A TUNING ALGORITHM FOR DIGITALLY PROGRAMMABLE CONTINUOUS–TIME FILTERS[1]

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Abstract

A tuning algorithm applicable to digitally programmable continuous-time filters is presented. This algorithm is based upon estimates of filter characterization parameters based upon the results of a system identification and subsequently calculated adjustments of the modelled controlling components. Simulations show this algorithm converges to a solution after two iterations in the ideal case and attains good speed and accuracy even when parasitic effects are included in the system model. Simulations of the tuning of a 6th-order elliptic low-pass filter with 0.5dB passband ripple designed to have a normalized cutoff frequency at 1(rad/sec) indicate that 0.501dB passband ripple and 0.015% accuracy in the 3dB band-edge can be achieved in the presence of typical parasitics.

1. Introduction

Monolithic continuous-time filters usually suffer from degraded performance due to inherent process parameter variations and the presence of large and uncontrollable parasitic components. Tuning is essential when high precision filtering is required.

Conventional approaches to tuning continuous-time filters have used analog tuning loops associated with the master-slave techniques [1]-[3] or standard optimization techniques [4],[5]. The performance of circuits based upon master-slave structures suffer from imprecise matching of desired and parasitic components between the master and slave circuit. Standard optimization algorithms have been generally applied to deterministic tuning and thus suffer from parasitics as well as algorithmically induced local minimum convergence problems.

In digitally programmable analog filters, extreme precision and accuracy can be achieved with good tuning algorithms. In the context of this paper, digital tuning consists of three phases: measurement, system identification, and adjustment. Measurements will be made by using a low speed analog to digital converter which converts signals grabbed by a high speed sample and hold circuit. To identify a deterministic linear time-invariant system in a robust way, we will use frequency-domain methods in which a number of sinusoidal excitations are used to obtain least-squares estimates of the transfer function. An iterative complex least squares method is used for the frequency-domain system identification.

The central idea of this tuning algorithm is to estimate the model parameters of the OTAs(Operational Transconductance Amplifiers) of the digitally programmable continuous-time filter structure [6],[7] based on the reliable results of the system identification and then to calculate and adjust the contol voltages for the OTAs in such a way as to minimize the errors between the identified system and the desired system response.

2. Digitally Programmable Analog Filter Architecture

The basic digitally programmable continuous-time filter architecture [6],[7] we will be using is shown in Fig. 1. This structure has been selected specifically as a test vehicle for investigating the performance of digital tuning algorithms. The system consists of an analog bus, a digital bus, a local digital controller, a performance monitor and a number of digitally programmable biquadratic sections. This filter structure is capable of realizing any of the standard even order filter functions. The structure of each biquadratic block is shown in Fig. 2. The ideal transfer function of the block is given by

$$\frac{V_{out}}{V_{in}} = \frac{(B_{hp})s^2 + (\frac{g_{m4}-g_{m3}B_{bp}}{C_7})s + (\frac{g_{m1}g_{m3}+g_{m2}g_{m3}B_{lp}}{C_6C_7})}{s^2 + (\frac{g_{m5}}{C_7})s + \frac{g_{m2}g_{m3}}{C_6C_7}} \quad (1)$$

where the B variables can be 0 or 1 depending upon the switch settings. If the transfer function of the system identification model for each biquad is given by

$$T(s) = \frac{s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \quad (2)$$

which has only 4 degrees of freedom for identification instead of 5 in ordinary second order rational transfer functions. The coefficients are given respectively by

$$a_1 = \frac{g_{m5}}{C_7} \quad (3)$$

$$a_0 = \frac{g_{m2}g_{m3}}{C_6C_7} \quad (4)$$

$$b_1 = \frac{g_{m4} - g_{m3}B_{bp}}{C_7} \quad (5)$$

$$b_0 = \frac{g_{m1}g_{m3} + g_{m2}g_{m3}B_{lp}}{C_6C_7} \quad (6)$$

From (3)-(6), it follows that we can get independent or sequential adjustment of the system charaterization parameters. This ideal model for the system is not exact because the actual physical system will have over-ordering problems due to the parasitic poles of the OTAs as well as other layout parasitics. Actually, it is impossible to obtain the exact model including all parasitics since we do not know even the order of the physical system. The iterative complex least squares algorithm which will be addressed in the next section makes it possible to use a low order model for identifying the higher-order physical system. Thus, we can maintain near independence of adjustment of key system charaterization parameters even in the presence of significant parasitics.

3. Iterative Complex Least Squares Method

The rudiments of an iterative complex least squares system identification algorithm are discussed in this section. Additional details appear in [8]. Consider an analog filter that has transfer function

$$T(s) = \frac{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots + b_n s^n} = \frac{N(s)}{D(s)} \quad (7)$$

Various methods are available for measuring $T(j\omega_i)$, $i = 1, 2, \ldots, N(N > n + m + 1)$. With any measurement method, there will be unavoidable measurement errors. We denote the results of the measurements by $T_M(j\omega_i)$, $i = 1, 2, \ldots, N$. If we

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denote the complex estimation error at the i'*th* frequency as e_i, then

$$e_i = \frac{N(j\omega_i)}{D(j\omega_i)} - T_M(j\omega_i) \quad (8)$$

$$= \frac{1}{D(j\omega_i)}[N(j\omega_i) - D(j\omega_i)T_M(j\omega_i)] \quad (9)$$

An iterative identification algorithm is generated by replacing the denominator of (9) with a previous estimate

$$e_{k,i} = \frac{1}{D_{k-1}(j\omega_i)}[N_k(j\omega_i) - D_k(j\omega_i)T_M(j\omega_i)] \quad (10)$$

Treating $D_{k-1}(j\omega_i)$ as a known polynomial gives a set of N linear complex equations which can be solved by a complex least squares method [9] to obtain estimates of $N_k(\cdot)$ and $D_k(\cdot)$. The coefficients of the initial denominator polynomial $D_0(j\omega)$ can be set to 1 or other values. The algorithm converges rapidly, taking less than 5 iterations in repeated simulations. It is also robust in the presence of over-ordering problems and measurement errors. As an example, a 6th-order elliptic lowpass filter which was assumed to have a 12th-order transfer function due to over-ordering was identified with a 6th-order model in the presence of 1.0% measurement errors and under the assumption that the dominant parasitic poles of the OTAs are located at 10 times the filter resonant frequency. The identification of the 12th-order system with a 6th-order model showed very good results exhibiting a 0.23% error of the passband gain estimate [8].

4. Tuning Algorithm

The control mechanism relating the g_m of the OTAs to their control voltage V_c and current mirror gain M, is characterized by the linear equations.

$$g_{mi}(V_{ci}, M_i) = M_i K_i' \frac{W_i}{L_i}[V_{ci} - V_{ss} - V_{Ti}] \quad (11)$$

where K_i' and V_{Ti} are the process dependent transconductance and threshold voltage, M_i is the controllable output stage mirror gain, and W_i, and L_i are the width and length of the input differential pair devices. This can also be expressed as,

$$g_{mi}(V_{ci}, M_i) = M_i m_i(K_i', W_i, L_i)[V_{ci} + n_i(V_{Ti})] \quad (12)$$

We consider V_c and M as the control parameters for tuning the filters. V_c will be used for smaller(fine) adjustment while the mirror gain M will be used for more significant(coarse) adjustment. Thus, the transconductance g_{mi} of i'*th* OTA of each biquad can be controlled by changing V_{ci} and M_i. The basic idea of the tuning algorithm is to calculate the control parameters V_{ci} and M_i for adjustments such that the identified g_{mi} of each OTA comes close to its design (nominal) value. The identified g_{mi} of each OTA can be obtained from identified coefficients through the relations (3) to (6).

The tuning procedure can be divided into three parts: initial implementation, first iteration and subsequent iterations. In the initial implementation part, the initial parameters m_i and n_i for $i = 1, 2, \ldots, 5$ of each biquad are set to their design values, and the initial control parameters V_{ci} and M_i are calculated. These control parameter values are used for initial implementaion of the filter. Each iteration consists of four steps as follows:

1. System identification using the iterative complex least squares algorithm
2. Estimation of process parameters m_i and n_i from the identified transfer function coefficients
3. Calculation of control parameters V_{ci} and M_i from the estimated m_i and n_i
4. Adjustments using the obtained control parameters

The estimation formulas of m_i and n_i for $i = 1, 3, 4, 5$ are summarized in Table 1 and 2 with special conditions $B_{bp} = 1$, and $B_{lp} = 0$. It is assumed that $C_6 = C_7 = C$ and $g_{m2} = g_{m3}$, so the expressions for m_2 and n_2 are the same as those for m_3 and n_3. At each iteration the control voltages can be calculated by using the equations shown in Table 1. If the control voltage V_{ci} exceeds a specified range the current mirror gain M_i should be adjusted to keep the control voltage within the controllable range. In the tables, the followings are should be noticed:

- $a_0^{(k)}, a_1^{(k)}, b_0^{(k)}$, and $b_1^{(k)}$: identified transfer function coefficients at (k+1)'*th* iteration
- $\bar{a}_0, \bar{a}_1, \bar{b}_0$, and $\bar{b}_1$: design (ideal) transfer function coefficients
- $V_{ci}^{(k)}$: control voltage of g_{mi} at k'*th* iteration
- $M_i^{(k)}$: current mirror gain of g_{mi} at k'*th* iteration
- $V_{ci}^{(0)}, M_i^{(0)}, m_i^{(0)}, n_i^{(0)}$, and C : design values
- $p1 = k - 1$ and $p2 = k - 2$

The following is a detailed procedure to adjust the coefficient a_1 of each biquad by this tuning algorithm.

[1] Set the initial control parameter g_{m5} and $C_7(= C)$ to their design values. From (11) and (12), the parameters $m_5^{(0)}$ and $n_5^{(0)}$ at the first iteration are given by

$$m_5^{(0)} = \frac{K_5'^{(0)} W_5^{(0)}}{L_5^{(0)}} \quad (13)$$

$$n_5^{(0)} = -(V_{ss} + V_{T5}^{(0)}) \quad (14)$$

where $K_5'^{(0)}, W_5^{(0)}, L_5^{(0)}$, and $V_{T5}^{(0)}$ are the nominal values. Also, set the initial current mirror gain $M_5^{(0)}$ to a proper value so that it may not exceed the specified range in which the good linearity of the transconductance is kept. Then, from (3) and (12), the control voltage $V_{c5}^{(0)}$ at the initial implementation becomes,

$$V_{c5}^{(0)} = \frac{\bar{a}_1 C}{m_5^{(0)} M_5^{(0)}} - n_5^{(0)} \quad (15)$$

where C and $\bar{a}_1$ are the nominal values.

[2] Obtain the identified coefficient $a_1^{(0)}$ from system identification of the physical filter.

[3] At the first iteration it will be assumed that $n_5^{(1)} = n_5^{(0)}$. From (3) and (12),

$$a_1 = \frac{1}{C} m_5 M_5 (V_{c5} + n_5) \quad (16)$$

Thus, we may approximate the estimate for m_5 by

$$m_5^{(1)} = \frac{a_1^{(0)} C}{M_5^{(0)}(V_{c5}^{(0)} + n_5^{(0)})} \quad (17)$$

[4] Obtain estimates of V_{c5} and M_5 from

$$M_5^{(1)} = M_5^{(0)} \quad (18)$$

$$V_{c5}^{(1)} = \frac{\bar{a}_1 C}{m_5^{(1)} M_5^{(1)}} - n_5^{(1)} \quad (19)$$

and test if $V_{c5}^{(1)}$ exceeds the specified range. If it does, then calculate new $V_{c5}^{(1)}$ and $M_5^{(1)}$.

[5] Obtain the identified coefficient $a_1^{(1)}$ from identification of

the actual filter with the updated control voltages and mirror gains.

[6] To obtain $m_5^{(2)}$ and $n_5^{(2)}$, observe from (16) that

$$a_1^{(0)} = \frac{1}{C} m_5 M_5^{(0)} (V_{c5}^{(0)} + n_5) \quad (20)$$

$$a_1^{(1)} = \frac{1}{C} m_5 M_5^{(1)} (V_{c5}^{(1)} + n_5) \quad (21)$$

Solving these two equations simultaneously, we obtain the next estimates of m_5 and n_5.

$$m_5^{(2)} = \frac{C(a_1^{(1)}/M_5^{(1)} - a_1^{(0)}/M_5^{(0)})}{V_{c5}^{(1)} - V_{c5}^{(0)}} \quad (22)$$

$$n_5^{(2)} = \frac{M_5^{(0)} a_1^{(1)} V_{c5}^{(0)} - M_5^{(1)} a_1^{(0)} V_{c5}^{(1)}}{M_5^{(1)} a_1^{(0)} - M_5^{(0)} a_1^{(1)}} \quad (23)$$

[7] Obtain new estimates of V_{c5} and M_5 from

$$M_5^{(2)} = M_5^{(1)} \quad (24)$$

$$V_{c5}^{(2)} = \frac{\bar{a_1} C}{m_5^{(2)} M_5^{(2)}} - n_5^{(2)} \quad (25)$$

and check again $V_{c5}^{(2)}$, and calculate new $V_{c5}^{(2)}$ and $M_5^{(2)}$ if necessary.

[8] Test whether the system is tuned and repeat the step [5], [6], and [7] until a tuned system is obtained.

Actually, the similar procedures for a_0, b_1, and b_0 are performed simultaneously. If the system model is ideal, this algorithm will converge after two iterations. In reality each OTA has parasitic poles and zeros which make the actual systems have over-ordered transfer functions. Thus, more iterations are needed to get a solution.

5. Simulation Results

In order to evaluate the tuning algorithm, a 6th-order elliptic lowpass filter which has a normalized cutoff frequency at 1(rad/sec) and 0.5dB passband ripple was chosen for tuning. The manufacturing process was simulated via Monte Carlo techniques with random values. It is assumed that the capacitors and the transconductors have ±5% deviations from their design values. Errors of the magnitude and phase measurements at each frequency points are assumed to be within ±1.0%.

Initially, an ideal case was simulated using the above conditions. In the initial simulations, parasitics in the OTAs were neglected. The results are displayed in the normalized frequency range from dc to 2(rad/sec) in Fig. 3. From the results, it can be seen that this algorithm converges to a solution after two iterations if the system model is ideal.

Fig. 4 shows the results of the tuning algorithm when the parasitic effects of the OTAs were considered. The OTAs were assumed to have a parasitic pole at 10 times higher frequency than the resonance frequency, so the initial actual system had a 12th order transfer function because of the over-ordering problem due to the parasitic poles. Measurement errors in the system identification were modelled at ±1.0%. In this case, more iterations were required for the algorithm to converge. From many simulations, it was observed that good convergence was obtained within 10 iterations. The simulation results showed that the filter tuned by this tuning technique had 0.501dB passband ripple, 0.015% accuracy in the cutoff frequency, and 2.83% dc error. This relatively big dc error was caused by the fact that the biquads of the current digitally programmable continuous-time filter structure [6],[7] don't offer a gain factor adjustment. This problem can be solved by changing the biquadratic structure to have a gain factor adjustment.

6. Conclusion

In this paper, an iterative tuning algorithm for digitally programmable continuous-time filters has been proposed. Simulation results have demonstrated that this algorithm offers improvements in speed and accuracy over other approaches using digital optimization techniques. Comparison with conventional analog master-slave techniques suggests this method offers potential for improvements in accuracy by well over one decade.

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i	$m_i^{(1)}$	$n_i^{(1)}$	$V_{ci}^{(k)}$
1	$\frac{(b_0^{(0)}/\sqrt{a_0^{(0)}})C}{M_1^{(0)}(V_{c1}^{(0)}+n_1^{(0)})}$	$n_1^{(0)}$	$\frac{\bar{b_0} C}{\sqrt{\bar{a_0}} m_1^{(k)} M_1^{(k)}} - n_1^{(k)}$
3	$\frac{\sqrt{a_0^{(0)}}C}{M_3^{(0)}(V_{c3}^{(0)}+n_3^{(0)})}$	$n_3^{(0)}$	$\frac{\sqrt{\bar{a_0}} C}{m_3^{(k)} M_3^{(k)}} - n_3^{(k)}$
4	$\frac{(b_1^{(0)}+\sqrt{a_0^{(0)}})C}{M_4^{(0)}(V_{c4}^{(0)}+n_4^{(0)})}$	$n_4^{(0)}$	$\frac{(\bar{b_1}+\sqrt{\bar{a_0}})C}{m_4^{(k)} M_4^{(k)}} - n_4^{(k)}$
5	$\frac{a_1^{(0)}C}{M_5^{(0)}(V_{c5}^{(0)}+n_5^{(0)})}$	$n_5^{(0)}$	$\frac{\bar{a_1} C}{m_5^{(k)} M_5^{(k)}} - n_5^{(k)}$

Table 1 Expressions for $m_i^{(1)}$ and $n_i^{(1)}$ at first iteration and for $V_{ci}^{(k)}$ at k'*th* iteration.

i	$m_i^{(k)}$
1	$\frac{C[b_0^{(p1)}/(M_1^{(p1)}\sqrt{a_0^{(p1)}})-b_0^{(p2)}/(M_1^{(p2)}\sqrt{a_0^{(p2)}})]}{V_{c1}^{(p1)}-V_{c1}^{(p2)}}$
3	$\frac{C(\sqrt{a_0^{(p1)}}/M_3^{(p1)}-\sqrt{a_0^{(p2)}}/M_3^{(p2)})}{V_{c3}^{(p1)}-V_{c3}^{(p2)}}$
4	$\frac{C[(b_1^{(p1)}+\sqrt{a_0^{(p1)}})/M_4^{(p1)}-(b_1^{(p2)}+\sqrt{a_0^{(p2)}})/M_4^{(p2)}]}{V_{c4}^{(p1)}-V_{c4}^{(p2)}}$
5	$\frac{C(a_1^{(p1)}/M_5^{(p1)}-a_1^{(p2)}/M_5^{(p2)})}{V_{c5}^{(p1)}-V_{c5}^{(p2)}}$
i	**$n_i^{(k)}$**
1	$\frac{M_1^{(p2)}\sqrt{a_0^{(p2)}}b_0^{(p1)}V_{c1}^{(p2)}-M_1^{(p1)}\sqrt{a_0^{(p1)}}b_0^{(p2)}V_{c1}^{(p1)}}{M_1^{(p1)}\sqrt{a_0^{(p1)}}b_0^{(p2)}-M_1^{(p2)}\sqrt{a_0^{(p2)}}b_0^{(p1)}}$
3	$\frac{M_3^{(p2)}\sqrt{a_0^{(p1)}}V_{c3}^{(p2)}-M_3^{(p1)}\sqrt{a_0^{(p2)}}V_{c3}^{(p1)}}{M_3^{(p1)}\sqrt{a_0^{(p2)}}-M_3^{(p2)}\sqrt{a_0^{(p1)}}}$
4	$\frac{M_4^{(p2)}(b_1^{(p1)}+\sqrt{a_0^{(p1)}})V_{c4}^{(p2)}-M_4^{(p1)}(b_1^{(p2)}+\sqrt{a_0^{(p2)}})V_{c4}^{(p1)}}{M_4^{(p1)}(b_1^{(p2)}+\sqrt{a_0^{(p2)}})-M_4^{(p2)}(b_1^{(p1)}+\sqrt{a_0^{(p1)}})}$
5	$\frac{M_5^{(p2)}a_1^{(p1)}V_{c5}^{(p2)}-M_5^{(p1)}a_1^{(p2)}V_{c5}^{(p1)}}{M_5^{(p1)}a_1^{(p2)}-M_5^{(p2)}a_1^{(p1)}}$

Table 2 Expressions for $m_i^{(k)}$ and $n_i^{(k)}$ at k'th iteration($k \geq 2$)

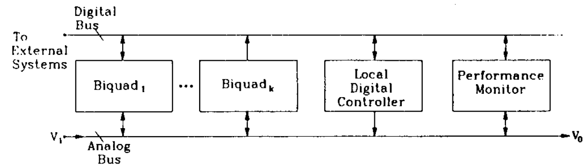

Fig. 1. Digitally Programmable Analog Filter Architecture[6]

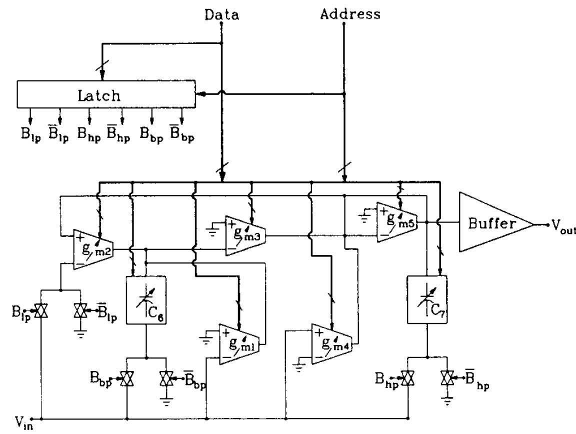

Fig. 2. Biquadratic Building Block[6]

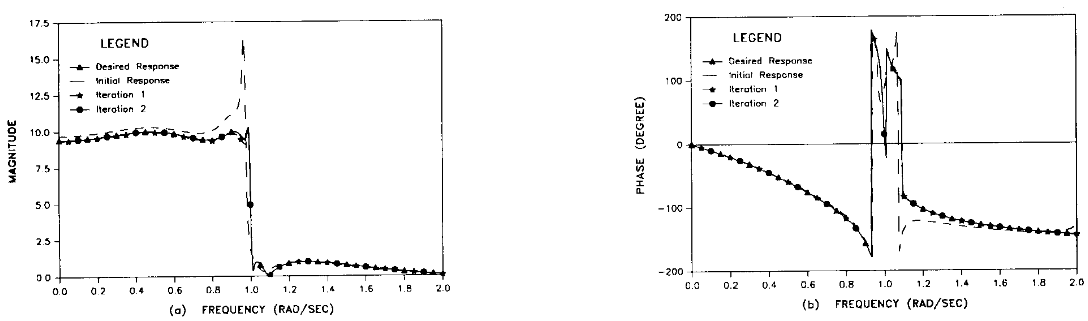

Fig. 3. Tuning process of the 6th–order elliptic lowpass filter with ideal OTAs (a) Magnitude responses (b) Phase responses

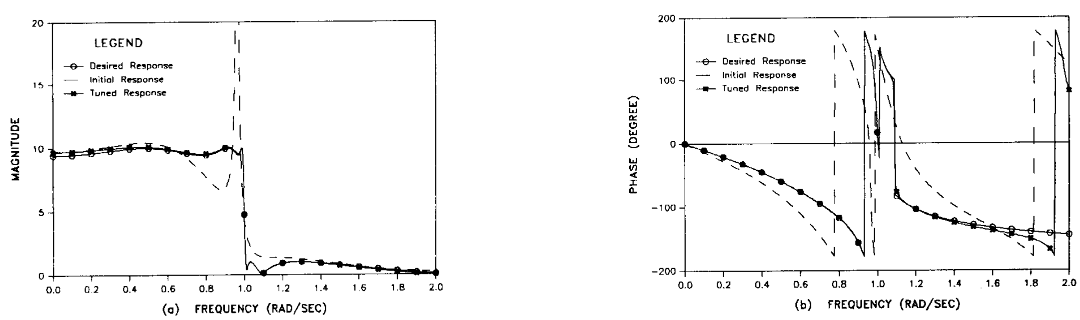

Fig. 4. Tuning results of the 6th–order elliptic lowpass filter with nonideal OTAs (a) Magnitude responses (b) Phase responses