Proceedings of the 29th Conference
on Decision and Control
Honolulu, Hawaii • December 1990

Nonlinear Adaptive Torque-Ripple Cancellation for Step Motors

Degang Chen
Department of Electrical and Computer Engineering
Brad Paden
Department of Mechanical and Environmental Engineering
University of California, Santa Barbara, CA 93106

Abstract: This paper considers the modeling of torque-ripple in hybrid step motors and its cancellation using adaptive linearization control. Although the nonlinear adaptive control of our problem can fit into a general framework we use a different representation of the torque-ripple which reduces the number of adapted parameters per torque-ripple harmonic by half. By doing so, we are able to prove conditions on exogenous signals to guarantee the persistency of excitation of the regressor, and hence the exponential stability of the unperturbed system. We also show that the adaptive system is robust to a class of state and parameter dependent modeling errors and disturbances even when the adaptation gain and convergence rate of the unperturbed system become small. Finally, the adapted parameter errors are proved to converge to a neighborhood of zero whose radius can be made small by slow adaptation. Our control scheme is verified in an experiment in which we observe a 32db reduction in torque-ripple component at the rotor pole frequency.

1. Introduction

The control of electric motors is a traditional control problem which has attracted new interest in the control theory community. The reasons for this are the development of theories of nonlinear geometric control and adaptive control, the low cost of high-performance digital control hardware, and the increase of demanding applications for electric motors. One such application is the actuation of direct-drive robots where high-torque and low torque-ripple (high linearity) are required. The application which motivated this work is the actuation of a cylindrical coordinate robot for silicon wafer transfer in a vacuum environment. In this case, torque-ripple must be eliminated to prevent excitation of structural vibrations and to reduce the risk of damage to wafers.

Torque-ripple in electric motors can be reduced either by design or by control. The desirable detent torque of step motors (which provides passive braking in the absence of power) is also reduced when a step motor is designed for small torque-ripple. Hence the reduction of torque-ripple through control when a step motor is actively powered is an attractive option leading to better overall performance. Interest in torque-ripple reduction in the control community is fairly recent. Le-Huy and Perret [1] make torque-ripple comparisons for brushless DC motor drives for two and three stator phases and several commutation waveforms. In Nagase *et al* [2] velocity-ripple is filtered through a band-pass filter and fed back to the current amplitude control loop to avoid structural resonances. To also address torque-ripple due to geometric imperfection, Murai *et al* [3] considered two types of non-sinusoidal flux distributions and proposed two heuristic switching strategy for torque-ripple reduction. In this paper, we initialize the adaptive controller with a sinusoidal commutation waveform and then proceed with the adaptation. This is the first systematic approach to torque-ripple reduction via adaptive control.

Globally linearizing control is another very promising approach to torque-ripple reduction and was first applied to variable reluctance motors by Taylor *et al* [4]. This methodology has the potential (in theory) to completely eliminate torque-ripple by introducing static nonlinear compensation in the commutation waveforms. This compensation depends on shaft angle and winding current. The results of Taylor's work prove the value of the linearization approach and encourages further research. Other work in this area is by Hemati and Leu [5] who study nonadaptive linearization of DC brushless motors and take saturation into account. However, the adaptive nonlinear motor control is new and includes this work and that of Marino *et al* [6] to appear, where adaptive partial linearization of the nonlinear current-flux interaction in induction motors is studied.

In addition to the previous work on motor control, recent nonlinear adaptive control theory is background for this paper. Sastry and Isidori [7] present a general adaptive control scheme for linearizable of systems with Lipschitz nonlinearities. This approach achieves convergence of tracking error. By introducing a matching condition Marino *et al* [8] succeed in eliminating the Lipschitz condition on the nonlinearity. With the same matching condition, or extended matching condition respectively, Taylor *et al* [9] and Kanellakopoulos *et al* [10] consider adaptive regulation of nonlinear systems and establish robustness to unmodeled stable dynamics. Pomet and Praley [11] work outside the framework of linearizable systems and have nonlinear adaptive control results for a class of stabilizable nonlinear systems.

The contributions of this paper to nonlinear adaptive control theory are the following. (1) Conditions on exogenous signals for persistency of excitation (PE) are established in the step motor control problem. (2) General techniques are devised for showing robustness of nonlinear adaptive systems to state and parameter dependent modeling error. (3) Parameter convergence in this nonlinear adaptive control scheme is analyzed using generalized harmonic analysis. Looking toward future applications, the adaptive controller presented here is relatively simple with only two adaptation parameters for each harmonic of the torque-ripple frequency to be cancelled.

The remainder of this paper is organized as follows. Section 2 gives a brief mathematical model of the motor dynamics which shows the source and structure of torque-ripple. In section 3, we first derive an adaptive control law for torque-ripple cancellation, then we establish a condition on exogenous signals to guarantee persistency of excitation of the regressor, and hence the exponential stability of the ideal system. Section 4 treats the actual system as a perturbed system by a disturbance term which is both state and parameter dependent, and establishes robust stability. Section 5 establishes the convergence of parameter error to a small neighborhood of zero at sufficiently slow adaptation. In the last section, some experimental results are presented which demonstrate a 32db reduction of torque-ripple at the pole frequency.

2. Modeling of Permanent Magnet Step Motor

A full model of a motor would consist of the electrical dynamics of the stator coils together with the shaft mechanical dynamics. However, the electric response is much faster than the mechanical response which allows us to consider the mechanical dynamics only. This approximation is further justified by the use of current amplifiers and the interest in low speed direct-drive applications. Additional assumptions used here include linear magnetic circuit and symmetry between the two phases.

With these assumptions, we can describe the mechanical component of the motor dynamics by the following equation

$$J\,\frac{d\omega}{dt} + T_l = T_m, \tag{2.1}$$

where J is moment of inertia, $\omega = \dot{\theta}$ is angular velocity, T_l load torque and friction and T_m is the induced magnetic torque. With linear magnetic materials, the coenergy, W', and energy, W, in the magnetic field are equal and can be written as:

$$W' = \frac{1}{2}\, i^t\, L\, i, \tag{2.2}$$

where $i = (i_a, i_b, i_f)^t$. Here i_a and i_b are the winding currents in phase a and phase b respectively, and $i_r = i_f$ is a fictitious rotor current provided by the permanent magnet. The inductance L in equation (2.2) is of the following form

$$L = \begin{bmatrix} L_{aa} & L_{ab} & L_{af} \\ L_{ab} & L_{bb} & L_{bf} \\ L_{af} & L_{bf} & L_{ff} \end{bmatrix}. \tag{2.3}$$

Then the induced magnetic torque T_m is the derivative of the co-energy W' with respect to rotor position:

$$T_m = \frac{\partial W'}{\partial \theta} = \frac{1}{2}\, i^t\, \frac{\partial L}{\partial \theta}\, i. \tag{2.4}$$

All the entries of the inductance matrix are periodic functions of rotor position θ. The basic frequencies of each element can be easily deduced from the symmetries of the motor and verified experimentally. Denoting the pole frequency by p, we can express the inductances as

$$\begin{aligned} L_{aa} &= L_0 + L_1\cos(2p\,\theta); \\ L_{bb} &= L_0 - L_1\cos(2p\,\theta); \\ L_{ab} &= -\frac{L_0}{2} + L_1\sin(2p\,\theta); \\ L_{af} &= L_{m0} + \sum_{j=1}^{n} L_{mj}\cos(jp\,\theta); \\ L_{bf} &= L_{m0} + \sum_{j=1}^{n} L_{mj}\sin(jp\,\theta); \\ L_{ff} &= L_{f0} + \sum_{j=4}^{n} L_{f1}\cos(jp\,\theta); \end{aligned} \tag{2.5}$$

where the upper limit, n, of the summation depends on the number of frequency components of the torque-ripple we wish to model and cancel. Following a standard approach, we first use the so-called d-q transforma-

CH2917-3/90/0000-3319$1.00 © 1990 IEEE

tion which transforms from the natural stator frame to a decoupled quadrature frame fixed to the rotor. The transformed decoupled and quadrature currents i_d and i_q are defined by:

$$\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} \cos p\theta & -\sin p\theta \\ \sin p\theta & \cos p\theta \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}. \tag{2.6}$$

The "decoupled" current i_d is so named since it is not related to torque production. Substituting equation (2.6) and (2.5) into (2.4) yields

$$T_m = -Ki_q + K_1 i_d i_q + i_d \sum_{j=1}^{n} K_{sdj}\sin jp\theta + K_{cdj}\cos jp\theta$$
$$+ i_q \sum_{j=1}^{n} K_{sqj}\sin jp\theta + K_{cqj}\cos jp\theta + i_f^2 \sum_{j=4}^{n} K_{fj}\sin jp\theta \tag{2.7}$$

where all the K's are constant parameters defined in terms of constants in (2.5). Since i_d does dot affect torque production, we set it to zero for maximum power efficiency. Then, the d–q transformation will result in a dynamic equation with only one input, i_q, which is to be determined in the next section. Also, the transformation in equation (2.6) determines i_a and i_b resulting in sinusoidal commutation. Substituting equation (2.7) into (2.1) yields the final dynamics equation for the motor:

$$\ddot{\theta} = k_0 i_q + i_q \sum_{j=1}^{n} k_{sj}\sin jp\theta + k_{cj}\cos jp\theta - t_l - \sum_{j=4}^{n} t_j \sin jp\theta \tag{2.8}$$

where k_0 is the nominal torque constant, and all other k's and t's are combinations of the previous coefficients in an obvious way. All those terms with sines and cosines are present due to geometric imperfection. Neglecting these and friction yields the ideal motor model. Equation (2.8) is the model used in deriving our adaptive controller for step motor. We make the reasonable assumptions that those non-ideal terms are bounded and that the nominal torque constant dominates the torque constant variations, i.e., $\left| \sum_{j=1}^{n} k_{sj}\sin jp\theta + k_{cj}\cos jp\theta \right| \ll k_0$.

3. Adaptive Control Laws and Basic Exponential Stability

In this section, we first derive an adaptive control scheme for torque-ripple cancellation using a Lyapunov approach. The ideal motor model is then used to establish a basic exponential stability result which is augmented in section 4. This result is obtained by taking advantage of the special regressor structure yielding conditions on exogenous signals to guarantee persistency of excitation.

Although the motor equation of (2.8) can be written in the form

$$\ddot{\theta} = f(\theta) + g(\theta) i_q,$$

with $g(\theta)$ and $f(\theta)$ bounded and periodic in θ, we do not parametrize f and g separately as is common in the nonlinear adaptive control literature. Instead, we isolate the "ideal motor part" in (2.8) and denote those non-ideal torque-ripple terms by $q(\theta)$. Hence, we rewrite (2.8) as

$$\ddot{\theta} = k_0 i_q + q(\theta), \tag{3.1}$$

where, in terms of f and g, $q(\theta) = (g(\theta) - k_0)i_q + f(\theta)$ and k_0 is the d.c. component of g.

Our goal is to achieve smooth motion by cancelling the term $q(\theta)$. The idea behind our control law is extremely simple. In order to cancel $q(\theta)$, we intentionally add some ripple to the input current to cancel the torque-ripple. Since the actual torque-ripple is taken as unknown, we choose a set of shape functions (which will become the regressor vector) and adaptively tune the coefficients. Then we need to solve two problems: design an parameter update law to ensure the asymptotic cancellation of $q(\theta)$, and design a tracking controller which will ensure good performance during and after adaptation.

To implement this idea, we first define a regressor vector

$$w^t = (1, \sin p\theta, \cos p\theta, \sin 2p\theta, \cdots, \cos np\theta), \tag{3.2}$$

a parameter vector, which is to be tuned adaptively,

$$\bar{P}^t = (\bar{k}, \bar{k}_{s1}, \bar{k}_{c1}, \bar{k}_{s2}, \cdots, \bar{k}_{cn}).$$

and let

$$i_q = \frac{1}{k_0}(v - w^t\bar{P}), \tag{3.3}$$

where $-w^t\bar{P}$ is the ripple added to the current, and v is the new control input. Substituting equation (3.3), equation (3.1) becomes

$$\ddot{\theta} = v + q(\theta) - w^t\bar{P}. \tag{3.4}$$

Next, we need to design a tracking controller to ensure good performance during and after adaptation. Let θ_d, $\dot{\theta}_d$ and $\ddot{\theta}_d$ be a bounded desired trajectory. We can choose a PD control

$$v = \ddot{\theta}_d + k_d(\dot{\theta}_d - \dot{\theta}) + k_p(\theta_d - \theta) \tag{3.5}$$

with $k_p, k_d > 0$ to ensure exponential tracking in the ideal case where $\ddot{\theta} = v$. Substituting (3.5) into (3.4) yields the error dynamics

$$\ddot{e} + k_d\dot{e} + k_p e + q(\theta) - w^t\bar{P} = 0, \tag{3.6}$$

where $e = \theta_d - \theta$ is the output error. Observe that in the absence of $q(\theta) - w^t\bar{P}$, $e \to 0$ exponentially. Setting $q(\theta) = 0$ gives the error dynamics of the ideal motor with parameter adaptation

$$\ddot{e} + k_d\dot{e} + k_p e + w^t\bar{P} = 0. \tag{3.7}$$

Finally, we design the parameter update law using a Lyapunov approach assuming $q(\theta) = 0$. Choose a Lyapunov function candidate

$$V = (\dot{e} + k_\alpha e)^2 + k_\beta e^2 + \bar{P}^t\Gamma^{-1}\bar{P}, \tag{3.8}$$

where Γ is a symmetric positive definite adaptation gain matrix (typically diagonal or simply γI; $\gamma > 0$), and k_α and k_β to be specified later. The derivative of V along solutions of equation (3.7) is

$$\dot{V} = -2(k_d - k_\alpha)\dot{e}^2 - 2k_\alpha k_p e^2 + 2(k_\beta + k_\alpha^2 - k_\alpha k_d - k_p)e\dot{e}$$
$$+ 2\bar{P}^t(\Gamma^{-1}\dot{\bar{P}} + (\dot{e} + k_\alpha e)w).$$

Choosing

$$k_\beta = k_p + k_\alpha k_d - k_\alpha^2 \tag{3.9}$$

and the adaptation law

$$\dot{\bar{P}} = -(\dot{e} + k_\alpha e)\Gamma w \tag{3.10}$$

leads to

$$\dot{V} = -2(k_d - k_\alpha)\dot{e}^2 - 2k_\alpha k_p e^2 \tag{3.11}$$

which will be negative semi-definite if k_α satisfies

$$k_d > k_\alpha > 0. \tag{3.12}$$

Remark 1: We will call equation (3.7) together with (3.10) the ideal system, and equation (3.6) together with (3.10) the perturbed system.

Remark 2: If $q(\theta)$ is in the range of w^t (a sufficient condition for this is that i_q is constant and f and g have finite number of spectral lines), then (3.6) will also be of the form (3.7) with $\bar{P}$ replaced by $\bar{P} - P^*$ where P^* contains the Fourier coefficients of $q(\theta)$.

Remark 3: The way we proceed from here is as follows. We first establish the exponential stability of the ideal system. Next we use a robustness analysis to establish the boundedness of all the internal signals in the perturbed system in section 4. Once we have the boundedness, we can define the desired parameter and study the parameter convergence of the perturbed system in section 5.

Lemma 3.1: In the ideal system described by (3.7) and (3.10)

i) the zero solution is globally stable in the sense of Lyapunov,

ii) $w^t\bar{P}$ is bounded,

iii) e and $\dot{e} \to 0$ as $t \to \infty$.

Proof: Choose positive definite V as defined by equations (3.9), (3.8) and (3.12). By (3.11), its derivative along the solution of (3.7) and (3.10) satisfies

$$\frac{d}{dt}V \le 0. \tag{3.13}$$

This implies i).

Due to the positivity of V, (3.13) immediately yields

$$V(0) \ge V(t) \ge 0.$$

Therefore, (3.8) $\Longrightarrow$ $e, \dot{e}, \bar{P} \in L^\infty$.

Since $\|w(t)\| \le \|(1, 1, ..., 1)\|$, $w^t\bar{P} \in L^\infty$. Thus, ii) holds.

With these, (3.7) $\Longrightarrow$ $\ddot{e} \in L^\infty$. Thus,

$$\frac{d}{dt}e, \frac{d}{dt}\dot{e} \in L^\infty \text{ and } e, \dot{e} \text{ are uniformly continuous.} \tag{3.14}$$

On the other hand, from equation (3.11), we have

$$2(k_d - k_\alpha)\int_0^T \dot{e}^2 dt + 2k_\alpha k_p \int_0^T e^2 dt = V(0) - V(T) \le V(0).$$

Hence, $e, \dot{e} \in L^2$. This and (3.14) $\Longrightarrow$ iii). □

Our next goal is to establish exponential stability of the ideal system (3.7) and (3.10). To do this, we first set up a condition on the exogenous signals to guarantee that the motor will have a minimum speed which, in turn, ensures the PE condition of the regressor.

Lemma 3.2: Given $\omega_m > 0$, there exist $T > 0$, ω_{dm}, ω_{dM}, ω_M, with $\omega_{dM} \ge \omega_{dm} > 0$ and $\omega_M \ge \omega_m > 0$ such that $\omega_{dM} \ge \dot{\theta}_d \ge \omega_{dm}$ $\forall t$ $\Longrightarrow$ $\omega_M \ge \dot{\theta} \ge \omega_m$ $\forall t \ge T$ if $w^t\bar{P}$ is bounded.

Proof: In equation (3.7), consider $w^t \bar{P}$ as the input and $\dot{e}$ as the output. Then, the corresponding transfer function is given by

$$H(s) = \frac{s}{s^2 + k_d s + k_p}.$$

Since H is exponentially stable, for given $\varepsilon_i > 0$, there exists $T > 0$ such that the contribution of initial conditions to $\dot{e}$ will be less than ε_i after T. Let ε_t be the bound on $|w^t \bar{P}|$, then $\forall t \geq T$

$$|\dot{\theta} - \dot{\theta}_d| = |\dot{e}| \leq \|H\|_\infty \varepsilon_t + \varepsilon_i. \qquad (3.15)$$

For given $\omega_m > 0$, choose

$$\omega_{dm} = \|H\|_\infty \varepsilon_t + \varepsilon_i + \omega_m > 0 \qquad (3.16)$$

any $\omega_{dM} \geq \omega_{dm}$ and

$$\omega_M = \|H\|_\infty \varepsilon_t + \varepsilon_i + \omega_{dM}. \qquad (3.17)$$

Then, from equation (3.15)

$$\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm} \quad \forall t \implies \omega_M \geq \dot{\theta} \geq \omega_m \quad \forall t \geq T. \ \square$$

Lemma 3.3: If $\omega_M \geq \dot{\theta} \geq \omega_m > 0$ for $t \geq T$ then w is PE $\forall t \geq T$.

Proof: We must establish the existence of α, $\beta > 0$ and T_0 such that

$$\alpha I \leq \int_t^{t+T_0} w w^t \, d\tau \leq \beta I.$$

Choose $T_0 = \frac{2\pi}{p\,\omega_m}$. Since $\dot{\theta} \geq \omega_m$, for $t \geq T$ we have

$$\theta(t + T_0) - \theta(t) = \int_t^{t+T_0} \dot{\theta} \, dt \geq \omega_m T_0 = \frac{2\pi}{p}.$$

Then

$$\int_t^{t+T_0} ww^t \, dt = \int_{\theta(t)}^{\theta(t+T_0)} ww^t \frac{1}{\dot{\theta}} \, d\theta$$

Since $\dot{\theta} \leq \omega_M$, we have

$$\int_t^{t+T_0} ww^t dt \geq \frac{1}{\omega_M} \int_{\theta(t)}^{\theta(t+T_0)} ww^t d\theta \geq \frac{1}{\omega_M} \int_{\theta(t)}^{\theta(t)+\frac{2\pi}{p}} ww^t d\theta \qquad (3.18)$$

where the last inequality follows from the fact that $ww^t \geq 0$ and that $\theta(t + T_0) \geq \theta(t) + \frac{2\pi}{p}$. In substituting w from equation (3.2) into (3.18), we obtain

$$\int_t^{t+T_0} ww^t dt \geq \frac{1}{\omega_M} \int_0^{\frac{2\pi}{p}} \begin{bmatrix} 1 & \sin p\theta & .. & \cos np\theta \\ \sin p\theta & \sin^2 p\theta & .. & \sin p\theta \cos np\theta \\ \cos p\theta & \sin p\theta \cos p\theta & .. & .. \\ .. & .. & .. & .. \\ \cos np\theta & .. & .. & \cos^2 np\theta \end{bmatrix} d\theta$$

$$= \frac{1}{\omega_M} \begin{bmatrix} \frac{2\pi}{p} & 0 & .. & 0 \\ 0 & \frac{\pi}{p} & .. & 0 \\ .. & .. & .. & .. \\ 0 & 0 & .. & \frac{\pi}{p} \end{bmatrix} \geq \frac{\pi}{p\,\omega_M} I_{2n+1}.$$

Similarly we can show that

$$\therefore \quad \int_t^{t+T_0} w w^t \, d\tau \leq \frac{2\pi}{p\,\omega_m} \left\lceil \frac{\omega_M}{\omega_m} \right\rceil I_{2n+1}.$$

$\therefore \quad w$ is PE on $[T, \infty)$. $\square$

We are now ready to present the main result of this section.

Theorem 3.1: Consider the system described by (3.7) and (3.10), there exist $\omega_{dM} \geq \omega_{dm} > 0$ such that $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm}$, $\forall t \implies e, \dot{e}, \bar{P} \to 0$ exponentially with convergence rate $\gamma k + O(\gamma^2)$ with some $k > 0$ as $t \to 0$.

Proof: Let $\varepsilon = \dot{e} + k_\alpha e$, and for simplicity, $\Gamma = \gamma I_3$, then our parameter update law becomes

$$\dot{\bar{P}} = -\gamma w \varepsilon.$$

Considering $w^t \bar{P}$ as an input in (3.7) and solving for ε, we can obtain

$$\varepsilon = H_e(w^t \bar{P}),$$

where

$$H_e(s) = \frac{s + k_\alpha}{s^2 + h_d s + k_p}. \qquad (3.20)$$

For k_α satisfying (3.13), we have

$$\mathrm{Re}(H_e(j\omega)) = \frac{k_\alpha k_p + (k_d - k_\alpha)\omega^2}{(k_p - \omega^2)^2 + \omega^2 k_d^2} > 0 \quad \forall \omega \in \mathbf{R} \qquad (3.21)$$

and so H_e is strictly positive real (SPR) by [12]. Using standard results [13] on exponential stability of adaptive systems, theorem 3.1 will be proved if w is PE.

To show this, fix a minimum speed $\omega_m > 0$. Then, we can choose $\dot{\theta}_{dm}$ according to equation (3.16) and $\omega_{dM} \geq \omega_{dm}$. By Lemma 3.2, there exist T and ω_M as in (3.17) such that $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm} > 0 \ \forall t \implies \omega_M \geq \dot{\theta} \geq \omega_m \ \forall t \geq T$. This, by Lemma 3.3, implies that w is PE $\forall t \geq T$ and the proof is completed. $\square$

4. Robustness to Modeling errors and Disturbances

In last section, we established the exponential stability of the ideal system described by (3.7) with parameter update law (3.10). Now the stability of the perturbed system (3.6) with the same update law can be considered as a robustness problem. By a result by Vidyasagar and Vannelli [14], an exponentially stable system has a finite input-output gain which is, roughly speaking, inversely proportional to the convergence rate. We can consider the perturbation as a block connected in parallel to the ideal system as in the small gain theorem [15]. If the product of the gain of the disturbance and that of the ideal system is less than one, stability of the perturbed system is guaranteed.

Unfortunately an extra complication arises in slow adaptation. Since, by theorem 3.1, the convergence rate of our ideal system is $\gamma k + O(\gamma^2)$, the input-output gain of our ideal system is almost inversely proportional to γ. Hence, if γ becomes small, the system gain becomes large, eventually causing the violation of the small gain theorem condition for a fixed form of disturbance term.

However, after a close examination of the structure of the adaptive system, we find out that this may not be always the case. This is due to the fact that we actually have two subsystems in the adaptive system: the fast error system and the slow parameter system. If the disturbances enter only the error system, the amount of disturbance that can be tolerated is roughly independent of the adaptation gain in slow adaptation. This will be made more precise by the following result.

Theorem 4.1: Consider the following linear adaptive system with nonlinear regressor and disturbance

$$\dot{x} = Ax + b(u + w^t \bar{P}) \qquad (4.1)$$

$$\dot{\bar{P}} = -\gamma w c^t x \qquad (4.2)$$

$$u = H_u(x, \bar{P}) + r \qquad (4.3)$$

where A, b, c are constant matrices and H_u is a nonlinear operator. Suppose that:

i) $H_e(s) = c^t(sI - A)^{-1} b$ is SPR;

ii) w is PE;

iii) H_u has bounded gain, that is, $\exists K_x, K_{\bar{P}}$, such that

$$\|H_u(x, \bar{P})\| \leq K_x \|x\| + k_{\bar{P}} \|\bar{P}\|. \qquad (4.4)$$

Under these conditions, if

$$\frac{m_x}{\rho_x}(\|c\| + \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \|cw^t\| \|H_e\|) K_x + \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \|H_e\| K_{\bar{P}} < 1 \quad (4.5)$$

where $\rho_x, \rho_{\bar{P}}, m_x$ and $m_{\bar{P}}$ are defined in the proof, then r bounded $\implies$ x, u and $\bar{P}$ are bounded.

Proof: Since H_e is SPR, A, b, c satisfy the positive real lemma [13]. Choose $V = x^t P x$ where $P > 0$ is as in the positive real lemma. Then its derivative along the solution of (4.1) is

$$\frac{d}{dt} V = x^t (A^t P + PA) x + 2x^t Pb(w^t \bar{P} + u)$$

$$= x^t(-2\rho_x I - QQ^t)x + 2x^t c(w^t \bar{P} + u)$$

$$\leq -2\rho_x \|x\|^2 + 2\|x\| \|c(w^t \bar{P} + u)\|$$

$$\leq -2\frac{\rho_x}{\bar{\sigma}(P)} V + \frac{2}{\underline{\sigma}^{1/2}(P)} V^{1/2} \|c(w^t \bar{P} + u)\|,$$

where ρ_x is strictly positive, and $\bar{\sigma}(P)$ and $\underline{\sigma}(P)$ are the maximum and minimum singular values of P respectively.

Let $v = V^{1/2}$. Then, we immediately have

$$\frac{d}{dt} v \leq -\frac{\rho_x}{\bar{\sigma}(P)} v + \frac{1}{\underline{\sigma}^{1/2}(P)} \|c(w^t \bar{P} + u)\|,$$

which yields

$$\|v(t)\| \leq v(0) + \frac{1}{\rho_x} \frac{\bar{\sigma}(P)}{\underline{\sigma}^{1/2}(P)} \|c(w^t \bar{P} + u)\| d\tau.$$

Since $\|x\| \le \frac{1}{\underline{\sigma}^{1/2}(P)} \|v\|$, we have

$$\|x\| \le \frac{m_x}{\rho_x} \|c\| \, \|u\| + \frac{m_x}{\rho_x} \|cw\| \, \|\bar{P}\| + \beta_x, \qquad (4.6)$$

where

$$m_x = \frac{\bar{\sigma}(P)}{\underline{\sigma}(P)} \quad \text{and} \quad \beta_x = \frac{1}{\underline{\sigma}^{1/2}(P)} v(0).$$

Now using (4.1), (4.2) can be rewritten as

$$\dot{\bar{P}} = -\gamma w H_e(w^t \bar{P}) - \gamma w H_e(u). \qquad (4.7)$$

Since H_e is SPR, and w is PE,

$$\dot{\bar{P}} = -\gamma w H_e(w^t \bar{P}) \qquad (4.8)$$

is exponentially stable with convergence rate $\gamma k + O(\gamma)$. By the converse Lyapunov theorem, there exists a Lyapunov function $V(t, \bar{P})$ such that

$$\|\bar{P}\|^2 \ge V(t, \bar{P}) \ge \alpha_1 \|\bar{P}\|^2, \qquad (4.9)$$

$$\frac{d}{dt} V(t, \bar{P}) \mid_{(4.8)} \le -\alpha_2 \|\bar{P}\|^2,$$

$$\|\frac{\partial}{\partial \bar{P}} V(t, \bar{P})\| \le \alpha_3 \|\bar{P}\|,$$

for some positive α_i, $i = 1, 2, 3$. Since the choice of Lyapunov function is not unique, we assume that with the above choice, α_2 will reflect the convergence rate of (4.8). That is, $\alpha_2 = \gamma k + O(\gamma^2)$. Therefore, we can choose $\rho_{\bar{P}} = \frac{1}{2}(k - \varepsilon)$ for some small ε such that

$$2\gamma\rho_{\bar{P}} \le \gamma k + O(\gamma^2) = \alpha_2$$

for small γ. Differentiating V along the solutions of (4.7) leads to

$$\frac{d}{dt} V(t, \bar{P}) \mid_{(4.7)} = \frac{d}{dt} V(t, \bar{P}) \mid_{(4.8)} - \frac{\partial}{\partial \bar{P}} V(t, \bar{P}) \gamma w H_e(u),$$

$$\le -\alpha_2 \|\bar{P}\|^2 + \alpha_3 \|\bar{P}\| \, \|\gamma w H_e(u)\|$$

$$\le -2\gamma\rho_{\bar{P}} \|\bar{P}\|^2 + \alpha_3 \|\bar{P}\| \, \|\gamma w H_e(u)\|$$

$$\le -2\gamma\rho_{\bar{P}} V(t, \bar{P}) + \frac{\alpha_3}{\alpha_1^{1/2}} V^{1/2} \|\gamma w H_e(u)\|.$$

setting $v \triangleq V^{1/2}$ yields

$$\frac{d}{dt} v \le -\gamma\rho_{\bar{P}} v + \gamma \frac{\alpha_3}{2\alpha_1^{1/2}} \|w\| \, \|H_e\| \, \|u\|,$$

and hence

$$0 \le v \le v(0) + \frac{1}{\rho_{\bar{P}}} \frac{\alpha_3}{2\alpha_1^{1/2}} \|w\| \, \|H_e\| \, \|u\|.$$

Using (4.9) and the definition of v, we obtain

$$\|\bar{P}\| \le \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \, \|H_e\| \, \|u\| + \beta_{\bar{P}} \qquad (4.10)$$

with $m_{\bar{P}} \triangleq \frac{\alpha_3}{2\alpha_1}$ and $\beta_{\bar{P}} \triangleq \frac{1}{\alpha_1^{1/2}} v(0)$. Substituting this into (4.6) gives

$$\|x\| \le \frac{m_x}{\rho_x}(\,\|c\| + \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|cw\| \, \|w\| \, \|H_e\|)\,\|u\| + \frac{m_x}{\rho_x} \beta_{\bar{P}} + \beta_x. \quad (4.11)$$

Furthermore, from (4.3) and (4.4), we have

$$\|u\| \le \|r\| + \|H_u(x, \bar{P})\| \le \|r\| + K_x \|x\| + K_{\bar{P}} \|\bar{P}\|. \qquad (4.12)$$

Call the right hand side of (4.5) γ_1. Substituting (4.10) and (4.11) into (4.12) results in

$$\|u\| \le \gamma_1 \|u\| + \|r\| + K_{\bar{P}} \beta_{\bar{P}} + K_x \beta_x + \frac{m_x}{\rho_x} K_x \beta_{\bar{P}}.$$

Since $\gamma_1 < 1$, $1 - \gamma_1 > 0$. Hence from the above inequality, if r is bounded, we immediately have u bounded and the bound is

$$\|u\| \le (1 - \gamma_1)^{-1} \{ \; \|r\| + K_{\bar{P}} \beta_{\bar{P}} + K_x \beta_x + \frac{m_x}{\rho_x} K_x \beta_{\bar{P}} \; \}. \quad (4.13)$$

Denoting the right hand side of (4.13) by γ_2 and substituting (4.13) into (4.10) and (4.11) yields

$$\|x\| \le \frac{m_x}{\rho_x}(\,\|c\| + \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \, \|cw^t\| \, \|H_e\|)\gamma_2 + \frac{m_x}{\rho_x} \|cw^t\| \, \beta_{\bar{P}} + \beta_x$$

$$\|\bar{P}\| \le \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \, \|H_e\| \, \gamma_2 + \beta_{\bar{P}}.$$

This completes the proof. □

Theorem 4.2: Consider the error dynamics (3.6) with parameter update law (3.10). If $\omega_M \ge \dot{\theta} \ge \omega_m$, then there exists $\varepsilon^* > 0$ such that $\frac{\|g(\theta) - k_0\|}{k_0} < \varepsilon^* \implies$ boundedness of internal signals in the perturbed system.

Proof: Equation (3.6) can be rewritten in the form of (4.1) with

$$x^t = (e \;\; \dot{e}), \quad A = \begin{bmatrix} 0 & 1 \\ -k_p & -k_d \end{bmatrix}, \quad b^t = (0 \;\; 1), \quad c^t = (\alpha \;\; 1),$$

and

$$u = q(\theta) = (g(\theta) i_q + f(\theta)$$

$$= \frac{g(\theta) - k_0}{k_0}(\ddot{\theta}_d + k_d \dot{e} + k_p e + w^t \bar{P}) + f(\theta)$$

$$= \frac{g(\theta) - k_0}{k_0} \ddot{\theta}_d + f(\theta) + \frac{g(\theta) - k_0}{k_0}(k_p \;\; k_d)x + \frac{g(\theta) - k_0}{k_0} w^t \bar{P}.$$

Hence, u is of the form of equation (4.3) with

$$r \triangleq \frac{g(\theta) - k_0}{k_0} \ddot{\theta}_d + f(\theta).$$

By the assumptions on f, g, and $\ddot{\theta}$, r is bounded, and so are K_x and $K_{\bar{P}}$ defined as follows

$$K_x = \frac{\|g(\theta) - k_0\|}{k_0} \|(k_p \;\; k_d)\|$$

$$K_{\bar{P}} = \frac{\|g(\theta) - k_0\|}{k_0} \|w\|.$$

Hence, u satisfies condition iii) in Theorem 4.1. The SPR condition is met is due to (3.21) in last section. Also, Lemma 3.3 and the condition on θ in this theorem ensures that the PE conditions will be satisfied. Thus, Theorem 4.1 applies.

Note that both K_x and $K_{\bar{P}}$ have the common factor $\frac{\|g(\theta) - k_0\|}{k_0}$. Substituting them into (4.5) and taking out the common factor yield

$$\frac{\|g(\theta) - k_0\|}{k_0} \{ \; \frac{1}{\rho_x}(\,\|c\| + \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \, \|cw^t\| \, \|H_e\|\,) \, \|(k_p \;\; k_d)\|$$

$$+ \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \, \|H_e\| \, \|w\| \; \} < 1.$$

Defining

$$\varepsilon^* \triangleq \{ \; \frac{m_x}{\rho_x}(\,\|c\| + \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\| \, \|cw^t\| \, \|H_e\|\,) \, \|(k_p \;\; k_d)\|$$

$$+ \frac{m_{\bar{P}}}{\rho_{\bar{P}}} \|w\|^2 \, \|H_e\| \; \}^{-1},$$

leads to $\frac{\|g(\theta) - k_0\|}{k_0} < \varepsilon^* \implies (4.5) \implies$ stability. □

Note that in the above theorem we assumed the condition on $\dot{\theta}$ in lemma 3.3 directly, whereas in theorem 3.1 a condition on exogenous signals is assumed. The reason is that lemma 3.2 cannot be used directly due to the presence of the term q. But with the theorem in the above, we can restate lemma 3.2 although the bounds ω_{dm} and ω_{dM} are now different. This implies that the PE condition can still be established with conditions on exogenous signals.

There are two basic ideas in the proof. First, in a small initial interval everything is bounded and we can choose the reference signal to accelerate the motor to a minimum speed. Second, once the motor is spinning, the regressor is PE, the parameter vector will be bounded and we can still choose the reference to maintain the minimum speed. Therefore, the proof parallels that of lemma 3.2 except that now we should have $\varepsilon_t = \max \{ \varepsilon_{t1}, \varepsilon_{t2} \}$, where ε_{t1} is the bound on $w^t \bar{P}$ - $q(\theta)$ during an initial interval, and ε_{t2} that by assuming theorem 4.2.

5. Parameter Convergence in Slow Adaptation

In our experiment we observed that the residual torque ripple is very small with slow adaptation. This indicates the convergence of parameter errors to a small neighborhood of zero. Motivated by this observation, we provide a mathematical treatment of parameter convergence in slow adaptation.

Lemma 5.1: Let

$$Q(t,T) = \frac{1}{T}\int_t^{t+T} w(\tau)\, H(w^t)(\tau)\, d\tau,$$

where $w: R_+ \to R^n$, and $H(s)$ is proper rational. If H is SPR, w is PE, stationary and bounded, then there exists T^* such that for $T > T^*$, the solution of

$$\dot{x}(t) = -Q(t,T)x(t); \qquad x(0) = x_0$$

converges to zero exponentially $\forall x_0 \in R^n$.

The proof uses generalized harmonic analysis and is omitted here due to space limitation. The proof of this and the following corollary can be found in [16].

Corollary 5.1: There exists T^* such that $\forall T > T^*$, $Q(t,T) + Q^t(t,T)$ is positive definite. In fact, T^* can be chosen such that

$$Q(t,T) + Q^t(t,T) \geq \frac{\sigma\alpha}{2T_0} I > 0. \tag{5.1}$$

Now if we re-examine the proof of Theorem 4.1, we notice that the bound on $\bar{P}$ is actually dependent on the correlation between the regressor and the disturbance term. Intuitively, the updating of the parameters acts to reduce the correlation until the filtered residual error is uncorrelated to the regressor. This motivates the following definition of the desired parameter.

Suppose the following two limits exist

$$r \triangleq \lim_{T\to\infty} \frac{1}{T}\int_0^T wH_e(q(\theta(t)))\, dt, \tag{5.2}$$

$$Q \triangleq \lim_{T\to\infty} \frac{1}{T}\int_0^T wH_e(w^t)\, dt. \tag{5.3}$$

By corollary 5.1, $Q(t,T) + Q(t,T)^t$ is positive definite when w is PE. Since Q is the limit of $Q(t,T)$, its symmetric part is also positive. Hence, Q is non-singular. Then, we define the desired parameter as

$$P^* \triangleq -Q^{-1} r. \tag{5.4}$$

Since H_e is linear and P^* is constant, we immediately have

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T wH_e(q(\theta) + w^t P^*)\, dt = 0. \tag{5.5}$$

Note that $q(\theta) - w^t P^*$ is the ideal (or desired) residual torque with the property that when filtered by H_e it is uncorrelated with w. In words, the above equation says that the desired parameter is such that the correlation between the regressor and the filtered residual torque-ripple is asymptotically eliminated. Hence, if we can show that $\bar{P}$ converges to a neighborhood of P^*, we will have good torque-ripple cancellation.

Theorem 5.1: Let P^* be defined as in equation (5.4). Suppose the conditions in Theorem 4.2 and Lemma 5.1 are satisfied. If the adaptation is slow enough (γ sufficiently small), $\bar{P}$ in the perturbed system of (3.7) and (3.10) will converge to a neighborhood of P^*. The radius of the neighborhood tends to zero as $\gamma \to 0$.

Proof: Let us denote the parameter error by ϕ, that is,

$$\phi = \bar{P} - P^*.$$

Since P^* is constant, $\dot{\phi} = \dot{\bar{P}}$. From (4.11), we have

$$\dot{\phi} = -\gamma w(\dot{e} + \alpha e) = -\gamma w H_e(w^t \bar{P} - q(\theta))$$
$$= -\gamma w H_e(w^t\phi + w^t P^* - q(\theta))$$
$$= -\gamma w H_e(w^t\phi) + \gamma w H_e(q(\theta) - w^t P^*).$$

We first define an average value of ϕ and show that it converges to a small neighborhood of zero.

By equation (5.5), given $\varepsilon > 0$, there exists T^* such that $T > T^* \Longrightarrow$

$$\left| \frac{1}{T}\int_{t_0}^{t_0+T} w\, H_e(q(\theta) - w^t P^*)\, dt \right| \leq \varepsilon \quad \forall t_0. \tag{5.6}$$

Choosing T such that both (5.6) and the condition in corollary 5.1 are satisfied, we consider the averaged value on an interval of ϕ given by

$$\phi_{av}(t) = \frac{1}{T}\int_t^{t+T} \phi(\tau)\, d\tau.$$

Then

$$\dot{\phi}_{av}(t) = \frac{1}{T}(\phi(t+T) - \phi(t)) = \frac{1}{T}\int_t^{t+T} \dot{\phi}(\tau) d\tau$$

$$= \frac{1}{T}\int_t^{t+T} -\gamma w H_e(w^t \dot{\phi}) d\tau + \frac{1}{T}\gamma \int_t^{t+T} wH_e(q - w^t P^*) d\tau. \tag{5.7}$$

Using the Swapping Lemma [17]

$$wH_e(w^t \dot{\phi}) = wH_e(w^t)\dot{\phi} - a(\dot{\phi}) \tag{5.8}$$

where $a(\dot{\phi}) \triangleq H_c(H_b(w^t)\dot{\phi})$, H_c and H_b as in the Swapping Lemma.

$$|a(\dot{\phi})| \leq \|H_c\|\, \|H_b\|\, \|w\|\, \|\dot{\phi}\|$$
$$\leq \gamma \|H_c\|\, \|H_b\|\, \|H_e\|\, \|w\|^2\, \|\dot{e} + \alpha e\| \triangleq \gamma a$$

where $\dot{e} + \alpha e$ is bounded by theorem 4.2, thus a finite.

Substituting equation (5.8) into equation (5.7) yields

$$\dot{\phi}_{av}(t) = \frac{1}{T}\int_t^{t+T} -\gamma w H_e(w^t)\phi d\tau$$
$$+ \frac{1}{T}\int_t^{t+T} \gamma a(\dot{\phi}) d\tau + \frac{1}{T}\gamma \int_t^{t+T} wH_e(q(\theta) - w^t P^*) d\tau. \tag{5.9}$$

For $\tau \in [t, t+T]$, we can define $b(\tau)$ by

$$\phi(\tau) = \phi_{av}(t) + b(\tau). \tag{5.10}$$

Then $\dot{b}(\tau) = -\gamma w(\dot{e} + \alpha e)$ and $\int_t^{t+T} b(\tau) d\tau = 0$. Hence,

$$|b(\tau)| \leq \gamma \|w\|\, \|\dot{e} + \alpha e\| T \triangleq \gamma T b,$$

for finite b. Since $\phi_{av}(t)$ is constant relative to τ, using equation (5.10), equation (5.9) becomes

$$\dot{\phi}_{av}(t) = -\gamma \{ \frac{1}{T}\int_t^{t+T} wH_e(w^t) d\tau \}\, \phi_{av}(t)$$

$$+ \frac{1}{T}\int_t^{t+T} \gamma a(\dot{\phi}) - wH_e(w^t) b(\tau) d\tau + \frac{1}{T}\gamma \int_t^{t+T} wH_e(q - w^t P^*) d\tau$$

$$= -\gamma Q(t,T)\phi_{av}(t) + \frac{1}{T}\gamma \int_t^{t+T} wH_e(q - w^t P^*) d\tau$$

$$+ \frac{1}{T}\int_t^{t+T} \gamma a(\dot{\phi}) - wH_e(w^t) b(\tau) d\tau.$$

Choosing a simple Lyapunov function $V = \phi_{av}^t(t)\phi_{av}(t)$, we have

$$\dot{V} = -\gamma\, \phi_{av}^t(t) \{ Q(t,T) + Q^t(t,T) \}\, \phi_{av}(t)$$
$$+ 2\gamma\, \phi_{av} \{ \frac{1}{T}\int_t^{t+T} w\, H_e(q(\theta) - w^t P^*)\, dt \}$$
$$+ 2\gamma\, \phi_{av} \{ \frac{1}{T}\int_t^{t+T} a(\dot{\phi}) - wH_e(w^t) b(\tau) d\tau \}$$
$$\leq -2\gamma \frac{\sigma\alpha}{2T_0} \|\phi_{av}\| \{ \|\phi_{av}\| - \frac{4\varepsilon T_0}{\sigma\alpha} - \frac{4\gamma T_0 (a + Tb\|w\|^2 \|H_e\|)}{\sigma\alpha} \},$$

where the last inequality is obtained by using equation (5.1), (5.6) and the bounds of $a(\dot{\phi})$ and $b(\tau)$. It follows that $\|\phi_{av}(t)\|$ converges to a neighborhood of zero exponentially with rate $2\gamma\alpha\sigma/2T_2$. The size of the neighborhood is proportional to ε and γ, thus can be made small by choosing ε small (hence T large) and γ small. Furthermore, since

$$|\phi(\tau)| = |\phi_{av}(t) + b(\tau)|$$
$$\leq |\phi_{av}(t)| + |b(\tau)| \leq |\phi_{av}(t)| + \gamma T b,$$

$|\phi(\tau)|$ will also become small if γ is small. Hence, $\bar{P}$ converges to a neighborhood of P^*.

1. Experimental results

The adaptive motor controller defined by (3.5) and (3.10), was successfully implemented in our lab for motor speed control (i.e. $\dot{\theta}_d$ = constant). The motor used is a 90-pole axial-gap hybrid step motor with stator and rotor saliency (teeth) and the shaft angle is sensed with an optical quadrature encoder. The encoder has 5000 lines and therefore generates 20,000 pulses per revolution. The controller hardware consists of an IBM/AT, a twelve-bit digital-to-analog board, a quadrature counter interface, and a pulse-width modulated current amplifier. The computing speed of the IBM/AT allowed a 2ms sample time and the capability of adapting three parameters corresponding to the "DC torque-ripple" and a single torque-ripple harmonic of the pole frequency.

For purposes of comparison the torque ripple spectrum of the motor was measured without adaptation and is shown in Figure 1. The commutation waveforms were the "ideal" sinusoidal signals, but torque

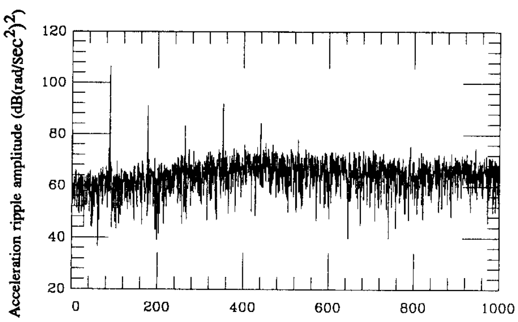

Figure 1. Acceleration-Ripple Spectrum with Sinusoidal Commutation and No Adaptation.

ripple is clearly seen at the pole frequency and its harmonics. This spectrum was produced with the following experimental procedure: First, a constant current i_q was applied to the motor with a low-gain velocity feedback loop to keep the motor running at roughly constant speed. Once a near steady-state motion was attained the motor current was, for our purposes, constant. Next, a series of shaft-angle measurements was stored at the sample rate. This series was interpolated and resampled uniformly in the spatial *shaft–angle* domain. Finally, 2048 samples corresponding to 2 revolutions of the motor were used to generate a (noisy) acceleration estimate which was FFT'd to generate Figure 1.

In a preliminary experiment, the adaptive control law was implemented with a low-gain PD controller to track a constant speed trajectory. The parameters are initialized to be zero, corresponding to sinusoidal commutation. After steady-state was attained, the same procedure was followed to generate the new torque ripple spectrum shown in Figure 2. Observe the dramatic reduction (32db) in the first torque-ripple harmonic. The reduction in the third harmonic cannot be explained by the adaptation since this harmonic was not adaptively cancelled. It appears that there is some unmodeled nonlinearities which couple the first and third harmonics. Finally, the initial adaptation process in time domain is illustrated by Figure 3. Note how the velocity ripple was gradually suppressed.

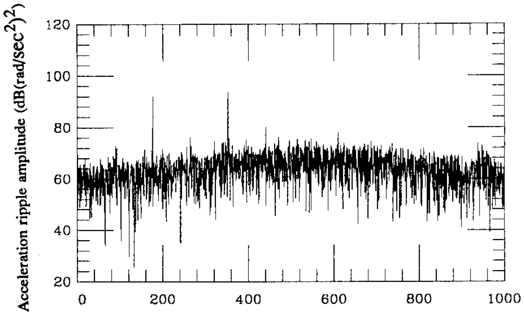

Figure 2. Acceleration-Ripple Spectrum after Adaptation at 2.3 Radians/Second.

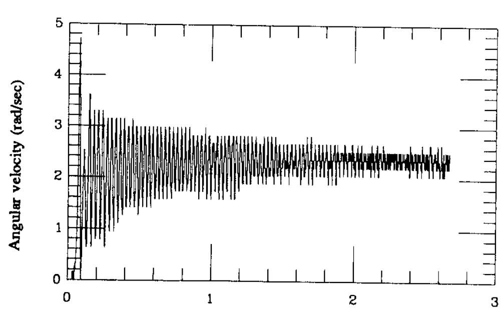

Figure 3. Time-Domain Plot of Velocity Showing Effect of Adaptation on Velocity Ripple.

7. Conclusion

In this paper we described a nonlinear adaptive control scheme which provides dramatic reduction in step-motor torque-ripple at frequencies specified by the control system designer. The scheme used is similar to that proposed by Sastry and Isidori [7] except for a nonlinear term in the state and parameter error that appears in the error dynamics. This term motivated our study of robustness to such nonlinear perturbations and we were able to show parameter convergence with this term present. This nonlinear term could have been eliminated had we parameterized torque-ripple in the two periodic function $f(\theta)$ and $g(\theta)$ separately, however persistency of excitation would have been lost.

The parameter convergence result of section 5 is illuminating though somewhat weak since it requires stationarity of signals internal to the nonlinear system and we do not yet have conditions on exogenous signals which guarantee this property. Ergodic theorems for nonlinear systems which would provide such conditions are not yet available but this is an active research area (see e.g. [18] and the references therein). Taking a more macroscopic perspective, it is intriguing to see that the apparently mundane problem of motor control leads us to fundamental mathematical questions.

Finally, the potential application of sophisticated control algorithms such as this one to motor control can not be underestimated. The rapid growth in microelectronics technology makes the use of new algorithms inevitable in the control of motors of all sizes.

References

1. H. Le-Huy and R. Perret, "Minimization of Torque Ripple in Brushless DC Motor Drives," *IEEE Trans. on Industrial Applications*, vol. 22, no. 4, pp. 748-755, July/Aug. 1986.
2. H. Nagase, T. Okuyama, J. Takahashi, and K. Saitoh, "A Method for Suppressing Torque Ripple of an AC Motor by Current Amplitude Control," *IEEE Trans. on Industry Electronics*, vol. 36, no. 4, pp. 504-510, Nov. 1989.
3. Y. Murai, Y. Kawase, K. Ohashi, K. Nagatake, and K. Okuyama, "Torque Ripple Improvement for Brushless DC Miniature motors," *IEEE Trans. on Industry Applications*, vol. 25, no. 3, pp. 441-449, 1989.
4. D.G. Taylor, M.J. Woolley, and M. Illic, "Design and Implementation of a Linearizing and Decoupling Feedback Transformation for Switched Reluctance Motors," *Proc. 17th Annual Symposium on Increamental Motion Control Systems and Devices*, pp. 173-184, June 1988.
5. N. Hemati and M.C. Leu, "Nonlinear Tracking Control of Brushless DC Motors for High-Performance Applications," *Proc. of the 28th IEEE Conference on Decision and Control*, vol. 1, pp. 527-530, Tempa, Florida, Dec. 1989.
6. R. Marino, S. Peresada, and P. Valigi, "Adaptive Partial Feedback Linearization of Induction Motors," *Proc. of IEEE CDC*, 1990.
7. S. Sastry and A. Isidori, "Adaptive control of linearizable systems," *IEEE TAC*, vol. 34 No. 11, pp. 1123-1131, November 1989.
8. R. Marino, I. Kannellakopoulos, and P. Kokotovic, "Adaptive Tracking for Feedback Linearizable SISO Systems," *Proc. of the 28th IEEE Conference on Decision and Control*, vol. 2, pp. 1002-1007, Tempa, Florida, Dec. 1989.
9. D.G. Taylor, P. V Kokotovic, R. Marino, and I. Kanellakopoulos, "Adaptive Regulation of Nonlinear Systems with Unmodelled Dynamics," *IEEE TAC*, vol. 34 No. 4, pp. 405-412, April 1989.
10. I. Kanellakopoulos, P.V. Kokotovic, and R. Marino, "Robustness of Adaptive Nonlinear Control under an Extended Matching Condition," *IFAC NOLCOS*, pp. 192-197, Capri, Italy, 1989.
11. J.-B. Pomet and L. Praly, "Adaptive Nonlinear Regulation: Equation Error From the Lyapunov Equation," *Proc. of the 28th IEEE Conference on Decision and Control*, vol. 2, pp. 1008-1013, Tempa, Florida, 1989.
12. P.A. Ioannou and G. Tao, "Frequency Domain Conditions for Strictly Positive Real Functions," *IEEE Trans. on Automatic Control*, vol. AC-32, pp. 53-54, 1987.
13. B.D.O. Anderson, R.R. Bitmead, C.R. Johson, Jr., P.V. Kokotovic, R.L. Kosut, I.M.Y. Mareels, L. Praly, and B.D. Riedle, *Stability of Adaptive Systems: passivity and averaging analysis*, the MIT Press, Cambridge, Massachusetts, 1986.
14. M. Vidyasagar and A. Vannelli, "New Relationships Between Input-Output and Lyapunov Stability," *IEEE Transactions on Automatic Control*, vol. AC-27, no. 2, pp. 481-483, April, 1982.
15. C.A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*, Academic Press, New York, 1975.
16. D. Chen and B. Paden, "Nonlinear Adaptive Torque Ripple Cancellation, Robustness and Parameter Convergence," *submitted to IEEE Trans. on Automatic Control.*
17. S. Sastry and M. Bodson, *Adaptive Control: stability, convergence, and robustness*, Prentice Hall, Englewood Cliffs, New Jersey, 1989.
18. W.M. Ruess and W.H. Summers, "Weak Almost Periodicity and the Strong Ergodic Limit Theorem for Periodic Evolution Systems," *to appear in Israel J. Math.*.