, the following algorithmic design strategy can be used to obtain a_1, a_2, b_1, and b_2.

Solving (2) for b_1 we obtain the expression

$$b_1 = (1 + b_2 - r^2)/r^2 \quad (4)$$

and

$$a_1a_2 = \frac{1+b_2}{r^2}(1 + r^2 - 2r\cos\theta). \quad (5)$$

It can be seen that b_2 is a design parameter that can be used to judiciously tailor either sensitivity or total capacitance. Since only the a_1a_2 product is fixed, the designer also has flexibility in specifying one of these parameters. Next, in order to relate s- and z-plane, we arbitrarily use for purpose of reference the impulse invariant response [1] to derive the sensitivity expressions. It can be shown [1] that

$$f_0 = \frac{f_s}{2\pi}\left[\theta + (\ln r)^2\right]^{1/2} \quad (6)$$

and

$$Q = -\frac{2\pi f_0}{f_s \ln r^2} \quad (7)$$

where f_s is the sampling (clock) frequency and f_0 is the center (cutoff) frequency.

If p is any capacitor ratio in the circuit of Fig. 1, it follows from (2), (6) and (7) that

$$S_p^{f_0} = \left(\frac{f_s}{\omega_0}\right)^2\left[\frac{\ln r + \theta/\tan\theta}{2} S_p^{r^2} - (\theta/\tan\theta) S_p^{2r\cos\theta}\right] \quad (8)$$

and

$$S_p^Q = \left(\frac{f_s}{\omega_0}\right)^2\left[\frac{-\theta^2 + \theta(\ln r)/\tan\theta}{2\ln r} S_p^{r^2} - (\theta/\tan\theta) S_p^{2r\cos\theta}\right]. \quad (9)$$

The sensitivities of $2r\cos\theta$ and r^2 that appear in these expressions can be obtained from (2) and (3). For high Q circuits the reader is cautioned to avoid selecting values of b_1 and b_2 that will result in very high Q sensitivities and the associated stability problems.

IV. Practical Design Considerations

There is a tradeoff, in designing SC filters, between the total capacitance C_T, Q sensitivity[1] and the op amp output voltage swings. It is particularly important to consider these tradeoffs in SC filter topologies which involve both positive and negative feedback. The tradeoffs for the circuit of Fig. 1 are illustrated in Table I in terms of the sensitivity measure

$$S_{\text{average}} = 1/2\left[|S_{b1}^Q| + |S_{b2}^Q|\right] \quad (10)$$

for a particular set of design specifications, i.e., $f_0/f_s = 1/50$ and $Q = 10$. Two cases are simultaneously considered for comparison purposes. In one case the op amp outputs are unbalanced, that is, V_{02} is fixed to 0 dB and V_{01} is variable. In the other case both outputs are scaled (by the choice of the parameter a_1) to 0 dB. C_{TU} and C_{TB} are the total capacitances for the unbalanced and balanced outputs, respectively.

The tradeoff between the C_T and Q sensitivities for balanced op amp outputs will now be considered. The total capacitance C_T

[1]We do not present results on $S_p^{f_0}$ since they are around 0.5, therefore, not critical.

TABLE I
BALANCED AND UNBALANCED OUTPUTS VERSUS Q SENSITIVITY AND C_T, WITH $V_{02} = 0$ dB

b_2	V_{o_1} (dB)	$S_{average}$	C_{TU}	C_{TB}
0	.02726	0.49844	114.91	114.97
.025	.1345	2.42720	81.214	81.416
.05	.23916	4.2641	54.725	55.013
.1	.44119	7.6875	41.953	42.416
.2	.81907	13.678	49.252	46.369
.4	1.4885	23.09	83.28	72.793
.8	2.5800	35.64	162.76	125.0

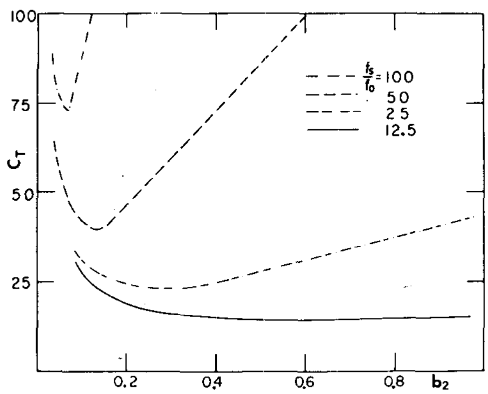

Fig. 2. Total capacitance versus the capacitor ratio b_2 for a fixed $Q = 10$, and different f_0/f_s values.

can be expressed as [3]:

$$C_T = [(a_2C + b_2C + b_1C + kC + C)/C_{\min 1}] + (a_1C_1 + C_1)/C_{\min 2}]\text{Cu} \quad (11)$$

where $C_{\min 1}$ and $C_{\min 2}$ are the smallest capacitors in the sets $\{C, kC, b_1C, b_2C, a_2C\}$ and $\{C_1, a_1C_1\}$, respectively. Since the variable a_1 has been used to obtain balanced outputs, it follows from (4) and (5) that C_T is actually only a function of the single variable b_2. The nonlinear nature of C_T in terms of b_2 is illustrated in Fig. 2 for a family of different f_s/f_0 and a fixed $Q = 10$.

Two practical situations are now considered.

i) Fig. 3 illustrates the compromise between the total capacitance C_T, and S_{average} for a family of f_0/f_s values.[2] Note that for this plot Q is fixed, in this case $Q = 10$. Another set of curves can be easily generated for any desired Q value. These curves can be used to obtain the tradeoffs between C_T and S_{average} for a given f_0/f_s value. These curves can be used to obtain the minimum C_T for a given maximum permissible sensitivity value or a minimum sensitivity for a given total capacitance.

ii) For a given value of Q, we can determine the compromise between S_{average} and C_T. This is shown in Fig. 4 for a family of Q values and a fixed ratio of $f_0/f_s = 1/10$.

[2]Figs. 3, 4, and 5 already include the scaling of both op amp outputs, for $V_{01} = V_{02} = 1$ V. (0 dB).

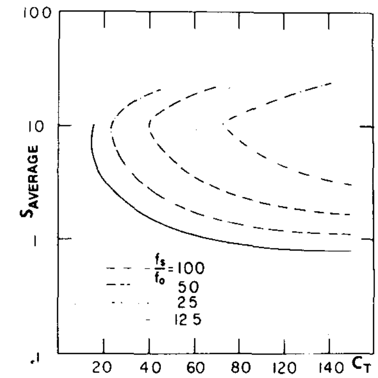

Fig. 3. Average Q sensitivity versus total capacitance for a fixed $Q = 10$, and different f_0/f_s values.

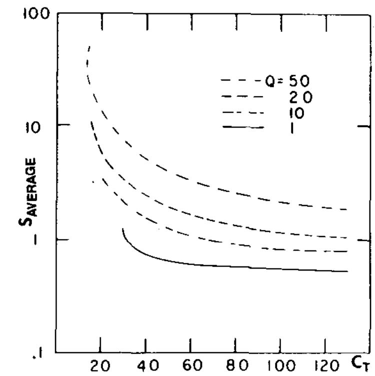

Fig. 4. Average Q sensitivity versus total capacitance for a fixed $f_0/f_s = 1/10$, and different Q values.

In either case considered, once the desired operating point is determined the required value of b_2 can be obtained from Fig. 2. It is important to emphasize that in Figs. 2, 3, and 4, C_T is bivalued and the value of S_{average} is not unique, b_2 should be chosen to minimize S_{average}.

V. EXAMPLES

The following examples illustrates the use of Figs. 2, 3, and 4. First, assume the design specifications are $f_0/f_s = 1/50$, $Q = 10$, and a center frequency gain of 0 dB and that the maximum S_{average} is 11. The problem is to determine C_T and the value of b_2 to satisfy the above conditions. By using Fig. 3, we find that $C_T = 40$ Cu. The value of b_2 can be determined by means of Fig. 2. Two values of b_2 are obtained for $C_T = 40$ Cu, $b_2 \in \{0.12, 0.15\}$.

Additional flexibility is available through the parameter a_1 if the requirement of unbalanced op amp outputs is relaxed. This is illustrated in Table II, where the case of no positive feedback $b_2 = 0$, is included for comparison. In particular, note that $b_2 = 0.12$ yields a smaller S_{average} than the case of $b_2 = 0.15$. For a very small S_{average}, i.e., when $b_2 = 0$, the total capacitance increases by nearly a factor of 3.

Consider for the second example the design specifications $f_0/f_s = 1/25$, $Q = 10$ and a center frequency peak gain of 0 dB. Assume that the maximum permitted value of C_T is 40 Cu. The problem is to determine S_{average} and b_2 for the above data. By using Fig. 4, the corresponding value of S_{average} becomes 2.9. Then $b_2 \in \{0.07, 0.90\}$ form Fig. 2. The smallest b_2 gives the smallest S_{average} of the two choices. Table III describes all the component values for the balanced and unbalanced output cases. The particular case of $b_2 = 0$ is included in the table. It is

TABLE II
C_T AND Q SENSITIVITIES FOR EXAMPLE 1

	Output	V_{o_2}(dBs)	b_1	a_1	a_2	k	C_T	$S_{average}$
b_2= 0	Unbalanced	.02726	.01264	.12598	.12598	.20147	114.91	0.4981
	Balanced	0	.01264	.12558	.12638	.20147	114.97	
b_2= .12	Unbalanced	.15944	.13416	.13332	.13332	2.3893	39.95	8.9712
	Balanced	0	.13416	.12556	.14154	2.3893	40.504	
b_2= .15	Unbalanced	.63424	.16454	.13509	.13509	3.008	41.40	10.813
	Balanced	0	.16454	.12558	.14533	3.008	39.712	

TABLE III
C_T AND Q SENSITIVITIES FOR EXAMPLE 2

		V_{o_2}(dBs)	b_1	a_1	a_2	k	C_T	$S_{average}$
b_2 = 0	Unbalanced	.054683	.025451	.25225	.25225	.2008	63.056	.4969
	Balanced	0	.025451	.25067	.25384	.2008	63.143	
b_2= .07	Unbalanced	.34852	.097233	.26093	.26093	.81472	36.87	3.0675
	Balanced	0	.097233	.25067	.27161	.81472	37.183	

observed in the latter case that $S_{average}$ is reduced by a factor of about 6, however the total capacitance is increased by nearly a factor of 2.

VI. CONCLUSIONS

We have presented design guidelines that consider the tradeoffs between the op amp voltage swings, Q sensitivity, and total capacitance for a biquadratic SC filter. We illustrated the approach using plots for certain particular design specifications. It can be seen that the advantage of using positive feedback is the reduced capacitance area at the expense of increased Q sensitivity. Equations were given which are suitable for any desired design specification.

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A Canonical Form for Lossless Multiport Transmission Lines

J. M. POTTER

Abstract—**The propagation of TEM waves on a reciprocal n-port lossless nonuniform transmission line, with absolutely continuous characteristic impedance and locally equal wavefront velocities in each port, is described in this note by a canonical system of wave equations. The essential feature of this canonical system is an $n \times n$ local reflection coefficient which generalizes the notion of a reflection coefficient for a single-port line. The reflection coefficient characterizes the local scattering behavior at any point on the line, and is unique up to a constant orthogonal transformation.**

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I. INTRODUCTION

There is a long historical interest in nonuniform, or tapered, lines—see the early bibliography of Kaufman [1]. The purpose of this note is to specify a canonical system of equations for reciprocal n-port lossless nonuniform transmission lines and to derive these equations from the usual transmission line equations. This derivation involves a nontrivial extension of the well-known procedure for single-port lines reviewed in Section II below.

The derivation of canonical forms for hyperbolic systems of equations is a standard mathematical technique (Courant and Hilbert [2]); however the particular form of the transmission-line equations considered here allow a modification of the standard technique entailing a particularly useful canonical form. The usefulness of this form centres around an $n \times n$ reflection coefficient $\Gamma(x)$ which characterizes the local scattering behaviour along the line. In fact, our motivation for this work arose from our investigation (in [3]) of the multivariable generalization of Krein's work on inverse scattering (see, e.g., Faddeyev [4] or Chadan and Sabatier [5]) and its application, *inter alia*, to transmission line synthesis. Krein's basic system of equations (as recorded in [4] e.g.) correspond to a wave variable formulation of the transmission-line equations, reviewed in Section II. We experienced difficulty in relating these equations to the transmission-line equations in the multivariable case; this note reports the resolution of this difficulty.

II. GROUNDWORK

Here we introduce our notation, list our assumptions for the types of transmission line under consideration, and briefly review the well-known derivation of the canonical form for the single-port transmission line.

We restrict attention to the propagation of TEM waves in reciprocal, n-port, lossless, nonuniform transmission lines, with