A FREQUENCY DOMAIN PARAMETRIC SYSTEM IDENTIFICATION ALGORITHM FOR CONTINUOUS–TIME LINEAR FILTERS[1]

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Abstract

The identification of rational polynomial transfer functions of continuous–time filters is performed in the context of tuning monolithic continuous–time filters to high precision using a low cost on-chip digital signal processing and control system. The original contribution detailed here is a robust s domain system identification algorithm which uses (low cost) noisy complex frequency domain measurements ($s_i = j\omega_i$). We use large over-sampling in frequency (e.g. $\times$ 10), while minimizing the time-domain sampling rate. We introduce an iterative method which solves a complex least-squares problem at each step. The algorithm is observed to converge rapidly and is robust in the presence of parasitic poles and zeroes.

Introduction

Monolithic analog filters typically require only 1% to 10% of the die area of comparable order and rate digital filters, but their use has been limited because of large component tolerances due to processing, temperature, and aging. We propose an on-chip digital tuning system, but to achieve low cost and high accuracy requires that transformation errors from the z to the s domain be minimal without resorting to high over-sampling in the time domain. Practical solution of this problem opens a host of reduced cost and new signal processing applications.

Previous approaches to tuning analog filters have used nonlinear optimization techniques [1]-[4], which are limited to ≈1% tuning range by local minima capture. On-line Identification(ID) time-domain techniques require a large number of time-domain samples, which are expensive and slow in our system. Our iterative method uses s domain complex data, which is related to the iterative least squares method using real data [5].

Problem Motivation

We consider the problem of digitally tuning continuous-time (analog) active filters to highly accurate design specifications. Operational Transconductance Amplifiers (OTA) with digital control of capacitive loading and g_m are configured to cover a wide tuning range [6]-[8]. Our work is motivated by the fact that active analog filters are known to require orders of magnitude less die area than comparable digital filters. However, component tolerances have in the past made them unsuited for many monolithic IC applications. We implement tuning by iteratively estimating (system ID) and adjusting the system in a quickly converging manner. The accuracy, stability, and convergence rate of tuning is thus highly dependent on the accuracy of the system ID algorithm, which must be insensitive to transformation errors and parasitics. To preserve the advantages of the analog filter, the digital identification and control must also have a very low cost implementation.

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While many ID methods are possible, our goal here is to ID (and to tune) filters quite close to their parasitic poles and zeroes. For an analog filter with bandwidth of $10MHz$, a relatively modest over-sampling $\times 2$ requires an *effective* $40MHz$ sampling rate, which is near the practical limit in inexpensive technologies. Thus the desire to avoid the $\times 10$ over-sampling typical in the ID of continuous-time systems by the ID of a z domain counterpart. To avoid the expense of a high speed A/D converter, which is a major expense of the all digital filter, we use a periodic excitation source, a narrow aperture sample and hold, and a low-cost low-speed A/D. Time domain sampling is thus low cost even at an effective high sampling rate, but data acquisition is slow, so traditional on-line ID methods are inappropriate. Many low cost excitation and measurement systems are possible, including sinusoids and pseudo random number generators (PRNG). A binary PRNG can be smoothed by an untuned passive RC filter designed to operate with large component tolerances. The period of the PRNG is chosen to generate many more harmonics than unknown parameters in the rational polynomial model. Magnitude and phase estimates at these frequencies, $T_M(j\omega_i)$, serve as inputs to the system ID algorithm.

Formulation of Iterative Identification Algorithm

If the ideal transfer function $T(s)$ of the system to be identified is given by

$$T(s) = \frac{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots + b_n s^n} = \frac{N(s)}{D(s)} \qquad (1)$$

then, the complex estimation error at the i'*th* frequency denoted by e_i becomes

$$e_i = \frac{N(j\omega_i)}{D(j\omega_i)} - T_M(j\omega_i) \qquad (2)$$

$$= \frac{1}{D(j\omega_i)}[N(j\omega_i) - D(j\omega_i)T_M(j\omega_i)] \qquad (3)$$

An iterative algorithm is generated by replacing the denominator in (3) with a previous estimate of $D(\cdot)$.

$$e_{k,i} = \frac{1}{D_{k-1}(j\omega_i)}[N_k(j\omega_i) - D_k(j\omega_i)\, T_M(j\omega_i)] \qquad (4)$$

Treating $D_{k-1}(\cdot)$ as a known polynomial gives a set of N linear complex equations in $n+m+1$ unknowns, where n and m are the number of poles and zeroes respectively. We choose $N > n+m+1$ so the equations are overdetermined and can be solved by a complex least squares method due to Levy [9]. Thus, at k'*th* iteration, the coefficients in $D_k(\cdot)$ and $N_k(\cdot)$ can be estimated by minimizing the sum of squares of the error terms, so the performance criterion is given by

$$J_k = \sum_{i=1}^{N} e_{k,i}^* e_{k,i} \qquad (5)$$

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where $*$ denotes complex conjugate transpose. Let $\frac{1}{D_{k-1}(j\omega_i)} = C_{k,i}$, then

$$\begin{aligned} e_{k,i} &= C_{k,i}[N_k(j\omega_i) - D_k(j\omega_i)T_M(j\omega_i)] &(6)\\ &= C_{k,i}[(a_0 + a_1 j\omega_i + \ldots + a_m(j\omega_i)^m) \\ &\quad -(1 + b_1 j\omega_i + \ldots + b_n(j\omega_i)^n)T_M(j\omega_i)] &(7)\\ &= -C_{k,i}T_M(j\omega_i) \\ &\quad -[C_{k,i}b_1 j\omega_i T_M(j\omega_i) + \ldots + C_{k,i}b_n(j\omega_i)^n T_M(j\omega_i) \\ &\quad -C_{k,i}a_0 - C_{k,i}a_1 j\omega_i - \ldots - C_{k,i}a_m(j\omega_i^m)] &(8) \end{aligned}$$

$$\begin{bmatrix} e_{k,1} \\ e_{k,2} \\ e_{k,3} \\ \vdots \\ e_{k,N} \end{bmatrix} = \begin{bmatrix} -C_{k,1}T_M(j\omega_1) \\ -C_{k,2}T_M(j\omega_2) \\ -C_{k,3}T_M(j\omega_3) \\ \vdots \\ -C_{k,N}T_M(j\omega_N) \end{bmatrix} - \begin{bmatrix} C_{k,1}j\omega_1 T_M(j\omega_1) & \cdots & C_{k,1}(j\omega_1)^n T_M(j\omega_1) & -C_{k,1} & \cdots & -C_{k,1}(j\omega_1)^m \\ C_{k,2}j\omega_2 T_M(j\omega_2) & \cdots & C_{k,2}(j\omega_2)^n T_M(j\omega_2) & -C_{k,2} & \cdots & -C_{k,2}(j\omega_2)^m \\ C_{k,3}j\omega_3 T_M(j\omega_3) & \cdots & C_{k,3}(j\omega_3)^n T_M(j\omega_3) & -C_{k,3} & \cdots & -C_{k,3}(j\omega_3)^m \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{k,N}j\omega_N T_M(j\omega_N) & \cdots & C_{k,N}(j\omega_N)^n T_M(j\omega_N) & -C_{k,N} & \cdots & -C_{k,N}(j\omega_N)^m \end{bmatrix} \begin{bmatrix} b_1 \\ \vdots \\ b_n \\ a_0 \\ \vdots \\ a_m \end{bmatrix} \quad (9)$$

where,

$$\begin{aligned} C_{k,1} &= \frac{1}{D_{k-1}(j\omega_1)} \\ C_{k,2} &= \frac{1}{D_{k-1}(j\omega_2)} \\ C_{k,3} &= \frac{1}{D_{k-1}(j\omega_3)} \\ \vdots &\;\vdots\; \vdots \\ C_{k,N} &= \frac{1}{D_{k-1}(j\omega_N)} \end{aligned}$$

For N measured data $T_M(j\omega)$ at $\omega_1, \omega_2, \ldots, \omega_N$, we can get the above matrix expression at k'*th* iteration. If a matrix/vector notation is used, then (9) can be denoted by

$$\mathbf{e}_k = \mathbf{b} - \mathbf{A}\mathbf{x}_k \quad (10)$$

The performance criterion of (5) becomes

$$\begin{aligned} J_k &= \sum_{i=1}^{N} e_{k,i}^* e_{k,i} \\ &= \mathbf{e}_k^* \mathbf{e}_k &(11)\\ &= (\mathbf{b} - \mathbf{A}\mathbf{x}_k)^*(\mathbf{b} - \mathbf{A}\mathbf{x}_k) &(12) \end{aligned}$$

The partial derivatives of J_k with respect to $\mathbf{x}_k$ can be expressed

$$\begin{aligned} \frac{\partial J_k}{\partial \mathbf{x}_k} &= \frac{\partial}{\partial \mathbf{x}_k}[(\mathbf{b} - \mathbf{A}\mathbf{x}_k)^*(\mathbf{b} - \mathbf{A}\mathbf{x}_k)] &(13)\\ &= \frac{\partial}{\partial \mathbf{x}_k}[(\overline{\mathbf{b} - \mathbf{A}\mathbf{x}_k})^T(\mathbf{b} - \mathbf{A}\mathbf{x}_k)] &(14)\\ &= -\mathbf{A}^*(\mathbf{b} - \mathbf{A}\mathbf{x}_k) - \mathbf{A}^T(\overline{\mathbf{b} - \mathbf{A}\mathbf{x}_k}) &(15)\\ &= -(\mathbf{A}^*\mathbf{b} + \mathbf{A}^T\overline{\mathbf{b}}) + (\mathbf{A}^*\mathbf{A} + \mathbf{A}^T\overline{\mathbf{A}})\mathbf{x}_k &(16) \end{aligned}$$

The value of $\hat{\mathbf{x}}_k$ minimizing J_k is found by setting the partial derivatives in (16) equal to zero, giving

$$\begin{aligned} \hat{\mathbf{x}}_k &= (\mathbf{A}^*\mathbf{A} + \mathbf{A}^T\overline{\mathbf{A}})^{-1}(\mathbf{A}^*\mathbf{b} + \mathbf{A}^T\overline{\mathbf{b}}) &(17)\\ &= (\mathbf{A}^*\mathbf{A} + \overline{\mathbf{A}^*\mathbf{A}})^{-1}(\mathbf{A}^*\mathbf{b} + \overline{\mathbf{A}^*\mathbf{b}}) &(18)\\ &= [\mathrm{Re}(\mathbf{A}^*\mathbf{A})]^{-1}[\mathrm{Re}(\mathbf{A}^*\mathbf{b})] &(19) \end{aligned}$$

Now, the coefficients of the denominator $(b_1, b_2, \ldots, b_n)$ can be extracted from $\hat{\mathbf{x}}_k$ and used for the next iteration.

Simulation Results

In order to evaluate this algorithm a 6th–order elliptic lowpass filter which has a normalized cutoff frequency at 1(rad/sec) was chosen for identification. It consists of three second–order lowpass notch filters. Its transfer function is given by

$$H(s) = \prod_{i=1}^{3} \frac{s^2 + B_{0i}}{s^2 + A_{1i}s + A_{0i}} \quad (20)$$

where,

i	A_{1i}	A_{0i}	B_{0i}	ω_o	Q_p
1	0.933855	0.611899	4.36790	0.7822	0.8376
2	0.156221	0.934830	1.19243	0.9669	6.1891
3	0.017576	0.990620	1.02486	0.9952	56.628

It was assumed that measurement was performed at uniformly spaced frequency points on the normalized frequency range from dc to 2 (rad/sec) to obtain NOD(Number Of Data) complex data. Measurement errors were modelled as complex independent Gaussian random variables with variance 0.1MN in passband and 0.01MN in the stopband. The stopband noise models the measurement system noise floor, while the passband noise models the relative resolution of large signals. Other measurement noise models including simple additive noise and multiplicative noise were found to give similar performance. The iteration limit was set to 20. The algorithm was considered to have converged if all the identified coefficients were within 0.1% of their previous values.

First, we assumed there were no over–ordering effects due to parasitics. The ideal 6th–order filter was identified with a 6th–order model. Different number of data were used and various measurement errors were considered in the identification. The results are shown in Table 1 and 2, and Fig. 1 and 2. From the results it can be shown that very accurate results and fast convergences were obtained when using 50 or more data points. The magnitude error between the true and identified system is seen in Table 1 to be less than 0.2% of the passband gain. It took less than 5 iterations. The accuracy is proportional to the number of data used for the identification but inversely proportional to the measurement error as seen in Table 2. We can still, however, attain fast convergence and error less than 1% even with 5% measurement error. Therefore, very accurate results can be attained with 50 or more data points and with reasonable measurement errors if the system model is ideal, i.e., if the transfer function of the actual system to be identified has the same order as that of the model which is used for the identification.

In reality, however, actual filters have parasitics which cause their real order to be higher than designed one. This is called the over–ordering problem. In our target architecture [7], there is no attempt to tune or cancel the parasitics themselves. A straightforward method is to then approximately identify the over–ordered real system with a low order model. Usually, parasitic effects in an OTA can be approximated by a single pole(ω_p) and the over–ordering factor can be defined by ω_o/ω_p, where ω_o is the desired system pole.

In the second simulation, the actual 12th–order system due to over–ordering effects was identified using an ideal 6th–order model. We assumed the over–ordering factor was 0.1, i.e., the parasitic pole was located at 10 times the desired system pole. The identified results are shown in Tables 3, and 4, and in Fig. 3 and 4. Note that zero error in ID is not possible because the ID is now approximating the over–ordered system. But these results show that very good results can be achieved in the presence of over–ordering effects. The passband error was less than 0.23% when 50 or more complex data was used and 1% measurement error was assumed. However, a significant measurement error (10%) gave a relatively big passband gain error and the iteration limit was exceeded. Comparison of the results at different iterations is given in Table 5. Comparing the result of the first iteration, which corresponds to Levy's algorithm [9], we see that our iterative algorithm gives great improvements in accuracy.

Conclusion

In this paper an iterative complex least squareis algorithm for frequency domain parametric system identification has been proposed. Extensive simulation results show that this algorithm converges rapidly and attains very accurate results even in the presence of large measurement errors. This algorithm is also robust in the presence of over–ordering effects due to parasitics, so it is possible to identify a high–order physical system with a low–order ideal model. Thus, this algorithm can be simply applied to various tuning algorithm.

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NOD	Number of Iterations	Max. passband Error (%)	3dB band-edge Error (%)
13	8	51 (1.62*)	2.23
25	4	30 (0.92*)	0.45
50	4	0.2 (0.2*)	0.015
100	4	0.18 (0.14*)	0.015

* Maximum error from dc to 0.9

Table 1 Identification results of the ideal 6th order system with NOD number of data points and MN = 1.0%

MN	Number of Iterations	Max. passband Error (%)	3dB band-edge Error (%)
0.1%	3	0.02	0.001
1%	4	0.2	0.015
5%	5	1.0	0.01
10%	6	2.1	0.02

Table 2 Identification results of the ideal 6th order system with MN% complex measurement error and NOD = 50 points

Number of Complex Data	Number of Iterations	Max. passband Error (%)
13	12	7.90
25	5	1.21
50	6	0.23
100	6	0.16

Table 3 Identification results of the 12th order system including parasitics with a 6th order model ($\omega_o/\omega_p = 0.1$, MN = 1.0%)

Measurement Error	Number of Iterations	Max. passband Error (%)
0.1%	6	0.13
1%	6	0.23
5%	20*	2.1
10%	20*	8.7

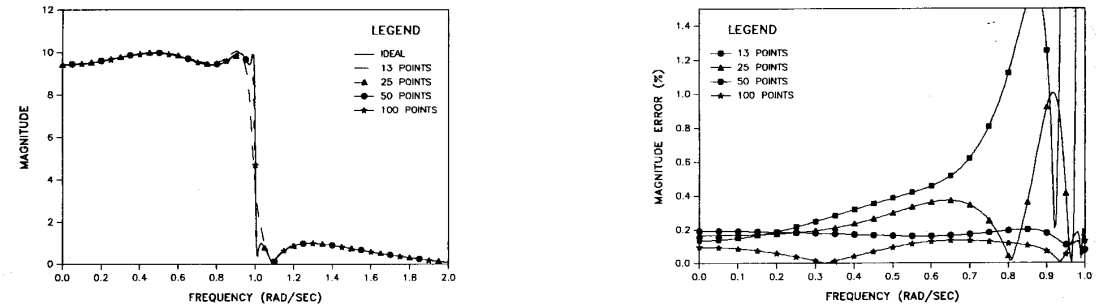
* Iteration limit was exceeded

Table 4 Identification results of the 12th order system including parasitics with a 6th order model ($\omega_o/\omega_p = 0.1$, NOD = 50)

Number of Iterations	Max. passband Error (%)	3dB band-edge Error (%)
1	29.7 (1.22*)	0.778
2	0.27 (0.20*)	0.015
3	0.20 (0.20*)	0.015
4	0.20 (0.20*)	0.015

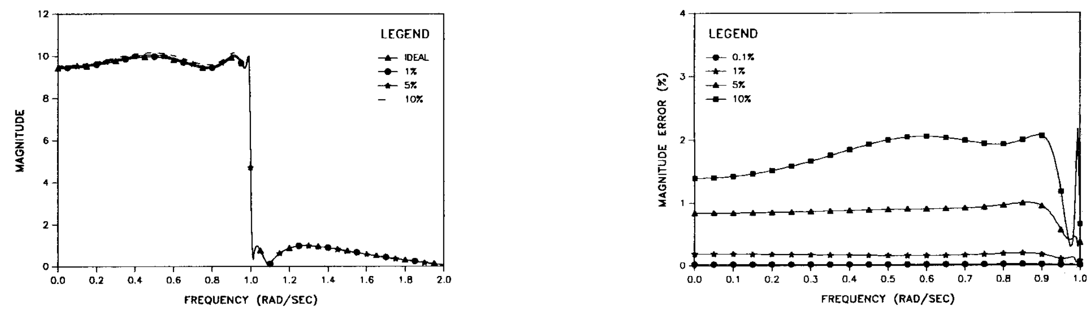
* Maximum error from dc to 0.9

Table 5 Iterative improvements of the ID algorithm for the ideal 6th order system (MN = 1.0%, NOD = 50)



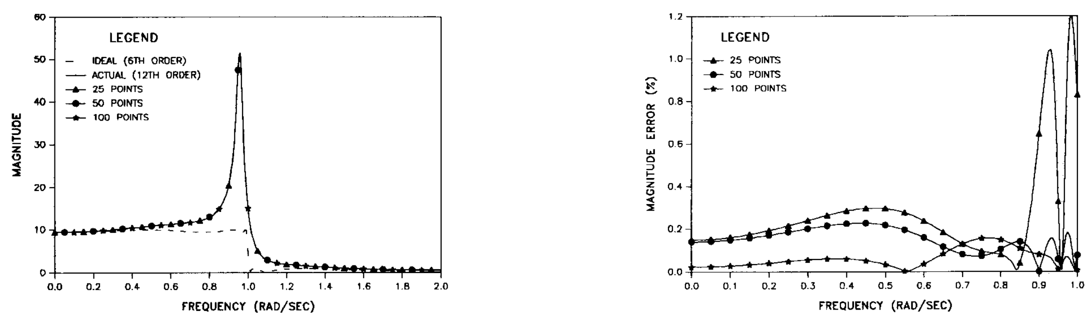
(a) Magnitude Response (b) Magnitude Error

Fig. 1. Identification results of the ideal 6th order system with NOD number of data points and MN = 1.0%



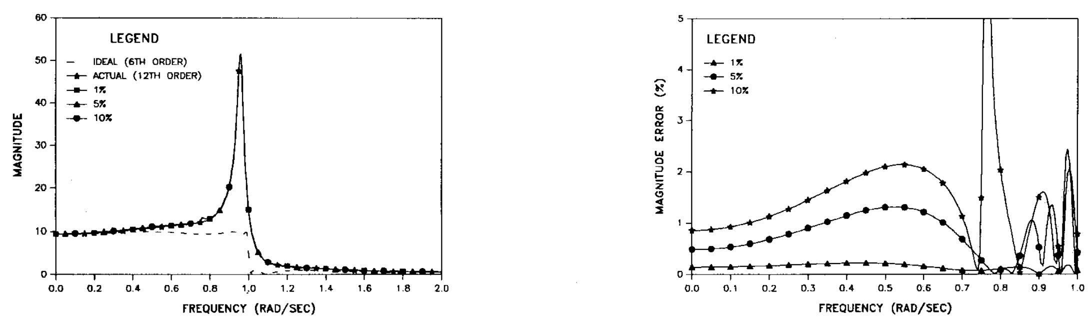
(a) Magnitude Response (b) Magnitude Error

Fig. 2. Identification results of the ideal 6th order system with MN% complex measurement error and NOD = 50



(a) Magnitude Response (b) Magnitude Error

Fig. 3. Identification results of the 12th order system with NOD number of data points ($\omega_o/\omega_p = 0.1$, MN = 1.0%)



(a) Magnitude Response (b) Magnitude Error

Fig. 4. Identification results of the 12th order system with MN% complex measurement error ($\omega_o/\omega_p = 0.1$, NOD = 50)