Analysis of Random Common-Mode Rejection Ratio in Op-Amps

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Abstract

One of the important nonideal factors causing op-amp errors is the common-mode rejection ratio (CMRR). The CMRR contains both deterministic and random components, and the random component of the CMRR is usually comparable to the deterministic component. However, little attention has been paid to the random CMRR. In this paper expressions for the random and deterministic CMRR of a CMOS op-amp architecture are derived, and the statistical characteristics of the CMRR are analyzed. Definition of the CMRR for processes is also made.

I. Introduction

The performance of practical op-amps is usually degraded by lots of nonideal effects. Among them, finite CMRRs along with nonzero offset voltages and finite open-loop gains are the major sources which limit the high-precision applications of amplifiers. The CMRR and offset are not totally deterministic but have both deterministic and random components. These random components due to device mismatches make it difficult to analyze the op-amp errors. The statistical characteristics of these parameters must be well understood to obtain high-precision performance. Several analyses of the random CMRR in differential amplifiers have been made [5],[6], but these analyses were made several decades ago for bipolar differential amplifiers. Moreover, they focused on the methods to increase the CMRR, not on its statistical characteristics.

The term, CMRR, is widely used and has appeared in texts for many years [1]-[4]. For a single sample amplifier, the term is defined as

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right| \tag{1}$$

where A_{dm} and A_{cm} are the small signal differential-mode and common-mode gains respectively. For the single sample amplifier, the CMRR is deterministic and can be readily measured in the laboratory. Of more importance than the CMRR of a single sample amplifier from an operational amplifier yield viewpoint is the CMRR of an amplifier architecture in a process. Unfortunetely, a rigorous definition of the CMRR has not appeared in the literature. Consequently, designers have been basing designs on inaccurate models and/or expensive "worst case" simulations. The impact has often resulted in designs that are overly conservative or designs that have substantially degraded performance.

In this paper the CMRR of CMOS op-amps and its statistical characteristics are thoroughly analyzed. In section 2 the random and deterministic CMRR of a two-stage CMOS op-amp shown in Fig.1 are derived, and in section 3 the statistical characteristics of the total CMRR are discussed. Definition of the CMRR for processes and some conclusions are given in section 4 and 5, respectively. The sample amplifier in Fig.1 has been designed for high-speed and high-precision applications. The simulated performances of the op-amp are shown in Table 1.

II. Derivation of the random and deterministic CMRR

The CMRR of the two-stage CMOS op-amp will be dominated by the first stage. The small signal equivalent circuit of the differential stage is shown in Fig.2. Ideally M1 and M2 are matched as are M3 and M4.

The small-signal output voltage is given by

$$v_o = A_{dm} v_d + A_{cm} v_c \tag{2}$$

where

$$v_d = v_{in1} - v_{in2}, \qquad v_c = \frac{v_{in1} + v_{in2}}{2}. \tag{3}$$

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The nodal equations at nodes (1), (2), and (3) are

$$\begin{aligned} (g_{m1} + g_{d1})v_1 - (g_{m3} + g_{d1})v_2 &= g_{m1} v_{in1} \\ (g_{m2} + g_{d2})v_1 - g_{m4} v_2 - (g_{d2} + g_{d4})v_{out} &= g_{m2} v_{in2} \end{aligned} \tag{4}$$

$$(g_{m1} + g_{m2} + g_{d1} + g_{d2} + g_o)v_1 - g_{d1} v_2 - g_{d2} v_{out} = g_{m1} v_{in1} + g_{m2} v_{in2}.$$

The model parameters are all random variables and can be expressed as

$$\begin{aligned} g_{mk} &= g_{mkN} + g_{mkR1} + g_{mkR2} \\ g_{dk} &= g_{dkN} + g_{dkR1} + g_{dkR2}, \quad k = 1,2,3,4 \end{aligned} \tag{5}$$

where the N subscript denotes the nominal value which is deterministic, the R1 subscript denotes a random component that is process dependent but which does not vary from device to device on a wafer and where the R2 subscript denotes a random component that varies randomly from device to device on a wafer. It will be assumed that process dependent random variables (those with an R1 subscript) are totally correlated and identical for matched devices and that the wafer-level random variables (those with an R2 subscript) are identically distributed for ideally matched devices but statistically uncorrelated. Assuming that $g_{mk} >> g_{dl}$, for all $k, l \in \{1,2,3,4\}$ and that M1 and M2 are nominally matched as are M3 and M4, we can obtain the expressions for the differential-mode gain A_{dm} and the common-mode gain A_{cm}, which are themselves random variables,

$$\begin{aligned} A_{dm} \simeq\ & [2g_{mi}^2 g_{ml} + g_{mi}^2(2g_{mlR1} + g_{m3R2} + g_{m4R2}) + 2g_{mi}g_{ml}(2g_{miR1} \\ & + g_{m1R2} + g_{m2R2})]/[2g_{mi}g_{ml}(g_{di} + g_{dl})] \end{aligned} \tag{6}$$

$$\begin{aligned} A_{cm} \simeq\ & [-g_{di}g_{mi}g_o + (2g_{di}g_{ml} + g_o g_{ml})(g_{m1R2} - g_{m2R2}) - 2g_{mi}g_{ml}(g_{d1R2} \\ & - g_{d2R2}) - g_o g_{mi}(g_{m3R2} - g_{m4R2})]/[2g_{mi}g_{ml}(g_{di} + g_{dl})], \end{aligned} \tag{7}$$

where

$$\begin{aligned} g_{mi} &= g_{m1N} = g_{m2N}, & g_{miR1} &= g_{m1R1} = g_{m2R1} \\ g_{ml} &= g_{m3N} = g_{m4N}, & g_{mlR1} &= g_{m3R1} = g_{m4R1} \\ g_{di} &= g_{d1N} = g_{d2N}, & g_{diR1} &= g_{d1R1} = g_{d2R1} \\ g_{dl} &= g_{d4N}, \end{aligned} \tag{8}$$

where the i subscript denotes the input transistors M1 and M2, and the l subscript denotes the load transistors M3 and M4.

Since the random component of the differential gain is very small compared to the deterministic component of the differential gain as can be seen in (6), the total differential-mode gain can be approximated by the deterministic gain only.

$$A_{dm} \simeq \frac{g_{mi}}{g_{di} + g_{dl}}. \tag{9}$$

The random component of the common-mode gain is, however, comparable in magnitude to the deterministic component of the common-mode gain. The deterministic and random common-mode gains, A_{cm}^D and A_{cm}^R, can be defined so that

$$A_{cm} = A_{cm}^D + A_{cm}^R \tag{10}$$

From (7), natural definitions of A_{cm}^D and A_{cm}^R are

$$A_{cm}^D = -\frac{g_{di}g_o}{2g_{ml}(g_{di} + g_{dl})} \tag{11}$$

$$\begin{aligned} A_{cm}^R =\ & \frac{1}{2(g_{di} + g_{dl})}\left[g_o\left(\frac{g_{m1R2} - g_{m2R2}}{g_{mi}} - \frac{g_{m3R2} - g_{m4R2}}{g_{ml}}\right)\right. \\ & \left. + 2g_{di}\left(\frac{g_{m1R2} - g_{m2R2}}{g_{mi}} - \frac{g_{d1R2} - g_{d2R2}}{g_{di}}\right)\right]. \end{aligned} \tag{12}$$

The ratios of (12) are readily obtained in terms of the geometric and process device parameters.

$$A^R_{cm} = \frac{1}{2(g_{di}+g_{dl})}\left[g_o\left(\frac{W_{1R2}-W_{2R2}}{W_i}+\frac{L_{2R2}-L_{1R2}}{L_i}+\frac{W_{3R2}-W_{4R2}}{W_l}+\frac{L_{4R2}-L_{3R2}}{L_l}+\frac{V_{T2R2}-V_{T1R2}}{V_{GSi}-V_{Ti}}+\frac{V_{T4R2}-V_{T3R2}}{V_{GSl}-V_{Tl}}\right)+2g_{di}\frac{V_{T1R2}-V_{T2R2}}{V_{GSi}-V_{Ti}}\right] \quad (13)$$

The CMRR, defined in (1) where A_{cm} is now a random variable, is itself a random variable. If we define

$$CMRR_D^{-1} = \frac{A^D_{cm}}{A_{dm}} \quad \text{and} \quad CMRR_R^{-1} = \frac{A^R_{cm}}{A_{dm}}, \quad (14)$$

then we have

$$\begin{aligned} CMRR &= \left|\frac{A_{dm}}{A_{cm}}\right| \\ &= \left|\frac{A_{dm}}{A^D_{cm}+A^R_{cm}}\right| \\ &= \left|\frac{1}{CMRR_D^{-1}+CMRR_R^{-1}}\right|. \end{aligned} \quad (15)$$

From (9), (11), (13) and (14), the deterministic and random CMRRs are given by

$$CMRR_D^{-1} = -\frac{g_{di}g_o}{2g_{mi}g_{ml}} \quad (16)$$

and

$$CMRR_R^{-1} = \frac{1}{2g_{mi}}\left[g_o\left(\frac{W_{1R2}-W_{2R2}}{W_i}+\frac{L_{2R2}-L_{1R2}}{L_i}+\frac{W_{3R2}-W_{4R2}}{W_l}+\frac{L_{4R2}-L_{3R2}}{L_l}\frac{V_{T2R2}-V_{T1R2}}{V_{GSi}-V_{Ti}}+\frac{V_{T4R2}-V_{T3R2}}{V_{GSl}-V_{Tl}}\right)+2g_{di}\frac{V_{T1R2}-V_{T2R2}}{V_{GSi}-V_{Ti}}\right]. \quad (17)$$

The deterministic CMRR given by (16) is as reported in [2] and [3]. From (17) we can see that the random component of the CMRR is caused by the nonzero output conductance of the bias current source, g_o, and the nonzero output conductance of the input transistors, g_{di}, as well as the mismatch of the paired devices. It can be seen that the effect due to g_o on the random CMRR are more dominant than that due to g_{di}.

At this stage, we will calculate a pseudo worst case CMRR to compare the magnitude of the random and deterministic components of the CMRR. To calculate the pseudo worst case CMRR of the op-amp shown in Fig.1 whose simulated parameter values are shown in Table 2, it is assumed that the wafer-level random component of L and W are normally distributed with zero mean and standard deviation

$$\sigma_L = \sigma_W = 0.014\mu m \quad (18)$$

and that the corresponding random component of V_T is normally distributed with zero mean and standard deviation

$$\sigma_{V_T} = \frac{k}{\sqrt{A}}, \quad (19)$$

where k=0.0236 Vμm. We define the pseudo worst case CMRR to be the sample CMRR that would result if all random variables comprising the CMRR are at the 3σ value and in the direction that they add. The corresponding σ values for width, length and threshold variations are summarized in Table 2. The deterministic CMRR calculated from (16) was 63.7dB which is close to the simulated one shown in Table 1. The pseudo worst case random CMRR calculated from (17) was 51.6dB which dominates the deterministic CMRR. The worst case total CMRR was thus 49.6dB.

III. Statistical characteristics of CMRR

In this section the statistical characteristics of the random variable, CMRR as defined by (15), will be investigated. For notational convenience we will define

$$\mathbf{c} = CMRR, \qquad \mathbf{x} = CMRR_R^{-1} \quad (20)$$

$$d = CMRR_D^{-1}, \qquad \mathbf{y} = \mathbf{x} + d \quad (21)$$

where the bold letters are used to denote random variables. From (15), the common-mode rejection ratio can be expressed as

$$\mathbf{c} = \left|\frac{1}{\mathbf{x}+d}\right| = \left|\frac{1}{\mathbf{y}}\right| = \frac{1}{|\mathbf{y}|}. \quad (22)$$

Equation (17) shows that the random variable $\mathbf{x} = CMRR_R^{-1}$ is a function of 12 random variables. These random variables are assumed independent and normally distributed with zero mean. Since $\mathbf{x}$ is the sum of 12 uncorrelated zero mean random variables, its mean will also be zero and its variance is equal to the sum of the variances. Thus, $\mathbf{x}$ is distributed as

$$\mathbf{x} \sim N(0, \sigma_x^2) \quad (23)$$

where

$$\sigma_x^2 = \frac{1}{2g_{mi}^2}\left[g_o^2\sigma_L^2\left(\frac{1}{L_i^2}+\frac{1}{L_l^2}\right)+g_o^2\sigma_W^2\left(\frac{1}{W_i^2}+\frac{1}{W_l^2}\right)+\frac{\sigma^2_{V_{Ti}}(g_o^2+4g_{di}^2)}{(V_{GSi}-V_{Ti})^2}+\frac{\sigma^2_{V_{Tl}}g_o^2}{(V_{GSl}-V_{Tl})^2}\right]. \quad (24)$$

Since d in (21) is deterministic, the random variable $\mathbf{y} = \mathbf{x} + d$ is normally distributed with mean d and variance σ_x^2,

$$\mathbf{y} \sim N(d, \sigma_x^2). \quad (25)$$

The mean of $|\mathbf{y}|$ can be expressed as [7].

$$E\{|\mathbf{y}|\} = \sigma_x\sqrt{\frac{2}{\pi}}\,e^{-d^2/2\sigma_x^2} + 2d\,P\left(\frac{d}{\sigma_x}\right) - d \quad (26)$$

where

$$P(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}dy. \quad (27)$$

The variance of $|\mathbf{y}|$ is then

$$\begin{aligned} \sigma^2_{|\mathbf{y}|} &= E\{|\mathbf{y}|^2\} - E^2\{|\mathbf{y}|\} \\ &= E\{\mathbf{y}^2\} - E^2\{|\mathbf{y}|\} \\ &= \sigma_x^2 + d^2 - E^2\{|\mathbf{y}|\}. \end{aligned} \quad (28)$$

The probability density function, $f_c(c)$, of the common mode rejection ratio $\mathbf{c}$ can be obtained as follows. We want to determine the density of $\mathbf{c}$ in terms of the density of $\mathbf{y}$. Since $c > 0$, $f_c(c) = 0\ \forall c \le 0$. The equation $c = |\frac{1}{y}|$ has two solutions for $c > 0$,

$$y_1 = \frac{1}{c}, \qquad y_2 = -\frac{1}{c}. \quad (29)$$

From the fundamental theorem of determining the density of a function of a random variable [7], the pdf of $\mathbf{c}$ is then

$$\begin{aligned} f_c(c) &= \frac{f_y(y_1)}{|g'(y_1)|} + \frac{f_y(y_2)}{|g'(y_2)|} \\ &= \frac{1}{c^2}\left[f_y\left(\frac{1}{c}\right) + f_y\left(-\frac{1}{c}\right)\right], \end{aligned} \quad (30)$$

where $f_y(y)$ is the pdf of $\mathbf{y}$, and $g(y) = |\frac{1}{y}|$. Since from (25) the pdf of $\mathbf{y}$ is

$$f_y(y) = \frac{1}{\sqrt{2\pi}\sigma_x}exp\left[-\frac{(y-d)^2}{2\sigma_x^2}\right], \quad (31)$$

the pdf of the common-mode rejection ratio $\mathbf{c}$ becomes

$$f_c(c) = \frac{1}{\sqrt{2\pi}\sigma_x c^2}\left[exp\left\{-\frac{(1-dc)^2}{2\sigma_x^2c^2}\right\} + exp\left\{-\frac{(1+dc)^2}{2\sigma_x^2c^2}\right\}\right], \quad c > 0 \quad (32)$$

The probability density curves of $\mathbf{c}$ are shown in Fig.3 where $r = |d/\sigma_x|$ and the $CMRR_D^{-1}$ of the op-amp in Fig.1 was used for d. These curves show that the pdf of $\mathbf{c}$ is similiar to a Gaussian density function, but the symmetry is somewhat skewed and the mean is finite. Since we know the pdf of $\mathbf{c}$, we can find the mean and variance from the expressions,

$$E\{\mathbf{c}\} = \int_0^\infty c f_c(c) dc \quad (33)$$

$$\sigma_c^2 = E\{\mathbf{c}^2\} - E^2\{\mathbf{c}\}. \quad (34)$$

If $|\mathbf{y}|$ is concentrated near its mean, then $E\{\mathbf{c}\}$ and σ_c^2 can be approximated from the procedure of estimating the mean and variance of the functions of a random variable [7]. The approximated $E\{\mathbf{c}\}$ and σ_c^2 are

$$E\{\mathbf{c}\} \simeq \frac{1}{E\{|\mathbf{y}|\}}\left[1 + \left(\frac{\sigma_{|\mathbf{y}|}}{E\{|\mathbf{y}|\}}\right)^2\right] \quad (35)$$

$$\sigma_c^2 \simeq \left(\frac{\sigma_{|\mathbf{y}|}}{E^2\{|\mathbf{y}|\}}\right)^2. \quad (36)$$

The mean and variance of $|\mathbf{y}|$ are given in (26) and (28). From (32)-(36), it is clear that the statistical characteristics of the common-mode rejection ratio, i.e., its mean, variance, and pdf, can be readily obtained if the variance of the process parameters are known.

The statistical parameters of the CMRR of the sample op-amp were calculated using the derived equations and the data in Table 2. The approximated equations (35) and (36) were used to calculate $E\{\mathbf{c}\}$ and σ_c. The calculated results are listed in Column A of Table 3. In order to investigate the correctness of these derived equations, 200 Gaussian random numbers with zero mean and variance d were generated and used to calculate the corresponding parameters. From these sample data of the random variable $\mathbf{x}$, the sample data of $|\mathbf{y}|$ and $\mathbf{c}$ can be obtained using (21) and (22). Their calculated mean and variance are shown in Column B. The $E\{|\mathbf{y}|\}$ and $\sigma_{|\mathbf{y}|}$ from the derived equations are very close to those from the generated sample data, but the $E\{\mathbf{c}\}$ and σ_c of Column A somewhat differ from those of Column B because the $E\{\mathbf{c}\}$ and σ_c were calculated from the approximated equations (35) and (36). The histogram of the generated random data of $\mathbf{x}$ and the CMRR histogram are shown in Fig.4 and Fig.5. Since the $r(=|d/\sigma_x|)$ of the sample op-amp in Fig.1 is 2.2, Fig.5 corresponds to the curve (r=2.2) of Fig.3. These two plots are very similiar and support the model of equation (32) for the pdf of $\mathbf{c}$.

IV. Definition of the CMRR for processes

The random offset of CMOS amplifiers has been defined for processes as three times its standard deviation. The reason is that the offset voltage has a Gaussian distribution, so 99.7% of a sample satisfies the specification. However, attention has not been paid to the random CMRR of CMOS amplifiers, and no definition of the CMRR including random components has been made. Thus, the CMRR of CMOS op-amps for processes will be defined in this section.

In the previous section we found the pdf $f_c(c)$ of the CMRR. We want to find $\hat{c}$ such that 99.86% of a sample set has their CMRR greater than $\hat{c}$. Integration of the pdf, $f_c(c)$, from $\hat{c}$ to infinity gives the following results:

$$\int_{\hat{c}}^\infty f_c(c) dc = P(a) + P(b) - 1 \quad (37)$$

where

$$a = \frac{1/\hat{c} - d}{\sigma_x} \quad \text{and} \quad b = \frac{1/\hat{c} + d}{\sigma_x}. \quad (38)$$

From the equation (38) we can see that a is always greater than b by $2|d/\sigma_x|$ since d is negative for the sample op-amp. Thus, $P(a)$ is also greater than $P(b)$ because the function $P(x)$ defined in (27) increases from 0.5 to 1.0 as x increases from 0 to ∞. Since we want to make

$$\int_{\hat{c}}^\infty f_c(c) dc \geq 0.9986, \quad (39)$$

$P(b)$ should be very close to 1.0. This means that $P(a)$ is almost 1.0. In most cases, $|d/\sigma_x| > 0.5$, so $a > b + 1$. Therefore, under the condition in (39), the approximation

$$\int_{\hat{c}}^\infty f_c(c) dc \simeq P(b) \quad (40)$$

can be used. From the equation (39) and (40),

$$P\left(\frac{1/\hat{c} + d}{\sigma_x}\right) \geq 0.9986 \quad (41)$$

and from the table for $P(x)$ in [8],

$$\frac{1/\hat{c} + d}{\sigma_x} \geq 3. \quad (42)$$

We can obtain an integer 3 and thus a nice and simple expression,

$$\hat{c} \leq (3\sigma_x - d)^{-1}. \quad (43)$$

This is the reason why we chose a figure of 0.9986 in (39). If we use $(3\sigma_x - d)^{-1}$ as the CMRR specification in designing CMOS amplifiers, then 99.86% of a large sample will satisfy the specification. If d is positive, then $P(b)$ is greater than $P(a)$ and finally we have

$$\hat{c} \leq (3\sigma_x + d)^{-1}. \quad (44)$$

Therefore, we can define the CMRR for processes as

$$CMRR = (3\sigma_x + |d|)^{-1} \quad (45)$$

where d and σ_x are $CMRR_D^{-1}$ and the standard deviation of $CMRR_R^{-1}$. The $CMRR_D^{-1}$ and $CMRR_R^{-1}$ were defined in (14). The calculated CMRR for the sample op-amp was $56.2dB$. Comparing with the density curve (r=2.2) in Fig.3, we can see that the value $56.2dB$ is very reasonable.

The CMRR definition for processes of (45) and the CMRR pdf of (32) are general for the op-amps whose deterministic and random components comparably contribute to the total CMRR. If an op-amp has a first stage with differential output, then its deterministic common-mode gain is significantly reduced by the next stage [6]. In this case the deterministic CMRR can be ignored, i.e., $d \simeq 0$ and the obove CMRR definition and the pdf must be changed. If d is near zero, then the pdf of the total CMRR is

$$f_c(c) = \frac{2}{\sqrt{2\pi}\sigma_x c^2} exp\left[-\frac{1}{2\sigma_x^2 c^2}\right], \qquad c > 0. \quad (46)$$

The integration of the pdf from $\hat{c}$ to ∞ becomes

$$\int_{\hat{c}}^\infty f_c(c) dc = 2P\left(\frac{1}{\sigma_x \hat{c}}\right) - 1. \quad (47)$$

Thus, the CMRR definition for processes is

$$CMRR = (3\sigma_x)^{-1} \quad (48)$$

where 99.73% of a sample set will be greater than $(3\sigma_x)^{-1}$.

V. Conclusions

The CMRR of a two-stage CMOS op-amp has been analyzed. Several equations representing the statistical characteristics have been derived. Using these equation, we can readily find the distribution, mean, and variance of the CMRR if the process parameter variations are given. The derived equations have shown that the CMRR pdf is distributed similiar to that of a Gaussian density, but the mean is finite and the symmetry is skewed. The CMRR is defined by $(3\sigma_x + |d|)^{-1}$ for the op-amps which have both dominant deterministic and random CMRR so that 99.86% of a large sample may be greater than the defined value. For the op-amps whose deterministic CMRR is near zero, $(3\sigma_x)^{-1}$ can be used for the definition of the CMRR, where 99.73% of a large sample satisfies the specification. The d is the ratio of the deterministic common-mode gain to the differential-mode gain and the σ_x is the standard deviation of the ratio of the random common-mode gain to the differential-mode gain.

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Table 1: Simulated performance

Specification	Performance
Settling Time (2V Step, 5mV)	16.5 nS
Systematic Input Offset Voltage	0.26 mV
Open Loop Voltage Gain	58.3 dB
Unit Gain Frequency (GB)	59 MHz
Phase Margin	75°
Power Dissipation	16.5 mW
CMRR	62.5 dB

Table 2: Simulated parameter values and component σ values

g_{mi}	$1030\mu A/V$	g_{ml}	$712\mu A/V$
g_o	43.7×10^{-6}	g_{di}	22.0×10^{-6}
$(V_{GS} - V_T)_i$	$0.393V$	$(V_{GS} - V_T)_l$	$0.542V$
σ_L	$0.014\mu m$	σ_W	$0.014\mu m$
$\sigma_{V_{Ti}}$	$1.17mV$	$\sigma_{V_{Tl}}$	$1.57mV$

Table 3: The CMRR statistical characteristics calculated from (A) derived equations (B) 200 generated random numbers.

	A	B
d	-6.55×10^{-4}	
σ_x	2.976×10^{-4}	2.763×10^{-4}
$E\{\lvert\mathbf{y}\rvert\}$	6.579×10^{-4}	6.612×10^{-4}
$\sigma_{\lvert\mathbf{y}\rvert}$	2.797×10^{-4}	2.691×10^{-4}
$E\{\mathbf{c}\}$	1.795×10^3 (65 dB)	2.017×10^3 (66 dB)
σ_c	6.462×10^2	1.847×10^3

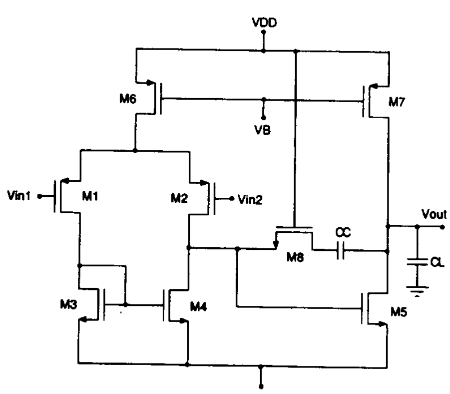

Figure 1: Two-stage CMOS operational amplifier

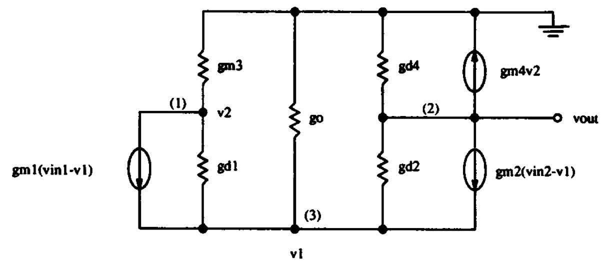

Figure 2: Small signal equivalent circuit of the differential stage

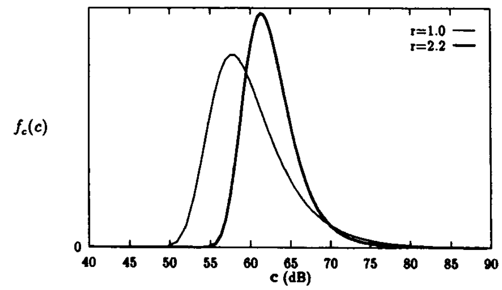

Figure 3: Probability density curves of CMRR ($r = |d/\sigma_x|$)

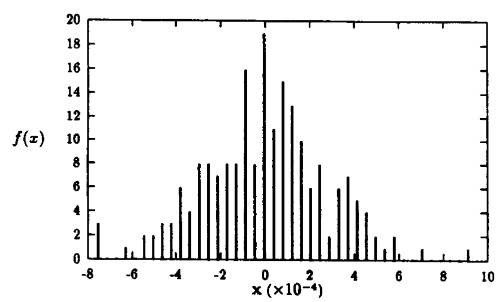

Figure 4: Histogram of the 200 samples generated for the random variable $\mathbf{x}$ ($\mathbf{x} = CMRR_R^{-1}$)

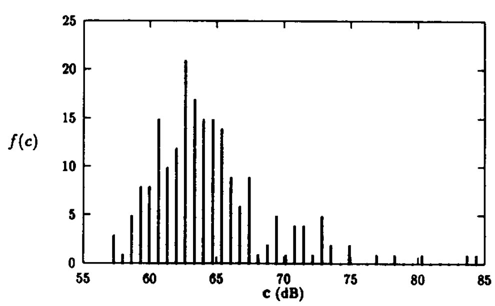

Figure 5: Histogram of the 200 samples calculated from the data in Fig.4 for the random variable $\mathbf{c}$ ($\mathbf{c} = CMRR$)