

Fig. 2. Magnitude Bode plots of (weighted) sensitivity function. (a) $\inf \|WS\|_\infty$ in all stabilizing controllers, (b1) S with a stable controller obtained by the proposed algorithm, and (b2) WS with a stable controller obtained by the proposed algorithm.

V. NUMERICAL EXAMPLE

We consider sensitivity reduction with stable controllers for the following plant satisfying PIP and the weighting function

$$P(s) = \frac{(s-1)(s-4)}{(s+1)(s-2)(s-3)} \quad W(s) = \left[\frac{s+1}{10s+1} \right]^3.$$

The specification is $\|WS\|_\infty < 0.1$. Taking $\lambda = \gamma = 0.1$, a solution of the N-P problem is

$$\eta(s) = \frac{-1.1564s^2 + 3.6119s - 5.3601}{1.1800s^2 + 3.7986s + 5.4417}$$

which interpolates $\eta(1) = -0.2787$, $\eta(4) = -0.2383$, and $\eta(\infty) = -0.9800$ given by (15). Then, using (17), S_W is given by

$$S_W(s) = 0.001 \frac{(s + 1.1022 \times 10^{-2})(s + 3.1399 \times 10^2)}{(s + 1.6095 - 1.4216i)(s + 1.6095 + 1.4216i)}.$$

Finally, by using (18) we can obtain a stable controller of degree five as

$$C(s) = -2.981 \times 10^2 \cdot \frac{(s+1)(s-0.7739)(s+0.3027)(s^2+0.3646s+9.901 \times 10^{-2})}{(s+0.1)^3(s+1.102 \times 10^{-2})(s+3.14 \times 10^2)}.$$

(27)

Bode plots of S and WS are shown in Fig. 2 (b1), (b2). We can see that $\|WS\|_\infty < 0.1$ is satisfied and a significant sensitivity reduction at low frequencies is attained. In this case, the infimum of λ obtained by binary search is 0.03881. This value is very close to 0.03879 which is the infimum in all stabilizing controllers.

VI. CONCLUSIONS

In this note, it has been shown that the \mathcal{H}^∞ optimal controller of weighted sensitivity minimization almost always has $l-1$ unstable poles when the plant has l unstable zeros. An algorithm for finding solutions to the sensitivity reduction problem with stable controllers has been proposed, which is based on

solving the N-P problem. In the algorithm, the upper bound of the weighted sensitivity can be minimized by using binary search.

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Application of Kharitonov's Theorem to Mechanical Systems

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Abstract—In this note, we apply Kharitonov's theorem to derive a robust stability condition for PID controlled multi-degree-of-freedom mechanical systems. The characteristic equation of such a system is given as a determinant of a third-order polynomial with matrix coefficients from which a scalar interval polynomial is obtained. We describe a simple procedure for designing PID controllers for these mechanical systems and prove a new Kharitonov-like result which states roughly that a controller designed for an upper bounding inertia matrix results in stable set-point regulation for all other inertias.

I. PROBLEM FORMULATION

Consider the dynamics of multi-degree-of-freedom mechanical system (e.g., robot manipulator) given by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = F \quad (1.1)$$

where θ is a vector of joint displacements, $M(\theta)$ is the configuration-dependent symmetric positive definite inertia matrix, $C(\theta, \dot{\theta})$ is the vector of centrifugal and coriolis forces (quadratic in $\dot{\theta}$), $G(\theta)$ is the gravity force vector, and F is the vector of applied joint forces. If a multivariable PID set-point controller

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with gravity compensation is used, the control is given by

$$F = -K_I \int_0^t [\theta(\tau) - \theta_d] d\tau - K_P [\theta - \theta_d] - K_D [\dot{\theta} - \dot{\theta}_d] + G(\theta) \quad (1.2)$$

where θ_d is the constant set-point and K_D , K_P , K_I are symmetric (usually diagonal in practice) controller gain matrices. Defining $\epsilon \triangleq \int_0^t [\theta(\tau) - \theta_d] d\tau$ and linearizing the combined dynamics of the robot and controller about the equilibrium point $(\epsilon, \theta, \dot{\theta}) = (0, \theta_d, 0)$ yields

$$M(\theta_d)\epsilon^{(3)} + K_D\dot{\epsilon} + K_P\epsilon + K_I\epsilon = 0. \quad (1.3)$$

If these linearized dynamics are exponentially stable, then Lyapunov's indirect method (see [1] page 179) implies the (local) exponential stability of the equilibrium point $(0, \theta_d, 0)$ for the combined nonlinear system (1.1) and (1.2). However, with θ_d time varying stability is not guaranteed. The interested reader is referred to [2] and [3] for approaches to the nonlinear tracking control problem.

The problem addressed in this note is that of finding set-point controller gains K_P , K_I , K_D such that (1.3) is stable for each fixed θ_d . Our results only provide theory for the set-point regulation problem. However, the results can provide guidance to practitioners who iteratively tune PID tracking controller gains at a family of set-points representing the range of robot inertias.

We begin by designing controller gains which stabilize the system (1.3) for each fixed θ_d . These dynamics are depicted graphically in Fig. 1. Since robots and other mechanical systems usually have revolute joints, or prismatic joints with limited motion, the set of inertias is assumed to be continuously parameterized by a parameter in a compact set, Θ . Thus, there exists positive definite symmetric matrices \bar{M} and \underline{M} such that $\underline{M} \leq M(\theta) \leq \bar{M} \forall \theta \in \Theta$. This raises two important Kharitonov-like stability questions. 1) Under what conditions will the PID regulated robot manipulator be stable for the whole class of inertias as depicted in Fig. 1? 2) How do we design a robust stabilizing controller if bounds on the inertia matrix are known?

Question 1) is answered in Section II where a simple sufficient condition for robust stability of a large class of PID regulated mechanical systems is derived. Question 2) is answered in Section III by a new procedure for designing a stabilizing controller. In addition it is shown that a controller designed based on an inertia matrix larger than all others in the family stabilizes the entire class. This is our new "Kharitonov-like" result. A two-link planar manipulator example is given in Section IV to illustrate the results. In Section V, we show via a counterexample that an intuitive extension of our result is not true. Our conclusions are made in Section VI.

II. APPLICATION OF KHARITONOV'S THEOREM

Kharitonov's theorem [4] provides a powerful criterion for the strict Hurwitz property of a family of polynomials with coefficients varying within given intervals. This well-known theorem states that the strict Hurwitz property of the entire family is equivalent to the strict Hurwitz property of four specially constructed vertex polynomials. This number can be reduced for polynomials of degree less than six [5], [6].

Here we apply the simplification of Kharitonov's theorem for third-order polynomials to find a robust stability condition for PID controlled robot manipulators. It is of interest to ascertain whether or not the stability of the family of polynomials can be

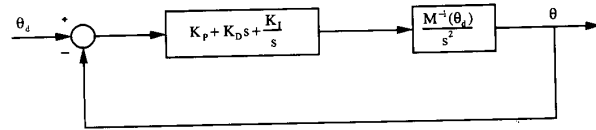


Fig. 1. PID control of a multi-DOF mechanical system.

determined by checking only some extremal polynomials. In Minnichelli *et al.*'s specialization of Kharitonov's theorem to third-order interval polynomials tests the stability of the whole family with just one vertex polynomial [6]. We now extend these ideas to our mechanical system (1.3).

The characteristic equation of (1.3) is easily computed to be

$$\chi(s) = \det [M(\theta_d)s^3 + K_D s^2 + K_P s + K_I]. \quad (2.1)$$

Let λ be a root of $\chi(s)$ for the fixed $\theta_d \in \Theta$. Then there exists an associated "mode shape" ν with unit 2-norm satisfying

$$[M(\theta_d)\lambda^3 + K_D\lambda^2 + K_P\lambda]\nu = 0. \quad (2.2)$$

Multiplying this equation on the left by the conjugate transpose of ν , ν^* , yields a polynomial in λ with real coefficients

$$a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0 \quad (2.3)$$

where

$$a_3 = \nu^* M(\theta_d) \nu, \quad a_2 = \nu^* K_D \nu, \quad a_1 = \nu^* K_P \nu, \quad a_0 = \nu^* K_I \nu. \quad (2.4)$$

Observe that

$$\begin{aligned} a_3 &\in [\lambda_{\min}(\underline{M}), \lambda_{\max}(\bar{M})] \triangleq [a_3, \bar{a}_3] \\ a_2 &\in [\lambda_{\min}(K_D), \lambda_{\max}(K_D)] \triangleq [a_2, \bar{a}_2] \\ a_1 &\in [\lambda_{\min}(K_P), \lambda_{\max}(K_P)] \triangleq [a_1, \bar{a}_1] \\ a_0 &\in [\lambda_{\min}(K_I), \lambda_{\max}(K_I)] \triangleq [a_0, \bar{a}_0]. \end{aligned} \quad (2.5)$$

Equation (2.3) is therefore an interval polynomial. The stability of this interval polynomial can be verified by checking just *one* of the Kharitonov polynomials. According to [6], (2.3) is Hurwitz if the following polynomial is Hurwitz

$$\chi(\lambda) = \bar{a}_3\lambda^3 + \underline{a}_2\lambda^2 + \underline{a}_1\lambda + \bar{a}_0. \quad (2.6)$$

The test can be further simplified by using the Routh-Hurwitz stability test which requires

$$\begin{aligned} &\text{i) } \bar{a}_0 > 0 \\ &\text{ii) } \underline{a}_2 > 0 \\ &\text{iii) } \bar{a}_3 > 0 \\ &\text{iv) } \frac{\underline{a}_1 \underline{a}_2 - \bar{a}_0 \bar{a}_3}{\underline{a}_2} > 0. \end{aligned} \quad (2.7)$$

Conditions i), ii), and iii) are satisfied if K_I , K_D , and \bar{M} are positive definite. In this case $\underline{a}_1 \underline{a}_2 > \bar{a}_0 \bar{a}_3$ implies iv). Therefore, under the assumption of symmetric positive definite gains, the closed-loop system given by (2.1) is stable if

$$\lambda_{\max}(\bar{M}) < \frac{\lambda_{\min}(K_D) \lambda_{\min}(K_P)}{\lambda_{\max}(K_I)}. \quad (2.8)$$

This is only a sufficient condition, but this condition is tight for some practical numerical experiments. The assumption of symmetric positive definite gains can be justified by the design procedure described in the next section.

III. CONTROLLER SYNTHESIS

In this section, we describe a simple procedure of choosing PID gains and show that a controller design based on an upper bounding inertia matrix stabilizes the entire class of inertias. Since the roots of the characteristic equation (2.1) (with M replaced by \bar{M}) remain unchanged if the matrix polynomial is pre and postmultiplied by $\bar{M}^{(-1/2)}(\theta_d)$, we have

$$\chi(s) = \det [Is^3 + K'_D s^2 + K'_P s + K'_I] \quad (3.1)$$

where

$$\begin{aligned} K'_D &= \bar{M}^{(-1/2)} K_D \bar{M}^{(-1/2)}, \\ K'_P &= \bar{M}^{(-1/2)} K_P \bar{M}^{(-1/2)}, \\ K'_I &= \bar{M}^{(-1/2)} K_I \bar{M}^{(-1/2)}. \end{aligned} \quad (3.2)$$

Since the transformed inertia matrix is the identity matrix, the design can be treated as a set of n decoupled PID controller designs which can be accomplished by, say, pole placement. We choose the primed gain matrices to be scalar multiples of the identity matrix:

$$K'_D = k_d I, \quad K'_P = k_p I, \quad K'_I = k_i I. \quad (3.3)$$

The controller gains used for implementation are therefore given by

$$K_D = k_d \bar{M}, \quad K_P = k_p \bar{M}, \quad K_I = k_i \bar{M}. \quad (3.4)$$

A surprising fact is summarized in the following theorem.

Theorem: Let $\det[\bar{M}s^3 + K_D s^2 + K_P s + K_I]$ be the characteristic equation of a stable PID controlled mechanical system with the controller gain matrices chosen based on the above procedure. Then the characteristic equation

$$\chi(s) = \det [Ms^3 + K_D s^2 + K_P s + K_I]$$

is Hurwitz whenever $0 < M \leq \bar{M}$.

Proof: Since $\det[\bar{M}s^3 + k_d \bar{M}s^2 + k_p \bar{M}s + k_i \bar{M}] = \det[(s^3 + k_d s^2 + k_p s + k_i) \bar{M}]$ is Hurwitz by design, the Routh-Hurwitz stability test yields the stability condition

$$k_i < k_d k_p. \quad (3.5)$$

Let λ be a root of the characteristic equation $\det[Ms^3 + k_d Ms^2 + k_p Ms + k_i M] = 0$. Then there exists a vector ν of unit 2-norm satisfying

$$\begin{bmatrix} M\lambda^3 + k_d M\lambda^2 + k_p M\lambda + k_i M \end{bmatrix} \nu = 0 \quad (3.6)$$

$$\Leftrightarrow \nu^* M \nu \lambda^3 + k_d \nu^* M \nu \lambda^2 + k_p \nu^* M \nu \lambda + k_i \nu^* M \nu = 0 \quad (3.7)$$

$$\Leftrightarrow \frac{\nu^* M \nu}{\nu^* M \nu} \lambda^3 + k_d \lambda^2 + k_p \lambda + k_i = 0. \quad (3.8)$$

Since $M < \bar{M}$, we have $\nu^* M \nu < \nu^* \bar{M} \nu$. This together with (3.5) gives

$$\frac{\nu^* M \nu}{\nu^* \bar{M} \nu} k_i < k_d k_p \quad (3.9)$$

which implies the stability of

$$\chi(s) = \det [Ms^3 + K_D s^2 + K_P s + K_I] \quad (3.10)$$

and the proof is complete. \square

Corollary: For $\chi(s)$ define the stability degree $\triangleq -\max_{\lambda_i} (\text{Re}(\lambda_i))$ where the λ_i are its roots. Let σ be the stability degree

of $\chi(s)$ with M replaced by \bar{M} . If $\sigma < k_p/2k_d$, then the stability degree of $\chi(s)$ is greater than or equal to σ .

Proof: This follows directly from a change of coordinates $s \rightarrow s + \sigma$.

IV. EXAMPLE

In this example, we use the controller design procedure described in Section III to examine the stability degree of the closed-loop system for various inertias. Consider the two-link planar manipulator with revolute joints and point masses at the distal end of the links as shown in Fig. 2. The dynamics of the manipulator are given by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) = F \quad (4.1)$$

where $F = [f_1 \ f_2]^T$ is the vector of applied joint forces and

$$M(\theta) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (4.2)$$

is the inertia matrix with

$$\begin{aligned} m_{11} &= (m_1 + m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(\theta_2) \\ m_{12} &= m_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2) \\ m_{22} &= m_2 l_2^2. \end{aligned} \quad (4.3)$$

$C(\theta, \dot{\theta}) = [c_1 \ c_2]^T$ is the vector of centrifugal and coriolis forces with

$$\begin{aligned} c_1 &= -2m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2) - m_2 l_1 l_2 \dot{\theta}_2^2 \sin(\theta_2) \\ c_2 &= m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2) \end{aligned} \quad (4.4)$$

and $G(\theta) = [G_1 \ G_2]^T$ is the gravity force vector with

$$\begin{aligned} G_1 &= (m_1 + m_2)gl_1 \cos(\theta_1) + m_2 l_2 g \cos(\theta_1 + \theta_2) \\ G_2 &= m_2 gl_2 \cos(\theta_1 + \theta_2). \end{aligned} \quad (4.5)$$

Suppose the desired set point is $(\epsilon, \theta, \dot{\theta}) = (0, \theta_d, 0)$, where $\epsilon = \int_0^t [\theta(\tau) - \theta_d] d\tau$ as defined in Section I. Linearizing about this point gives $M(\theta_d)\ddot{\theta} = F$. Taking $m_1 = m_2 = 1$ and $l_1 = l_2 = 1$, the inertia matrix simplifies to

$$M(\theta_d) = \begin{bmatrix} 3 + 2\cos(\theta_2) & 1 + \cos(\theta_2) \\ 1 + \cos(\theta_2) & 1 \end{bmatrix}. \quad (4.6)$$

An upper bound on the inertia matrix over a given range of θ_2 can be found with the help of the following lemma.

Lemma: Let M_1, M_2 be symmetric positive definite matrices. Let U be the transformation matrix such that $U^T M_1 U = \Sigma_1$ and $U^T M_2 U = \Sigma_2$ are diagonal. Let $\bar{\Sigma}$ be the diagonal matrix defined by $(\bar{\Sigma})_{ii} \triangleq \max((\Sigma_1)_{ii}, (\Sigma_2)_{ii})$. If $\bar{M} = (U^{-1})^T \bar{\Sigma} U^{-1}$, then

$$M_1 \leq \bar{M}, \quad M_2 \leq \bar{M}. \quad (4.7)$$

Furthermore, there is no "smaller" M satisfying this condition, i.e., $\nexists M$ such that

$$M_1 \leq M, \quad M_2 \leq M, \quad \text{and } M < \bar{M}. \quad (4.8)$$

Proof: The proof is straightforward and hence omitted.

This lemma can be used to generate a numerical upper bound on the family $M(\theta_d)$ by 1) discretizing the set, 2) choosing two members and finding an upper bound on these, 3) choosing another member and finding bound on the previous upper bound and the new member etc. The upper bound generated will, in general, depend on the order that the elements are scanned, but can be used in the design procedure none-the-less.

Next, we design a robust controller based on an upper bounding inertia for the two-link manipulator. Suppose the desired set point θ_d has the property that $\theta_2 \in [0, \pi/2]$. For this particular example, an upper bound \bar{M} is generated using the above

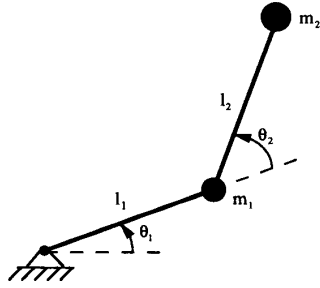


Fig. 2. Two-link planar manipulator.

lemma just once with $M_1 = M(\theta_2 = 0)$ and $M_2 = M(\theta_2 = \pi/2)$. The upper bound is given by

$$\bar{M} = \begin{bmatrix} 5.0607 & 1.8536 \\ 1.8536 & 1.3536 \end{bmatrix}. \quad (4.9)$$

If we place the closed-loop poles for each of the decoupled systems at $-1, -0.1 + j, -0.1 - j$, then the required controller gains are

$$\begin{aligned} K'_D &= \begin{bmatrix} 1.2000 & 0.0000 \\ 0.0000 & 1.2000 \end{bmatrix}, & K'_P &= \begin{bmatrix} 1.2100 & 0.0000 \\ 0.0000 & 1.2100 \end{bmatrix}, \\ K'_I &= \begin{bmatrix} 1.0100 & 0.0000 \\ 0.0000 & 1.0100 \end{bmatrix}. \end{aligned} \quad (4.10)$$

Using (3.4)

$$\begin{aligned} K_D &= \begin{bmatrix} 6.0728 & 2.2243 \\ 2.2243 & 1.6243 \end{bmatrix}, & K_P &= \begin{bmatrix} 6.1234 & 2.2428 \\ 2.2428 & 1.6378 \end{bmatrix}, \\ K_I &= \begin{bmatrix} 5.1113 & 1.8721 \\ 1.8721 & 1.3671 \end{bmatrix}. \end{aligned} \quad (4.11)$$

To check that the design is indeed stable for all $\theta_d \in [0, \pi/2]$ we compute the eigenvalues of the closed-loop system as a function of θ_d and plot the stability degree of the system as a function of θ_d (see Fig. 3). Note that the system is stable for $0 \leq \theta_d \leq \pi/2$ (i.e., $M \leq \bar{M}$) with a stability degree greater than the stability degree corresponding to \bar{M} . When θ_d is increased beyond $\pi/2$, increasing the inertia above \bar{M} , the stability degree decreases and the system (1.3) is eventually destabilized.

V. COUNTEREXAMPLE

Following the positive results of the theorem, it is natural to conjecture that Kharitonov's theorem can be generalized to polynomials with symmetric matrix coefficients lying in matrix intervals. The answer is negative as illustrated by the following counterexample. Let

$$\begin{aligned} K_D &= \begin{bmatrix} 0.4970 & 0.2752 \\ 0.2752 & 0.3153 \end{bmatrix}, & K_P &= \begin{bmatrix} 0.5235 & 0.4767 \\ 0.4767 & 0.8830 \end{bmatrix}, \\ K_I &= \begin{bmatrix} 0.4387 & 0.4827 \\ 0.4827 & 1.1720 \end{bmatrix}, \\ \bar{M} &= \begin{bmatrix} 0.6260 & 0.6930 \\ 0.6930 & 0.8624 \end{bmatrix}, & \underline{M} &= \begin{bmatrix} 0.0673 & 0.0701 \\ 0.0701 & 0.1258 \end{bmatrix}, \\ M &= \begin{bmatrix} 0.2008 & 0.2677 \\ 0.2677 & 0.4234 \end{bmatrix}. \end{aligned} \quad (5.1)$$

It is easy to check that $\underline{M} < M < \bar{M}$ and that the equations

$$\chi(s) = \det[\bar{M}s^3 + K_Ds^2 + K_Ps + K_I] \quad (5.2)$$

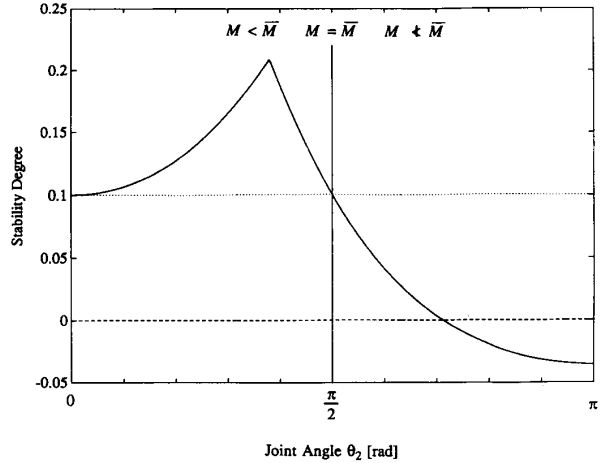


Fig. 3. Stability degree versus regulated joint angle θ_2 [rad].

and

$$\chi(s) = \det[Ms^3 + K_Ds^2 + K_Ps + K_I] \quad (5.3)$$

are Hurwitz. But

$$\chi(s) = \det[Ms^3 + K_Ds^2 + K_Ps + K_I] \quad (5.4)$$

is not. Hence, it is not sufficient to check the extremal polynomials 5.2 and 5.3. Our result is therefore very particular to mechanical systems and our design procedure.

VI. CONCLUSION

In this note, a simple sufficient condition for robust stability of a large class of PID-controlled mechanical systems was derived. This adds to the works of Shiel *et al.* [7] who found the conditions for the stability of second-order matrix polynomials. This is one of a few realistic applications of Kharitonov's theorem and serves as additional motivation for pursuing results of this kind. A procedure for designing a stabilizing controller is outlined and it is shown that a controller designed based on an upper bounding matrix stabilizes all other inertias. Furthermore, an example is presented to show that Kharitonov's theorem for interval polynomials cannot be generalized to polynomials with symmetric matrix coefficients lying in matrix intervals.

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