

# Adaptive Linearization of Hybrid Step Motors: Stability Analysis

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**Abstract**—The adaptive linearization of step motors is an attractive method for improving motor performance at relatively low cost. However, due to the lack of robustness, a direct application of a standard approach does not work well. In this paper we present a new adaptive linearization scheme for torque-ripple cancellation and establish stability and robustness. By taking a new approach in parameterizing the motor dynamics, we reduce the number of adapted parameters by a factor of two relative to the standard approach. This new parameterization and the unique periodic property of the motor enable us to find conditions on exogenous signals which guarantee persistency of excitation. As with many challenging control applications, the adaptive motor linearization problem raises new and fundamental theoretical questions. For example, we develop a new robustness result which, roughly speaking, shows that the allowable model perturbation does not decrease in size as the adaptation rate is slowed. This is accomplished with a unique dual-Lyapunov-function technique. Also, the kind of perturbations we consider include nonlinear dependence on state and parameter error. Finally, this nonlinear adaptive control scheme has been successfully implemented. Experimental results demonstrate over 30 db reduction in torque ripple.

## I. INTRODUCTION

THE control of electric motors is a traditional control problem which has attracted new interest in the control theory community. The reasons for this are the development of adaptive nonlinear control theory, the low cost of high-performance digital control hardware, and an increase in demanding applications for electric motors. One such application is the actuation of direct-drive robots where high torque and high linearity (low torque ripple) are required. The application which motivated this work is the actuation of a cylindrical coordinate robot for silicon wafer transfer in a vacuum environment. In this case, torque ripple must be reduced to prevent excitation of structural vibrations and to reduce the risk of damage to wafers.

Torque ripple in electric motors can be reduced either by design or by control. The desirable detent torque of step motors (which provides passive braking in the absence of power) is also reduced when a step motor is designed for small torque ripple. Hence, the reduction of torque ripple through control is an attractive option lead-

ing to better overall performance. Interest in torque-ripple reduction in the control community is fairly recent. Le-Huy and Perret [1] make torque-ripple comparisons for brushless DC motor drives for two and three stator phases and several commutation waveforms, while de la Ree and Boules [2] give a detailed analysis of torque production with rectangular current excitation. In Nagase *et al.* [3] velocity ripple is filtered through a band-pass filter and fed back to the current-amplitude control loop to avoid structural resonances. Torque ripple due to geometric imperfection is addressed by Murai *et al.* [4] where two types of nonsinusoidal flux distributions are considered and two heuristic switching strategies for torque-ripple reduction are proposed. In this paper, we use a sinusoidal commutation waveform together with adaptive linearization which adaptively adds periodic signals to the motor input current. This appears to be the first systematic approach to torque-ripple reduction via adaptive control.

Globally linearizing control is another very promising approach to torque-ripple reduction and is first applied to variable reluctance motors by Taylor *et al.* [5], [6]. This methodology has the potential (in theory) to completely eliminate torque ripple by introducing static nonlinear compensation in the commutation waveforms. This compensation depends on shaft angle and phase currents. The results of Taylor's work prove the value of the linearization approach and encourage further research. Other work in this area is by Hemati and Leu [7], [8] who also study nonadaptive linearization of brushless DC motors and take saturation into account. Although linear model-reference adaptive control has been used in motion control [9], [10], nonlinear adaptive motor control is new and is addressed here. Marino *et al.* [11] have also made contributions to adaptive partial linearization of the nonlinear current-flux interaction in induction motors.

In addition to the previous work on motor control, recent nonlinear adaptive control theory is background for this study. Sastry and Isidori [12] present a general adaptive control scheme for linearizable systems with Lipschitz nonlinearities. This approach achieves convergence of tracking error. By introducing a certain matching condition, Marino *et al.* [13] succeed in eliminating the Lipschitz condition on the nonlinearity. With the same matching condition, Taylor *et al.* [14] consider adaptive regulation of nonlinear systems and establish robustness to unmodeled stable fast dynamics. Using a back-stepping

Manuscript received June 1, 1992; revised October 5, 1992. Paper recommended by Past Associate Editor, G. C. Verghese.

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IEEE Log Number 9208595.

technique, Kanellakopoulos *et al.* extend this result to systems satisfying an extended matching condition [15], or in a more general parametric-pure-feedback form [16]. Pomet and Praly [17] work outside the framework of linearizable systems and have nonlinear adaptive control results for a class of stabilizable nonlinear systems.

Except in the case of Pomet and Praly [17], where parameters are guaranteed to be bounded by using a projection method, other schemes provide only partial stability (convergence of tracking error) and, in [14], [15], limited robustness. Persistency of excitation, parameter convergence and robustness to other types of perturbations are not guaranteed and impose potential instability problems in practical implementations. In the motor control case, these adaptive linearization schemes reduce to essentially the same scheme, which is hereafter called the *standard approach*. Unfortunately, it does not work well for motor speed control due to nonpersistent excitation. As a consequence, unmodeled nonlinear dynamics cause instability of the parameter estimate. Motivated by this, we introduce a different approach in this paper. A different, easy-to-implement parameterization is proposed. Although the analysis of our scheme is complex, stability, robustness and parameter convergence can be guaranteed and an experimental implementation has been successful [18].

The contributions of this study are in motion control and nonlinear adaptive control. In the area of motion control we contribute a stable adaptive control scheme for adaptively linearizing the nonlinearities of step motors. Although structurally very simple, experimental results have shown that it is very effective in reducing torque ripple. The particular parameterization we use reduces the number of adapted parameters by a factor of two relative to the standard approach in the motor control problem. Moreover, the scheme provides the “intelligence” for learning the torque ripple of a motor and cancelling it on-line. It is also possible to use our scheme for adaptive identification.

The particular structure of the motor control problem has also motivated our study of some basic problems of nonlinear adaptive control: First, we present conditions on exogenous signals guaranteeing persistency of excitation in a nonlinear system; these follow from the periodic dynamics of motors. Second, in showing stability we present a new robustness analysis for nonlinear adaptive control systems with nonlinear modeling errors in the state and parameters. This later analysis complements the work of Taylor *et al.* [14] where robustness to unmodeled stable dynamics is addressed.

The remainder of this paper is organized as follows. In the next section, we derive a mathematical model for two-phase hybrid step motors which identifies the source and structure of torque ripple. In Section III, we develop an adaptive linearizing controller using the standard approach and discuss its deficiencies. In Section IV, we take a different parameterization approach and derive a new adaptive linearization controller which has a simple struc-

ture and uses a reduced number of parameters. Section V establishes conditions on exogenous signals which guarantee persistency of excitation and hence lead to exponential stability of the simplified system. In Section VI a general robustness result in slow adaptation is developed using two coupled Lyapunov functions. Applying this result to the motor dynamics, robustness of the model system is established in Section VII. Tracking error and parameter error are shown to converge to a neighborhood of zero with size depending on the size of the residual torque ripple. Section VIII presents experimental results showing over 30 db reduction of torque ripple. Finally, Section IX concludes the paper with some remarks.

## II. MODELING OF HYBRID STEP MOTORS

Step motors have been widely used in many control applications [19]. A full model of a two-phase hybrid step motor consists of the electrical dynamics of the stator coils together with the shaft mechanical dynamics. However, the electric response is much faster than the mechanical response, allowing us to consider the mechanical dynamics only. The use of current amplifiers and the robustness to electrical dynamics results of Taylor *et al.* [14] are further justifications. Additional assumptions used here are that of linear magnetic materials and symmetry between the two motor phases.

With these assumptions, the dynamic equation of the motor shaft angle is given by

$$J\ddot{\theta} = \frac{1}{2}i^t \frac{\partial L}{\partial \theta} i - T_l \quad (1)$$

where  $J$  is the equivalent inertia seen by the rotor shaft,  $\theta$  is the angular position of the shaft,  $T_l$  is the load torque and friction,  $L$  is the  $3 \times 3$   $\theta$ -dependent inductance matrix,  $i^t = (i_a, i_b, i_r)$ ,  $i_a$  and  $i_b$  are the currents in phase  $a$  and phase  $b$ , respectively, and  $i_r$  is a fictitious equivalent rotor current due to the permanent magnet used in field generation.

Before we evaluate the electric torque, we first follow a common practice of applying the so-called  $d-q$  transformation as an initial step towards linearization. This transformation transforms from the natural stator frame to a decoupled quadrature frame fixed to the rotor. The transformed decoupled and quadrature currents  $i_d$  and  $i_q$  are defined by:

$$\begin{pmatrix} i_a \\ i_b \\ i_r \end{pmatrix} = \begin{pmatrix} \cos p\theta & -\sin p\theta & 0 \\ \sin p\theta & \cos p\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} i_d \\ i_q \\ i_r \end{pmatrix}$$

where  $p$  is the number of pole pairs. The “decoupled” current  $i_d$  is so named since it does not contribute to torque production in the ideal motor. Setting  $i_d = 0$ , the above transformation reduces to

$$\begin{aligned} i_a &= -\sin p\theta i_q, \\ i_b &= \cos p\theta i_q. \end{aligned}$$

This way of determining the phase currents from the quadrature current  $i_q$  is termed *sinusoidal commutation*. This commutation, in the ideal motor, generates an electric torque proportional to the quadrature current.

For an ideal motor, each entry of the inductance matrix is a constant offset plus a pure sinusoidal function of  $\theta$  whose frequency is determined by the symmetries of the motor. However, due to the nonsinusoidal gap saliency in real motors, these inductances contain phase shifts and higher order harmonics. This geometric imperfection is the source of torque ripple.

Expanding the inductance in a Fourier series and setting  $i_d = 0$ , (1) is equivalent to

$$\begin{aligned} \ddot{\theta} &= l_0 + \sum_{j=1}^{\infty} [l_{sj} \sin jp\theta + l_{cj} \cos jp\theta] + k_0 i_q \\ &+ i_q \sum_{j=1}^{\infty} [k_{sj} \sin jp\theta + k_{cj} \cos jp\theta] \\ &\triangleq f(\theta) + g(\theta) i_q \end{aligned} \quad (2)$$

where  $k_0$  is the nominal torque constant and

$$f(\theta) = l_0 + \sum_{j=1}^{\infty} [l_{sj} \sin jp\theta + l_{cj} \cos jp\theta], \quad (3)$$

$$g(\theta) = k_0 + \sum_{j=1}^{\infty} [k_{sj} \sin jp\theta + k_{cj} \cos jp\theta].$$

Here we have ignored the nonideal terms in the self and mutual inductances of the stator phases  $L_{aa}$ ,  $L_{bb}$ , and  $L_{ab}$  for simplification. These terms contribute to the variable reluctance torque which is usually made small in hybrid step motors by winding the phase coils properly. This simplification eliminates terms quadratic in  $i_q$  (also periodic in  $\theta$ ) in the above equation.

All sinusoidal terms in (2) are due to geometric imperfection. (Neglecting these and friction yields the ideal motor model). Equation (2) is the model used in deriving our adaptive controller for hybrid step motors. To conclude this section, we make the reasonable assumptions that the nominal torque constant dominates the torque constant variations, i.e.,

$$\left| \sum_{j=1}^{\infty} k_{sj} \sin jp\theta + k_{cj} \cos jp\theta \right| \ll k_0.$$

### III. STANDARD APPROACH TO ADAPTIVE MOTOR LINEARIZATION

For the motor dynamics described in (2), the approach by Sastry and Isidori and that by Taylor *et al.* are equivalent. In this study, we call their approach the *standard approach* for brevity. Note that (2) satisfies the so-called matching condition defined in Taylor *et al.* [14], that is, the control variable  $i_q$  and the torque-ripple uncertainty enter the dynamics in the same place. In this section, we will follow the standard approach and develop an adaptive linearization controller for step-motor control. The rea-

sons why this controller fails to work well in practice are discussed.

Suppose we want to cancel the first  $n$  harmonics ( $n$  can be determined by analyzing the torque-ripple spectrum in an open-loop measurement) and the DC component of the torque ripple. Then, we can define a vector containing shape functions

$$w^t = (1, \sin p\theta, \cos p\theta, \sin 2p\theta, \dots, \cos np\theta), \quad (4)$$

and two parameter vectors

$$P_1 = (l_0, l_{s1}, l_{c1}, \dots, l_{cn})^t, \quad (5)$$

$$P_2 = (k_0, k_{s1}, k_{c1}, \dots, k_{cn})^t.$$

Equation (3) can be rewritten as

$$f(\theta) = w^t P_1 + (f(\theta) - w^t P_1),$$

$$g(\theta) = w^t P_2 + (g(\theta) - w^t P_2),$$

and the motor dynamics of (2) rewritten as

$$\ddot{\theta} = w^t P_1 + (w^t P_2) i_q + \zeta, \quad (6)$$

where

$$\zeta = f(\theta) - w^t P_1 + (g(\theta) - w^t P_2) i_q \quad (7)$$

is the residual torque ripple not cancelled by the controller.

Suppose  $\bar{P}_1$  and  $\bar{P}_2$  are the estimates of  $P_1$  and  $P_2$ , respectively. Ignoring the term  $\zeta$  and using the certainty equivalence [20] principle, a linearizing control law is given by

$$i_q = (w^t \bar{P}_2)^{-1} (v - w^t \bar{P}_1). \quad (8)$$

Substituting (8) into (6) results in

$$\ddot{\theta} = v - w^t \bar{P}_1 - i_q w^t \bar{P}_2 + \zeta, \quad (9)$$

where  $\bar{P}_i \triangleq \bar{P}_i - P_i$ ,  $i = 1, 2$ . Using a simple PD controller for tracking, the error equation becomes

$$\ddot{e} + k_d \dot{e} + k_p e - w^t \bar{P}_1 - i_q w^t \bar{P}_2 + \zeta = 0. \quad (10)$$

Parameter update laws can then be derived using a Lyapunov approach. To do this we first ignore  $\zeta$  to get

$$\ddot{e} + k_d \dot{e} + k_p e - w^t \bar{P}_1 - i_q w^t \bar{P}_2 = 0. \quad (11)$$

Choose the Lyapunov function

$$V = (\dot{e} + k_\alpha e)^2 + k_\alpha (k_d - k_\alpha) e^2 + \bar{P}_1^t \Gamma_1^{-1} \bar{P}_1 + \bar{P}_2^t \Gamma_2^{-1} \bar{P}_2.$$

Its derivative along the solution of (11) is

$$\begin{aligned} \dot{V} &= -2(k_d - k_\alpha) \dot{e}^2 - k_\alpha k_p e^2 \\ &+ \left( \dot{\bar{P}}_1^t \Gamma_1^{-1} + (\dot{e} + k_\alpha e) w^t \right) \bar{P}_1 \\ &+ \left( \dot{\bar{P}}_2^t \Gamma_2^{-1} + (\dot{e} + k_\alpha e) i_q w^t \right) \bar{P}_2. \end{aligned}$$

The last two terms are made zero by choosing

$$\dot{\bar{P}}_1 = \dot{\hat{P}}_1 = -\Gamma_1 (\dot{e} + k_\alpha e) w, \quad (12)$$

$$\dot{\bar{P}}_2 = \dot{\hat{P}}_2 = -\Gamma_2 (\dot{e} + k_\alpha e) i_q w. \quad (13)$$

Equation (8) together with the update law of (12) and (13) defines the standard approach to adaptive linearizing control of step motors. Since the update laws are obtained by making  $\dot{V}$  nonpositive in a Lyapunov approach, boundedness of  $\dot{e}$ ,  $e$ ,  $\bar{P}_1$ , and  $\bar{P}_2$  are guaranteed automatically for the error dynamics equation (11).

However, since  $i_q$  involves division by  $\bar{P}_2$ , its boundedness, and therefore the boundedness of  $\ddot{e}$ , is not ensured. Consequently, error convergence of  $e$ ,  $\dot{e} \rightarrow 0$  is not guaranteed. This is usually fixed in the following way. Since we know that  $g(\theta)$  is always positive definite, we can limit the variations of  $\bar{P}_2$  during updating in such a way that  $w^i \bar{P}_2 \geq \epsilon > 0$ . Then  $i_q$  in (8) is bounded, which leads to bounded  $\ddot{e}$  in (11). And the convergence of  $e$  and  $\dot{e}$  follows by a standard argument (see e.g., [27]).

Convergent as the above  $e$  and  $\dot{e}$  may be, parameter convergence is difficult to obtain. The reason is that the same error signal  $\dot{e} + k_\alpha e$  is used for the updating of both  $\bar{P}_1$  and  $\bar{P}_2$ . Intuitively, if there is one torque-ripple harmonic causing the error signal, it is difficult to determine whether it is due to harmonics of  $f$  or  $g$ . Hence, making corresponding corrections to  $\bar{P}_1$  or  $\bar{P}_2$  is difficult.

Without parameter convergence, the adaptive system is not robust to uncertainties. The addition of the term  $\zeta$ , which is not cancelled by the control, will cause parameter drift and instability in practice. Another drawback of overparameterization in the standard approach is the large number of parameters used. This poses computational problems in real-time implementation.

#### IV. ADAPTIVE LINEARIZATION USING REDUCED PARAMETERIZATION

In this section, we design a new adaptive linearization controller for torque-ripple cancellation using reduced parameterization. The controller adaptively introduces some ripple into the input current to cancel the torque ripple of the motor. The update law for tuning the adapted parameters is derived using a Lyapunov approach.

Although the motor equation of (2) is in the standard affine-in-control form, we do not parametrize  $f$  and  $g$  separately as in the standard approach. Instead, we isolate the "ideal motor part" in (2) from those nonideal torque-ripple terms, and rewrite (2) as

$$\ddot{\theta} = k_0 i_q + q, \quad (14)$$

where  $k_0$  is the d.c. component of  $g$  and  $q = (g(\theta) - k_0) i_q + f(\theta)$ .

Our goal is to achieve smooth motion by cancelling the "nearly periodic" term  $q$ . The idea behind our control law is extremely simple. In order to cancel  $q$ , we intentionally add some ripple to the input current to cancel the torque ripple. Since the actual torque ripple is taken as unknown, we choose a linearly independent set of periodic shape functions (which will become the regressor vector) and adaptively tune the coefficients. We have two problems to solve: 1) design a parameter update law to ensure the asymptotic cancellation of  $q$ , and 2) design a tracking

controller which will ensure good performance during and after adaptation.

To implement this idea, we first define a regressor vector

$$w^t = (1, \sin p\theta, \cos p\theta, \sin 2p\theta, \dots, \cos np\theta), \quad (4)$$

a parameter vector, which is to be tuned adaptively,

$$\bar{P}^t = (\bar{k}, \bar{k}_{s1}, \bar{k}_{c1}, \bar{k}_{s2}, \dots, \bar{k}_{cn}),$$

and let

$$i_q = \frac{1}{k_0} (v - w^t \bar{P}), \quad (15)$$

where  $-w^t \bar{P}$  is the ripple added to the current, and  $u$  a new control input. Substituting (15) into (5) yields

$$\ddot{\theta} = v + q - w^t \bar{P}. \quad (16)$$

We think of  $w^t \bar{P}$  as an approximation to the first  $n$  harmonics of the torque ripple  $q$  and we tune  $\bar{P}$  to cancel them. Also, we need to design a tracking controller to ensure good performance during and after adaptation. Let  $\theta_d$  be a desired trajectory with  $\dot{\theta}_d$  and  $\ddot{\theta}_d$  bounded. Choose a PD control

$$v = \ddot{\theta}_d + k_d (\dot{\theta}_d - \dot{\theta}) + k_p (\theta_d - \theta) \quad (17)$$

with  $k_p, k_d > 0$  to ensure exponential tracking when  $q - w^t \bar{P} = 0$ . Substituting (17) into (16) yields the error dynamics

$$\ddot{e} + k_d \dot{e} + k_p e + q - w^t \bar{P} = 0,$$

where  $e = \theta_d - \theta$  is the output error. Now let  $P^*$  be the unknown desired parameter vector. Let  $\zeta = q - w^t P^*$  denote the residual torque ripple when the parameters are set to the desired values. Finally, let  $\bar{P} = \bar{P} - P^*$  denote the parameter error vector. Then the above equation can be written as the following:

$$\ddot{e} + k_d \dot{e} + k_p e - w^t \bar{P} + \zeta = 0. \quad (18)$$

As we did in the last section, we first treat the case  $\zeta = 0$  which represents a simplified situation when  $q$  contains only  $n$  torque-ripple harmonics. Setting  $\zeta = 0$  the error dynamics equation becomes

$$\ddot{e} + k_d \dot{e} + k_p e - w^t \bar{P} = 0. \quad (19)$$

Note that this is of the same form as the error equation in the standard approach of last section, except 1) the number of parameters is  $2n + 1$  instead of  $4n + 2$ , 2) the regressor contains only periodic functions of  $\theta$  and hence is bounded.

Now we design the parameter update law for  $\bar{P}$  (or equivalently for  $\bar{P}$  since  $P^* = 0$ ) in (19) using a Lyapunov approach. Choose a positive definite Lyapunov function

candidate

$$V = (\dot{e} + k_\alpha e)^2 + (k_p + k_\alpha(k_d - k_\alpha))e^2 + \tilde{P}'\Gamma^{-1}\tilde{P}, \quad (20)$$

where  $\Gamma$  is a symmetric positive definite adaptation-gain matrix (typically diagonal or simply  $\gamma I$ ;  $\gamma > 0$ ), and  $k_\alpha > 0$  satisfies

$$k_d > k_\alpha > 0. \quad (21)$$

The derivative of  $V$  along solutions of (19) is

$$\begin{aligned} \dot{V} = & -2(k_d - k_\alpha)\dot{e}^2 - 2k_\alpha k_p e^2 \\ & + 2\tilde{P}'(\Gamma^{-1}\dot{\tilde{P}} + (\dot{e} + k_\alpha e)w). \end{aligned}$$

Choosing the adaptation law

$$\dot{\tilde{P}} = \dot{\tilde{P}} = -(\dot{e} + k_\alpha e)\Gamma w \quad (22)$$

leads to

$$\dot{V} = -2(k_d - k_\alpha)\dot{e}^2 - 2k_\alpha k_p e^2 \quad (23)$$

which is negative semidefinite when (21) is satisfied. The update law in (22) will be used in two adaptive systems defined by the following.

*Definition 1:* We call (18) together with (22) the *model system*, and (19) together with (22) the *simplified system*.

Both of these systems share the structure shown schematically in Fig. 1. If  $q$  has a finite number of frequency components all represented in the regressor vector (a sufficient condition for this is that  $i_q$  is constant and  $f$  and  $g$  have a finite number of spectral lines), then by taking  $P^*$  to be the vector of Fourier coefficients for  $q$ , (18) becomes (19) since the residual torque ripple  $\zeta$  will be zero. In this ideal case, our results for the simplified system hold for the model system.

The Lyapunov analysis immediately leads us to the following.

*Lemma 2:* In the simplified system:

- i) the zero solution is globally stable in the sense of Lyapunov,
- ii)  $w'\tilde{P}$  is bounded,
- iii)  $e$  and  $\dot{e} \rightarrow 0$  as  $t \rightarrow \infty$ .

*Proof:* The proof is included for completeness.

i) is an immediate consequence of (23). Since i) implies  $e, \dot{e}, \tilde{P} \in L^\infty$  and  $\|w(t)\| \leq \|(1, 1, \dots, 1)\|$ , we have  $w'\tilde{P} \in L^\infty$  as required by ii). With these, (19) implies  $\ddot{e} \in L^\infty$ . Thus,

$$\frac{d}{dt}e, \frac{d}{dt}\dot{e} \in L^\infty \quad \text{hence } e, \dot{e} \text{ are uniformly continuous.} \quad (24)$$

Moreover, from (23), we have

$$\begin{aligned} 2(k_d - k_\alpha) \int_0^T \dot{e}^2 dt + 2k_\alpha k_p \int_0^T e^2 dt \\ = V(0) - V(T) \leq V(0). \end{aligned}$$

Hence,  $e, \dot{e} \in L^2$ . This and (24) lead to iii) and the proof is completed.

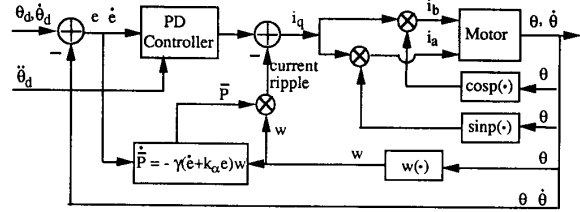


Fig. 1. Structure of adaptive linearizing motor control system.

In the above derivation, we have used a simple PD controller. However, any passive controller will work, and all the following analysis holds with little modification.

## V. EXPONENTIAL STABILITY OF THE SIMPLIFIED SYSTEM

It is well known in the adaptive control community that adaptive systems with only partial error convergence are not robust to modeling error and other uncertainties [21], [22]. Even for linear systems, various instabilities can occur [23], [24]. One way to ensure robust stability is by persistency of excitation of the regressor which guarantees exponential stability for common adaptive control systems. Although Dasgupta *et al.* [25] presented results on persistency of excitation in identifiable bilinear systems, general nonlinear systems and closed-loop adaptive control of nonlinear systems are extremely difficult to analyze.

Fortunately in the motor control case, the regressor functions are all continuous and periodic and hence bounded. By taking advantage of these special properties, we are able to find conditions on exogenous signals to guarantee persistency of excitation and establish robustness of our motor system.

This section establishes the exponential stability of the simplified system. The key to this result is the establishment of conditions on exogenous signals guaranteeing persistency of excitation. Such conditions are impossible to find for general nonlinear systems, but the special structure of the motor dynamics leads to this exceptional result. The development begins with the following Lemma.

*Lemma 3 (Guaranteed Minimum Speed):* For the simplified system, given  $\omega_m > 0$ , there exist  $T > 0$ ,  $\omega_{dM} \geq \omega_{dm} > 0$  and  $\omega_M \geq \omega_m$  such that  $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm}$  for all  $t$  implies  $\omega_M \geq \dot{\theta} \geq \omega_m$  for all  $t \geq T$ .

*Proof:* In (19), consider  $w'\tilde{P}$  as the input and  $\dot{e}$  as the output. Then, the corresponding transfer function is given by

$$H(s) = \frac{s}{s^2 + k_d s + k_p}.$$

Since  $H$  is exponentially stable, for given initial condition and  $\epsilon_i > 0$ , there exists  $T > 0$  such that the zero-input response will be less than  $\epsilon_i$  after  $T$ . Let  $\epsilon_i$  be the bound on  $|w'\tilde{P}|$ , then for all  $t \geq T$

$$|\dot{\theta} - \dot{\theta}_d| = |\dot{e}| \leq \|H\| \epsilon_i + \epsilon_i. \quad (25)$$

For a given  $\omega_m > 0$ , choose

$$\omega_{dm} = \|H\|\epsilon_t + \epsilon_t + \omega_m \quad (26)$$

and  $\omega_{dM} \geq \omega_{dm}$  and

$$\omega_M = \|H\|\epsilon_t + \epsilon_t + \omega_{dM}. \quad (27)$$

Then, from (25),  $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm}$  for all  $t$  implies  $\omega_M \geq \dot{\theta} \geq \omega_m$  for all  $t \geq T$  and the proof is complete.

The significance of the lemma is that it specifies a condition on an exogenous signal which guarantees a desired minimum motor speed. This minimum speed in turn, by the next lemma, ensures persistency of excitation of the regressor which is essential for robust adaptive systems.

In control system design, the lemma provides qualitative, rather than quantitative, guidelines for tuning the adaptive system. For example, by (26) and (27)

$$\omega_M - \omega_m = 2\|H\|\epsilon_t + 2\epsilon_t + (\omega_{dM} - \omega_{dm}).$$

Clearly, increasing  $k_d$  and  $k_p$  reduces motor speed variations, since doing so reduces  $\|H\|$  and reduces  $\epsilon_t$  for fixed  $T$ . Also apparent is that decreasing fluctuation in the reference decreases fluctuations in motor velocity.

**Lemma 4 (Motor Persistency of Excitation):** If  $\omega_M \geq \dot{\theta} \geq \omega_m > 0$  for  $t \geq T$  then  $w$  defined in (4) is persistently exciting (PE) for all  $t \geq T$ .

**Remark 5:** This result is intuitive yet very useful. It simply says that by running the motor at a minimum speed we can "see" the entire torque-ripple spectrum. If

motor speed control. The inclusions of a dead-band in our adaptation law around zero velocity may enable us to handle more general adaptive motion control problems.

*Proof:* By Definition A.1, we must establish the existence of  $\alpha, \beta > 0$  and  $T_0$  such that

$$I \leq \int_t^{t+T_0} ww^t d\tau \leq \beta I \quad \text{for all } t \geq T.$$

Choose  $T_0 = (2\pi/p\omega_m)$ . Since  $\dot{\theta} \geq \omega_m$ , for  $t \geq T$  we have

$$\theta(t+T_0) - \theta(t) = \int_t^{t+T_0} \dot{\theta} dt \geq \omega_m T_0 = \frac{2\pi}{p}.$$

Then

$$\int_t^{t+T_0} ww^t dt = \int_{\theta(t)}^{\theta(t+T_0)} ww^t \frac{1}{\dot{\theta}} d\theta$$

Since  $\dot{\theta} \leq \omega_M$ , we have

$$\begin{aligned} \int_t^{t+T_0} ww^t dt &\geq \frac{1}{\omega_M} \int_{\theta(t)}^{\theta(t+T_0)} ww^t d\theta \\ &\geq \frac{1}{\omega_M} \int_{\theta(t)}^{\theta(t) + \frac{2\pi}{p}} ww^t d\theta \end{aligned} \quad (28)$$

where the last inequality follows from the fact that  $ww^t \geq 0$  and  $\theta(t+T_0) \geq \theta(t) + (2\pi/p)$ . In substituting  $w$  from (4) into (28), we obtain

$$\begin{aligned} \int_t^{t+T_0} ww^t dt &\geq \frac{1}{\omega_M} \int_0^{\frac{2\pi}{p}} \begin{pmatrix} 1 & \sin p\theta & \cdots & \cos np\theta \\ \sin p\theta & \sin^2 p\theta & \cdots & \sin p\theta \cos np\theta \\ \cos p\theta & \sin p\theta \cos p\theta & \cdots & \cos p\theta \cos np\theta \\ \cdots & \cdots & \cdots & \cdots \\ \cos np\theta & \sin p\theta \cos np\theta & \cdots & \cos^2 np\theta \end{pmatrix} d\theta \\ &= \frac{1}{\omega_M} \begin{pmatrix} 2\pi/p & 0 & \cdots & 0 \\ 0 & \pi/p & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \pi/p \end{pmatrix} \geq \frac{\pi}{p\omega_M} I_{2n+1}. \end{aligned}$$

motor speed is too low, updating should be turned off to prevent parameter drifting. Note that although here we specified a positive speed, the direction of rotation does not matter. This lemma and the previous one allow us to specify conditions on exogenous signals that will guarantee persistency of excitation. In relying on the condition in this lemma we are restricting ourselves, more or less, to

On the other hand

$$\begin{aligned} \theta(t+T_0) &= \theta(t) + \int_t^{t+T_0} \dot{\theta} dt \leq \theta(t) + \omega_M T_0 \\ &= \theta(t) + \frac{\omega_M}{\omega_m} \frac{2\pi}{p} \leq \theta(t) + \left\lceil \frac{\omega_M}{\omega_m} \right\rceil \frac{2\pi}{p}, \end{aligned}$$

where  $\lceil x \rceil$  denotes the least integer greater than or equal

to  $x$ . Therefore,

$$\begin{aligned} \int_t^{t+T_0} ww' d\tau &\leq \int_{\theta(t)}^{\theta(t)+\left\lceil \frac{\omega_M}{\omega_m} \right\rceil \frac{2\pi}{p}} ww' \frac{1}{\theta} d\theta \leq \int ww' \frac{1}{\omega_m} d\theta \\ &= \frac{1}{\omega_m} \int \begin{pmatrix} 1 & \sin p\theta & \cdots & \cos np\theta \\ \sin p\theta & \sin^2 p\theta & \cdots & \sin p\theta \cos np\theta \\ \cos sp\theta & \sin p\theta \cos p\theta & \cdots & \cos p\theta \cos np\theta \\ \cdots & \cdots & \cdots & \cdots \\ \cos np\theta & \sin p\theta \cos np\theta & \cdots & \cos^2 np\theta \end{pmatrix} d\theta \\ &= \frac{1}{\omega_m} \left\lceil \frac{\omega_M}{\omega_m} \right\rceil \begin{pmatrix} 2\pi/p & 0 & \cdots & 0 \\ 0 & \pi/p & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \pi/p \end{pmatrix} \leq \frac{2\pi}{p\omega_m} \left\lceil \frac{\omega_M}{\omega_m} \right\rceil I_{2n+1}. \end{aligned}$$

Therefore, by Definition A.1,  $w$  is PE on  $[T, \infty)$  and the proof is complete.

Roughly speaking, a larger  $(\alpha/T_0)$  means that the regressor is "more exciting" and a larger  $(\alpha/\beta)$  means that the excitement is "more persistent," one expect that parameter convergence is better when these are large. From the proof,  $(\alpha/T_0)$  is proportional to  $(\omega_m/\omega_M)$  and  $\alpha/\beta$  to  $\omega_m^2/\omega_M^2$ .

We are now ready to present the main result of this section.

**Theorem 6 (Exponential Stability of Simplified System):** For the simplified system, there exist  $\omega_{dM} \geq \omega_{dm} > 0$  such that  $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm}$  for all  $t$  implies  $e, \dot{e}, \dot{P} \rightarrow 0$  exponentially with convergence rate  $\gamma k + O(\gamma^2)$  for some  $k > 0$ , where  $\gamma$  is the adaptation rate.

*Proof:* Let  $\epsilon = \dot{e} + k_\alpha e$ , and without loss of generality,  $\Gamma = \gamma I_3$ . Then our parameter update law becomes

$$\dot{\tilde{P}} = -\gamma w \epsilon.$$

Considering  $w' \tilde{P}$  as an input in (19) and solving for  $\epsilon$ , we obtain

$$\epsilon = H_e(w' \tilde{P}),$$

where

$$H_e(s) = \frac{s + k_\alpha}{s^2 + h_d s + k_p}.$$

For  $k_\alpha$  satisfying (21), we have

$$\operatorname{Re}(H_e(j\omega)) = \frac{k_\alpha k_p + (k_d - k_\alpha) \omega^2}{(k_p - \omega^2)^2 + \omega^2 k_d^2} > 0$$

$$\text{for all } \omega \in \mathbb{R} \quad (29)$$

and so  $H_e$  is strictly positive real (SPR, see [26, definition A.2]). Using standard results (see Lemma A.3 in the Appendix) on exponential stability of adaptive systems, Theorem 6 will be proved if  $w$  is PE.

To show this, fix a minimum speed  $\omega_m > 0$ . Then, we can choose  $\omega_{dm}$  according to (26) and  $\omega_{dM} \geq \omega_{dm}$ . By Lemma 3, there exist  $T$  and  $\omega_M$  as in (27) such that  $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm} > 0$  for all  $t$  implies  $\omega_M \geq \dot{\theta} \geq \omega_m$  for

all  $t \geq T$ . This, by Lemma 4, implies that  $w$  is PE for all  $t \geq T$  and the proof is complete.

This theorem is unique to motor control since it uses the periodicity of the motor dynamics. Although it is developed for the simplified system, it is of great practical importance for proving robust stability of the model system. Such results would be very difficult, if not impossible, had we taken the standard approach.

## VI. ROBUSTNESS IN SLOW ADAPTATION

With the exponential stability of the simplified system, we are now ready to study the stability of the model system. The basic idea is to think of  $\zeta$  as an unmodeled perturbation to the simplified system and derive what amounts to a robustness result.

There are some established robustness results in adaptive control aimed at these kinds of problems (see e.g., [27]). Such robustness results are based on a small-gain argument. The unperturbed adaptive system is viewed as an open-loop forward system and the perturbation is viewed as a feedback block. If the product of the input-output gain of the forward system and that of the perturbation block is less than one, the perturbed system remains stable by the small gain theorem [28]. Therefore, for a fixed controller and adaptation gain, a sufficient condition can be given on the allowable perturbation. However, this does not work very well if the adaptation gain is allowed to change, since the allowable perturbation goes to zero as the adaptation gain is reduced. The reason is that the gain of the forward system is, roughly speaking, inversely proportional to the adaptation gain in slow adaptation.

On the other hand, this loss of robustness is not observed in simulation or experiments. A closer look at the structure of the adaptive system reveals that the perturbation only enters the error subsystem directly, not the parameter update loop. It affects the parameter subsystem after being scaled down by the adaptation gain. These observations lead to the hypothesis that robust stability should be preserved in slow adaptation and independent of the adaptation gain.

Here, we take an approach which exploits the detailed structure of the adaptive controller and removes the limitations of the existing techniques which suggest that fast adaptation must be used for robust stability. This is done through a new two Lyapunov function approach which proves to be very effective. The technique of Vidyasagar and Vanelli [29] is used to construct Lyapunov function dynamics for the error and parameter dynamics. The contribution here is proving stability by the use of coupled Lyapunov function dynamics. The resulting theorem is the following.

*Theorem 7:* Consider the following adaptive system with linear error equation, nonlinear regressor  $w$  and unmodeled disturbance  $\zeta$  due to modeling error:

$$\dot{x} = Ax + b(\zeta + w'\tilde{P}) \quad (30)$$

$$\dot{\tilde{P}} = -\gamma wc'tx \quad (31)$$

$$\zeta = H_\zeta(x, \tilde{P}, r) \quad (32)$$

where  $A, b, c$  are constant matrices and  $H_\zeta$  is a nonlinear operator. Suppose the following:

- i)  $H_e(s) = c'(sI - A)^{-1}b$  is SPR;
- ii)  $w$  is PE;
- iii) there exist  $\beta_\zeta, K_x, K_{\tilde{P}},$  and  $K_r,$  such that

$$\|H_\zeta(x, \tilde{P}, r)\| \leq K_x\|x\| + K_{\tilde{P}}\|\tilde{P}\| + K_r\|r\| + \beta_\zeta. \quad (33)$$

Under these conditions, there exist  $\gamma^*$ , and constants  $\rho_1, \rho_2 > 0$  (independent of  $\gamma$ ) such that if  $\gamma \in (0, \gamma^*), K_x$  and  $K_{\tilde{P}}$  satisfy

$$\rho_1 K_x + \rho_2 K_{\tilde{P}} < 1, \quad (34)$$

and  $r$  is bounded, then  $\zeta, \tilde{P}$  and  $x$  are bounded.

*Remark 8:* In words, this theorem says that for arbitrarily slow adaptation, if the perturbation is sufficiently small relative to the stability of the matrix  $A$  and the degree of persistency of excitation of  $w$ , the perturbed system remains stable.

*Remark 9:*  $H_\zeta$  can be simply a nonlinear function. It is then required to be sector bounded.

*Remark 10:* When (34) cannot be satisfied globally, a local version of the result can be obtained.

*Proof:* Since  $H_e$  is SPR,  $A, b, c$  satisfy the positive-real lemma (see Lemma A.4 in the Appendix). Choose  $V_1 \triangleq x'(t)Px(t)$  where  $P > 0$  is as in the positive-real lemma. Then its derivative along the solution of (30) is

$$\begin{aligned} \dot{V}_1^{(30)} &= x'(A'P + PA)x + 2x'Pb(w'\tilde{P} + \zeta) \\ &= x'(-2\rho_x I - QQ')x + 2x'c(w'\tilde{P} + \zeta) \\ &\leq -2\rho_x\|Vx(t)\|^2 + 2\|x(t)\|\|c(w'\tilde{P}(t) + \zeta(t))\| \\ &\leq -2\frac{\rho_x}{\bar{\sigma}(P)}V_1 + \frac{2}{\underline{\sigma}^{1/2}(P)}V_1^{1/2}\|c(w'\tilde{P}(t) + \zeta(t))\|, \end{aligned}$$

where  $\rho_x$  is given in the PR lemma. Let  $v_1 = V_1^{1/2}$ . Then, we immediately have

$$\frac{d}{dt}v_1(t) \leq -\frac{\rho_x}{\bar{\sigma}(P)}v_1(t) + \frac{1}{\underline{\sigma}^{1/2}(P)}\|c(w'\tilde{P}(t) + \zeta(t))\|, \quad (35)$$

which yields

$$v_1(t) \leq v_1(0) + \frac{1}{\rho_x} \frac{\bar{\sigma}(P)}{\underline{\sigma}^{1/2}(P)}\|c(w'\tilde{P}(t) + \zeta(t))\|. \quad (36)$$

This equation describes the dynamics of the Lyapunov function  $V_1$ . Note that it is coupled to  $\tilde{P}$  and  $\zeta$ . (We will also derive similar dynamics for a Lyapunov function which is positive definite in the parameter  $\tilde{P}$ . Together with a small-gain type argument, they give the desired bounds.) First, we use the BIBO stability of (36) to get a bound on  $\|x\|$ . Since  $\|x(t)\| \leq (1/\underline{\sigma}^{1/2}(P))v_1(t)$  we have

$$\|x\| \leq \frac{m_x}{\rho_x}\|c\|\|\zeta\| + \frac{m_x}{\rho_x}\|w\|\|\tilde{P}\| + \beta_x, \quad (37)$$

where

$$m_x = \frac{\bar{\sigma}(P)}{\underline{\sigma}(P)} \quad \text{and} \quad \beta_x = \frac{1}{\underline{\sigma}^{1/2}(P)}v_1(0).$$

Now using (30), (31) can be rewritten as

$$\dot{\tilde{P}} = -\gamma wH_e(w'\tilde{P}) - \gamma wH_e(\zeta). \quad (38)$$

Since  $H_e$  is SPR, and  $w$  is PE,

$$\dot{\tilde{P}} = -\gamma wH_e(w'\tilde{P}) \quad (39)$$

is exponentially stable with convergence rate  $\gamma k + O(\gamma)$  for some  $k > 0$  independent of  $\gamma$ . By the converse Lyapunov theorem (Theorem A.5), there exists a Lyapunov function  $V_2(t, \tilde{P})$  such that

$$\begin{aligned} \|\tilde{P}(t)\|^2 &\geq V_2(t, \tilde{P}) \geq \alpha_1\|\tilde{P}(t)\|^2, \quad (40) \\ \dot{V}_2^{(39)}(t, \tilde{P}) &\leq -\alpha_2\|\tilde{P}(t)\|^2, \\ \left\| \frac{\partial}{\partial \tilde{P}} V_2(t, \tilde{P}) \right\| &\leq \alpha_3\|\tilde{P}(t)\|, \end{aligned}$$

for all  $t \geq 0$ , some  $\alpha_1 > 0, \alpha_2 = \gamma k_1 + O(\gamma^2), \alpha_3 > 0$ , and some  $k_1 > 0$ . Let  $\rho_{\tilde{P}} = k_1/4$  then there exist  $\gamma^*$  such that for all  $\gamma \in (0, \gamma^*)$

$$2\gamma\rho_{\tilde{P}} \leq \gamma k_1 + O(\gamma^2) = \alpha_2.$$

Differentiating  $V_2$  along the solutions of (38) leads to

$$\begin{aligned} \dot{V}_2^{(38)}(t, \tilde{P}) &= \dot{V}_2^{(39)}(t, \tilde{P}) - \frac{\partial}{\partial \tilde{P}} V_2(t, \tilde{P})\gamma wH_e(\zeta)(t), \\ &\leq -\alpha_2\|\tilde{P}(t)\|^2 + \alpha_3\|\tilde{P}(t)\|\|\gamma wH_e(\zeta)(t)\| \\ &\leq -2\gamma\rho_{\tilde{P}}\|\tilde{P}(t)\|^2 + \alpha_3\|\tilde{P}(t)\|\|\gamma wH_e(\zeta)(t)\| \\ &\leq -2\gamma\rho_{\tilde{P}}V_2(t, \tilde{P}) + \frac{\alpha_3}{\alpha_1^{1/2}}V_2^{1/2}\|\gamma wH_e(\zeta)(t)\|. \end{aligned}$$



setting  $v_2 \triangleq V_2^{1/2}$  yields

$$\frac{d}{dt}v_2(t) \leq -\gamma\rho_{\bar{p}}v_2(t) + \gamma\frac{\alpha_3}{2\alpha_1^{1/2}}\|wH_e(\zeta)(t)\|, \quad (41)$$

hence

$$0 \leq v_2(t) \leq v_2(0) + \frac{1}{\rho_{\bar{p}}}\frac{\alpha_3}{2\alpha_1^{1/2}}\|w\| \|H_e(\zeta)(t)\|.$$

(Note that the  $\gamma$  dependence drops out here!) Using (40) and the definition of  $v_2$ , we obtain

$$\|\tilde{P}\| \leq \frac{m_{\bar{p}}}{\rho_{\bar{p}}}\|w\| \|H_e\| \|\zeta\| + \beta_{\bar{p}} \quad (42)$$

with  $m_{\bar{p}} \triangleq (\alpha_3/2\alpha_1)$  and  $\beta_{\bar{p}} \triangleq (1/\alpha_1^{1/2})v_2(0)$ . Substituting this into (37) gives

$$\|x\| \leq \frac{m_x}{\rho_x} \left( \|c\| + \frac{m_{\bar{p}}}{\rho_{\bar{p}}}\|c\| \|w\|^2 \|H_e\| \right) \|\zeta\| + \frac{m_x}{\rho_x}\beta_{\bar{p}} + \beta_x. \quad (43)$$

Furthermore, from (32) and (33), we have

$$\|\zeta\| \leq K_r\|r\| + K_x\|x\| + K_{\bar{p}}\|\tilde{P}\| + \beta_{\zeta}. \quad (44)$$

Define  $\rho_1 \triangleq (m_x/\rho_x)(\|c\| + (m_{\bar{p}}/\rho_{\bar{p}})\|c\| \|w\|^2 \|H_e\|)$ ,  $\rho_2 \triangleq (m_{\bar{p}}/\rho_{\bar{p}})\|w\| \|H_e\|$ , and  $\gamma_1 \triangleq \rho_1 K_x + \rho_2 K_{\bar{p}}$ . (Note that  $\gamma_1$  is assumed to be less than 1 in the statement of the theorem.) Substituting (42) and (43) into (44) results in

$$\|\zeta\| \leq \gamma_1\|\zeta\| + K_r\|r\| + \beta_{\zeta} + K_{\bar{p}}\beta_{\bar{p}} + K_x\beta_x + \frac{m_x}{\rho_x}K_x\beta_{\bar{p}}.$$

Since  $\gamma_1 < 1$ ,  $1 - \gamma_1 > 0$ . Hence, from the above inequality, if  $r$  is bounded, we immediately have  $\zeta$  bounded:

$$\|\zeta\| \leq (1 - \gamma_1)^{-1} \cdot \left\{ K_r\|r\| + \beta_{\zeta} + K_{\bar{p}}\beta_{\bar{p}} + K_x\beta_x + \frac{m_x}{\rho_x}K_x\beta_{\bar{p}} \right\}. \quad (45)$$

Denoting the right-hand side of (45) by  $\gamma_2$  and substituting (45) into (42) and (43) yields

$$\|\tilde{P}\| \leq \frac{m_{\bar{p}}}{\rho_{\bar{p}}}\|w\| \|H_e\| \gamma_2 + \beta_{\bar{p}},$$

$$\|x\| \leq \frac{m_x}{\rho_x} \left( \|c\| + \frac{m_{\bar{p}}}{\rho_{\bar{p}}}\|c\| \|w\|^2 \|H_e\| \right) \gamma_2 + \frac{m_x}{\rho_x}\|c\| \|w\| \beta_{\bar{p}} + \beta_x.$$

This completes the proof.

From the proof we see that reducing  $\rho_1$  and  $\rho_2$  improve robustness. This is equivalent to increasing the convergence rates  $\rho_x$  and  $\rho_{\bar{p}}$  and reducing the overshoot  $m_x$  and  $m_{\bar{p}}$ . This can be accomplished by increasing the controller gain and improving the persistency of excitation of the regressor.

*Corollary 11 (Continuous Degradation):* Under the con-

ditions of Theorem 7,  $x$  and  $\tilde{P}$  converge to neighborhoods of zero with radii  $\kappa_x(1 - \gamma_1)^{-1}\|r\|$  and  $\kappa_{\bar{p}}(1 - \gamma_1)^{-1}\|r\|$  (which go to zero as  $r$  goes to zero), respectively, for some  $\kappa_x > 0$  and  $\kappa_{\bar{p}} > 0$ .

*Proof:* By inspection, the corollary follows immediately from (45), (41), and (35) since the  $\beta$ 's are due to initial conditions, and  $\kappa_x$  and  $\kappa_{\bar{p}}$  are given by

$$\kappa_x = \|x\| \leq \frac{m_x}{\rho_x} \left( \|c\| + \frac{m_{\bar{p}}}{\rho_{\bar{p}}}\|c\| \|w\|^2 \|H_e\| \right) K_r,$$

$$\kappa_{\bar{p}} = \frac{m_{\bar{p}}}{\rho_{\bar{p}}}\|w\| \|H_e\| K_r.$$

## VII. STABILITY OF THE MODEL SYSTEM

Applying Theorem 7 to our model system immediately leads to the following results.

*Lemma 12:* Consider the model system (18) and (22). If  $w$  is PE and  $\ddot{\theta}_d$  is bounded, then there exists  $\epsilon^* > 0$  such that  $(\|g(\cdot) - k_0\|/k_0) < \epsilon^*$  implies boundedness of  $e$ ,  $\dot{e}$ , and  $\tilde{P}$  in the model system.

*Proof:* Equation (18) can be rewritten in the form of (30) with

$$x' = (e \ \dot{e}), \quad A = \begin{pmatrix} 0 & 1 \\ -k_p & -k_d \end{pmatrix},$$

$$b' = (0 \ 1), \quad c' = (k_\alpha \ 1), \quad (46)$$

and

$$-\zeta = q - w'P^* = (g(\theta) - k_0)i_q + f(\theta) - w'P^* \quad (47)$$

$$= \frac{g(\theta) - k_0}{k_0} (\ddot{\theta}_d + k_d\dot{e} + k_p e + w'\tilde{P}) + f(\theta) - w'P^*$$

$$= \frac{g(\theta) - k_0}{k_0} \ddot{\theta}_d + f(\theta) - \frac{g(\theta)}{k_0} w'P^*$$

$$+ \frac{g(\theta) - k_0}{k_0} [k_p \ k_d]x + \frac{g(\theta) - k_0}{k_0} w'\tilde{P}.$$

Hence,  $\zeta$  is of the form of (32) with  $K_r = 1$ ,  $\beta_{\zeta} = 0$  and

$$r \triangleq \frac{g(\theta) - k_0}{k_0} \ddot{\theta}_d + f(\theta) - \frac{g(\theta)}{k_0} w'P^*. \quad (48)$$

By the assumptions on  $f$ ,  $g$ , and  $\ddot{\theta}_d$ ,  $r$  is bounded, and so are  $K_x$  and  $K_{\bar{p}}$  defined as follows:

$$K_x = \frac{\|g(\cdot) - k_0\|}{k_0} \|[k_p \ k_d]\| \quad (49)$$

$$K_{\bar{p}} = \frac{\|g(\cdot) - k_0\|}{k_0} \|w\|. \quad (50)$$

Hence,  $\zeta$  satisfies condition iii) in Theorem 7. Also the SPR condition is met due to (29) in Section V. Thus, Theorem 7 applies. Note that both  $K_x$  and  $K_{\bar{p}}$  have the common factor  $(\|g(\cdot) - k_0\|/k_0)$ . Substituting them into

(34) and taking out the common factor yields

$$\frac{\|g(\cdot) - k_0\|}{k_0} \{ \rho_1 \| [k_p \quad k_d] \| + \rho_2 \|w\| \} < 1.$$

Defining

$$\epsilon^* \triangleq \{ \rho_1 \| [k_p \quad k_d] \| + \rho_2 \|w\| \}^{-1},$$

then  $(\|g(\cdot) - k_0\|/k_0) < \epsilon^*$  implies (34) and therefore implies boundedness of  $e$ ,  $\dot{e}$ , and  $\tilde{P}$ .

Finally, we can state the stability of the adaptive controller with conditions on exogenous signals in the following.

**Theorem 13 (Stability of the Model System):** For the model system described by (18) and (22), there exist  $\omega_{dm} \geq \omega_{dm} > 0$ ,  $\epsilon^*$  and  $\gamma^*$  such that  $\omega_{dm} \geq \dot{\theta}_d \geq \omega_{dm}$  for all  $t$ ,  $\gamma \in (0, \gamma^*)$  and  $(\|g(\cdot) - k_0\|/k_0) < \epsilon^*$  imply boundedness of  $e$ ,  $\dot{e}$ , and  $\tilde{P}$ . Moreover,  $e$ ,  $\dot{e}$ , and  $\tilde{P}$  converge to a neighborhood of zero with radius proportional to

$$\left\| \frac{g(\cdot) - k_0}{k_0} \ddot{\theta}_d + f(\cdot) - \frac{g(\cdot)}{k_0} w' P^* \right\|.$$

**Remark 14:** Here we observe another way of having parameter (and also tracking error) convergence. If we choose  $\ddot{\theta}_d = 0$ , i.e., constant velocity tracking, and if  $f(\cdot)$  is in the range of  $w'(\cdot)$ , then there exists  $P^*$  such that the radius of the above neighborhood is zero. In practice, we can only remove a finite number of frequency components from the torque ripple so that  $f(\theta) - (g(\theta)/k_0)w'(\theta)P^*$  will contain higher frequency components. The theorem tells us that the tracking error and parameter error converge to a neighborhood of zero with radius proportional to the size of residual torque ripple.

We do not actually calculate any of those bounding numbers such as  $\epsilon^*$ ,  $\gamma^*$ ,  $\omega_{dm}$ ,  $K_x$ ,  $K_{\tilde{P}}$ , and so on. Their qualitative role in stabilizing the system is more important. For example, if the system is stable at a certain velocity with a particular controller gain and adaptation gain, then it is also stable if we increase motor speed or controller gain, or decrease adaptation gain.

**Proof:** The boundedness follows immediately from Lemma 12 if we have PE. The convergence follows from Corollary 11 and (48). Hence, we only need to establish that  $w$  is PE which, modulo the  $\zeta(\theta)$  term, can be accomplished by the choice of reference signals by Lemmas 3 and 4. Therefore, we show that with nonzero  $\zeta$  Lemma 3 still holds, i.e., a minimum motor speed can be achieved by choice of reference signal.

First, choose the Lyapunov function in (20). Differentiating it along the solutions of (18) yields

$$\dot{V}^{(18)} \leq c' x \zeta,$$

where  $c$  and  $x$  are as in (46). Using (47), (49), and (50) yields

$$\dot{V}^{(18)} \leq \|c\| \|r\| \|x\| + \|c\| \|K_x\| \|x\|^2 + \|c\| \|K_{\tilde{P}}\| \|x\| \|\tilde{P}\|, \quad (51)$$

where  $\|r\|$ ,  $K_x$  and  $K_{\tilde{P}}$  are finite. Noting that

$$l_1 \| (x'(t), \tilde{P}'(t)) \|^2 \leq V(t) \leq l_2 \| (x'(t), \tilde{P}'(t)) \|^2$$

for some  $l_1, l_2 > 0$ , (51) reduces to

$$\dot{V}^{(18)} \leq 2l_3 V^{1/2} + 2l_4 V$$

for some  $l_3, l_4 > 0$ . Substituting in  $v^2 = V$  yields

$$\dot{v}^{(18)} \leq l_3 + l_4 v.$$

Integrating this differential inequality and observing that  $\|(x', \tilde{P}')\| \leq l_1^{-1/2} v$ , we have  $\|(x'(t), \tilde{P}'(t))\| \leq l_5 e^{l_4 t} + l_6$  for some  $l_5, l_6 > 0$ .

Now fix a minimum speed  $\omega_m$ , let  $T_1$  be finite and

$$\epsilon_t \triangleq \sup_{t \in [0, T_1]} \|w' \tilde{P}(t) - \zeta(t)\|.$$

Parallel to the proof of Lemma 3, choosing  $\omega_{dm}$  and  $\omega_{dM}$  according to (28) and (29) ensures that  $\omega_{dM} \geq \dot{\theta}_d \geq \omega_{dm}$  for all  $t \in [0, T_1]$  implies  $\omega_m \geq \dot{\theta} \geq \omega_m$  for all  $t \in [T, T_1]$ , where  $T$  is as in Lemma 3.

Thus, the proof will be finished if we can find a bound on  $\epsilon_t$  which is independent of  $T_1$  since allowing  $T_1 \rightarrow \infty$  yields a uniform bound on  $w' \tilde{P}(t) - \zeta(t)$ . This is done as follows:

$$\begin{aligned} \epsilon_t &= \sup_{t \in [0, T_1]} \|w' \tilde{P}(t) - \zeta(t)\| \\ &= \max \left\{ \sup_{t \in [0, T]} \|w' \tilde{P}(t) - \zeta(t)\|, \right. \\ &\quad \left. \sup_{t \in [T, T_1]} \|w' \tilde{P}(t) - \zeta(t)\| \right\} \\ &\leq \max \left\{ l_5 e^{l_4 T} + l_6, \sup_{t \in [T, T_1]} \|w' \tilde{P}(t) - \zeta(t)\|, \right. \\ &\quad \left. \sup_{t \in [T_1, \infty)} \{ \|w' \tilde{P}(t) - \zeta(t)\| \dot{\theta}(\tau) \} \right. \\ &\quad \left. \in [\omega_m, \omega_M] \text{ for all } \tau \geq T_1 \right\} \\ &= \max \left\{ l_5 e^{l_4 T} + l_6, \sup_{t \in [T, \infty)} \{ \|w' \tilde{P}(t) - \zeta(t)\| \dot{\theta}(\tau) \} \right. \\ &\quad \left. \in [\omega_m, \omega_M], \text{ for all } \tau \geq T \right\} \end{aligned}$$

where the last term is finite by Lemma 12 and depends on  $T$  through the initial condition of  $x$  and  $\tilde{P}$  at  $T$  but not on  $T_1$ . Hence, the proof is completed.

### VIII. EXPERIMENTAL RESULTS

So far it has been shown that with an appropriate controller the motor can be run at a guaranteed minimum speed by the choice of reference signals, which in turn

ensures the boundedness of all the internal signal. Building on this, further results on parameter convergence in slow adaptation can be obtained [18], for the model system in the presence of residual torque-ripple harmonics which are not cancelled by the controller. Performance of the adaptive controller is evaluated in simulation and experiments, and results are included here to show the efficacy of the proposed controller.

Fig. 2 shows the experimentally measured torque-ripple spectrum of a 90-pole 2-phase hybrid step motor before adaptation. The commutation waveforms were the "ideal" sinusoidal signals, but torque ripple is clearly seen at the pole frequency and its harmonics. This spectrum was produced with the following experimental procedure: First, a constant current  $i_q$  was applied to the motor with a low-gain velocity feedback loop to keep the motor running at roughly constant speed on the average. Once a near steady-state motion was attained the motor current was, for our purposes, constant. Next, a series of shaft-angle measurements was stored at the sample rate. This series was interpolated and resampled uniformly in the spatial shaft-angle domain. Finally, 2048 samples corresponding to 2 revolutions of the motor were used to generate a (noisy) acceleration estimate which was FFT'd to generate Fig. 2.

Next, the adaptive control law proposed in this paper was implemented with a low-gain PD controller to track a constant speed trajectory. The parameters were initialized to be zero, corresponding to sinusoidal commutation. After steady state was attained, the same procedure was followed to generate the new torque-ripple spectrum shown in Fig. 3. Observe the dramatic removal of all the torque-ripple spikes, the reduction was over 30 db. Here, the removal of the spikes serves as an indirect proof, and may be the best proof, of parameter convergence.

Fig. 4 illustrates the initial adaptation process in time domain. Low PD gains and low adaptation gains were used. Note how the velocity ripple was gradually suppressed. The velocity takes discrete values due to the use of a digital encoder. As predicted by the analysis of Section VII, slow adaptation did not destroy robust stability. In fact, final parameter convergence was made better by reducing adaptation gains. In Fig. 4, the initial velocity variations can be reduced by increasing the PD gains, while increasing the adaptation gains speeds up the velocity convergence.

While our scheme is very easy to implement, the standard approach to adaptive linearization of Section III is not. Due to the division by  $w'P_2$  as we mentioned before, it is very difficult to stabilize the system, in experiments or simulation. One way to improve is to use projection or simple bounding to limit the variation in  $P_2$  if we have a good initial guess of the range of  $P_2$ . Another way is to largely reduce the adaptation gain  $\Gamma_2$  for  $P_2$ , for example, one thousand times smaller than that of  $\Gamma_1$ . Fig. 5 shows a simulation result of the standard approach using reduced  $\Gamma_2$ . Velocity converges very nicely, but parameters keep drifting slowly due to the lack of persistency of excitation.

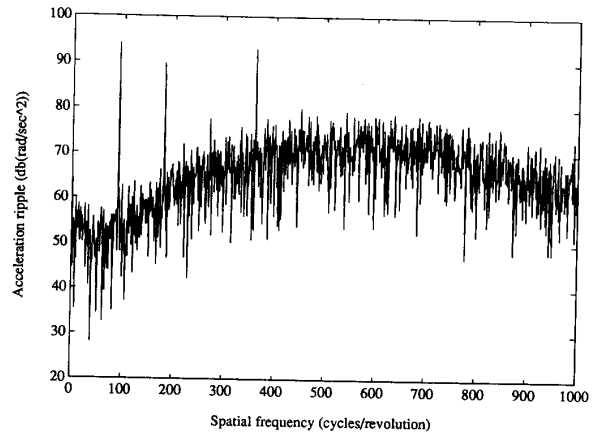


Fig. 2. Torque-ripple spectrum before adaptation.

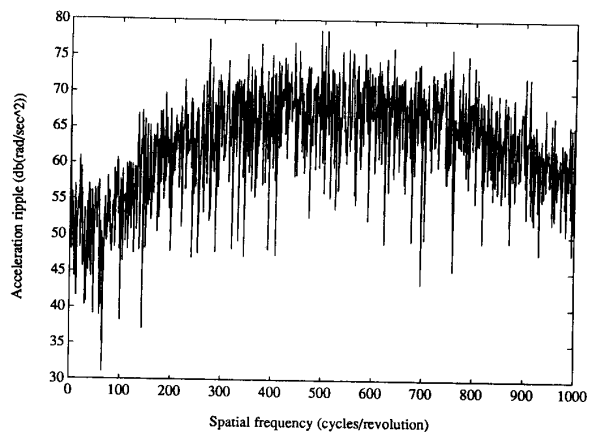


Fig. 3. Torque-ripple spectrum after adaptation.

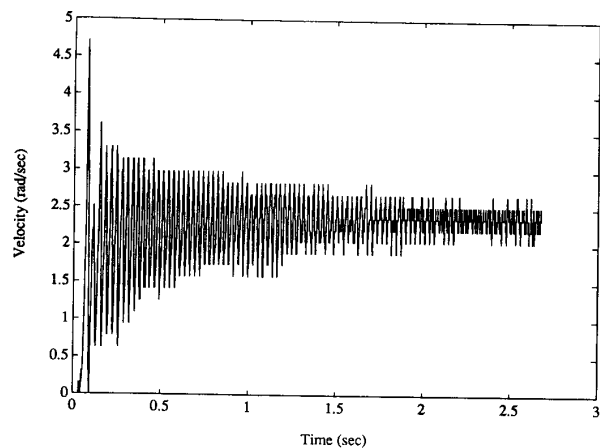


Fig. 4. Time-domain response of motor velocity during initial adaptation.

This drifting reflects instability in the system. To compare, Fig. 6 shows parameter convergence, using our reduced parameterization approach, in the presence of residual torque ripple of higher order harmonics.

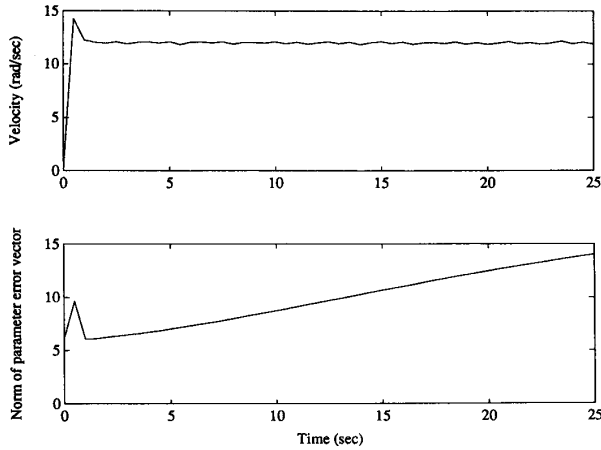


Fig. 5. Parameter drifting in standard approach.

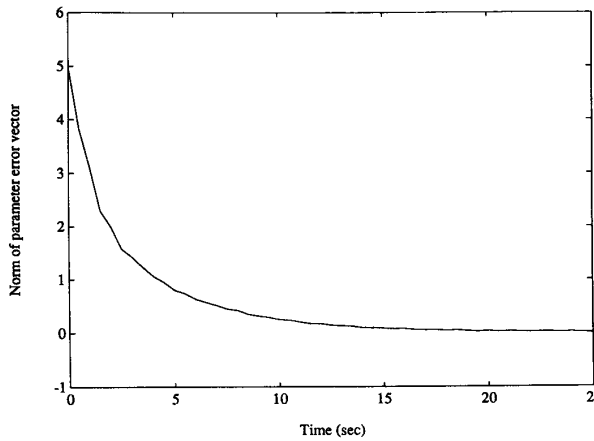


Fig. 6. Parameter convergence in our approach.

IX. CONCLUSION

The parameterization taken in the standard approach is simple and intuitive. However, it has three fundamental deficiencies. First, it uses twice the number of parameters to cancel the same number of torque-ripple harmonics as required by our approach. This may be a significant problem in real-time implementation if only limited computational power is available. Second, due to this over parameterization, parameter convergence is not guaranteed. In fact, persistency of excitation is lost in motor speed control, whereas in our approach PE can be guaranteed by conditions on exogenous signals. Third, due to the inversion in (8), which involves estimated parameters, a non-sector-bounded nonlinearity is undesirably introduced. This causes problems in stability as well as robustness.

Due to our reduced parameterization, we are able to establish strong stability results and robustness to nonlinear modeling errors. The results in Sections V and VII are special to our motor system since we have made explicit use of the periodicities in the motor dynamics. The result of Section VI, that robustness is preserved in

slow adaptation, is obtained in a fairly general setting, using a novel technique of two coupled Lyapunov function dynamics. The modeling errors that can be handled include nonlinear operators and nonlinear functions in tracking error and parameters. This robustness result complements that of Taylor *et al.* [14] where a nonlinear adaptive control scheme is shown to be robust to unmodeled fast stable dynamics. The successful implementation indicates the value of our approach for actuator linearization. In fact, our scheme can be readily applied to brushless DC motors and other rotating machines.

Although our robustness result is proved in a general framework and is useful outside motion control, it does not apply to the adaptive system of Section III based on the standard approach. There are two reasons for this. First, Theorem 7 requires persistency of excitation which is not ensured in the standard approach. Second, the sector-boundedness of equation (33) cannot be guaranteed for the perturbation term  $\zeta$  in (7).

APPENDIX

- $\mathbb{R}, \mathbb{R}_+$  real numbers, and nonnegative real numbers
- $\mathbb{C}$  complex numbers
- $\mathbb{R}^n, \mathbb{R}^{n \times m}$   $n$ -dimensional Euclidean space, and  $n \times m$  matrices with real entries
- PE persistency of excitation, or, persistently exciting
- SPR strictly positive real
- $\lambda_i(A)$  the  $i$ th eigenvalue of  $A \in \mathbb{R}^{n \times n}$
- $\bar{\sigma}(A)$  the largest singular value of  $A \in \mathbb{R}^{n \times m}$ ,  $\bar{\sigma}(A) = \sqrt{\max_{i=1}^n \lambda_i(AA^t)}$
- $\underline{\sigma}(A)$  the smallest singular value of  $A \in \mathbb{R}^{n \times m}$ ,  $\underline{\sigma}(A) = \sqrt{\min_{i=1}^n \lambda_i(AA^t)}$
- $H$  a transfer function, or a linear operator with transfer function  $H$
- $H(s)$  the transfer function evaluated at  $s$ , when  $s \in \mathbb{C}$
- $H(x)$  the output signal when passing  $x$  through a linear operator  $H$  with suitable dimension and zero initial condition, where  $x: \mathbb{R}_+ \rightarrow \mathbb{R}^n$
- $H(x)(t)$  the vector value of  $H(x)$  at time  $t$
- $\|a\|$  the vector 2-norm if  $a \in \mathbb{R}^n$ , or,
- $\|x\|$  the  $L^\infty$ -norm if  $x: \mathbb{R}_+ \rightarrow \mathbb{R}^n$ , i.e.,  $\|x\| = \sup_{t \in \mathbb{R}_+} \|x(t)\|$
- $\|A\|$  matrix 2-norm if  $A \in \mathbb{R}^{n \times m}$ , i.e.,  $\|A\| = \max_{\|x\|=1} \|Ax\|$
- $\|H\|$   $L^\infty$ -induced norm of an operator:  $\|H\| = \sup_{x \in L^\infty, \|x\| \neq 0} \|H(x)\|/\|x\|$
- $\lfloor a \rfloor$  floor of  $a \in \mathbb{R}$ :  $\lfloor a \rfloor = \max\{i \in \mathbb{Z}: i \leq a\}$
- $\lceil a \rceil$  ceiling of  $a \in \mathbb{R}$ :  $\lceil a \rceil = \min\{i \in \mathbb{Z}: i \geq a\}$
- $L^\infty \triangleq \{x: \mathbb{R}_+ \rightarrow \mathbb{R}^n \mid \lim_{T \rightarrow \infty} \sup_{t \in [0, T]} \|x(t)\| < \infty\}$
- $L^p \triangleq \{x: \mathbb{R}_+ \rightarrow \mathbb{R}^n \mid \int_0^\infty \|x(t)\|^p dt < \infty\}$ , where  $1 \leq p < \infty$

Definition A.1: Persistency of Excitation (PE) [27].

A vector valued function  $w: \mathbb{R}_+ \rightarrow \mathbb{R}^n$  is said to be persistently exciting (PE), if there exist constants  $T_0, \alpha, \beta$

> 0 such that

$$\beta I_n \geq \int_t^{t+T_0} w(\tau)w'(\tau) d\tau \geq \alpha I_n > 0 \quad \text{for all } t. \quad (\text{A.1})$$

**Definition A.2: Positive Realness [30].** A rational function  $H(s)$  is said to be positive real (PR) if

i) the  $j\omega$ -axis poles of  $H(s)$  are simple with nonnegative residual,

ii) for all  $\omega \in R$  for which  $j\omega$  is not a pole of  $H(s)$

$$\operatorname{Re} [H(j\omega)] \geq 0. \quad (\text{A.2})$$

$H(s)$  is said to be strictly positive real (SPR) if  $H(s - \zeta)$  is positive real for some  $\zeta > 0$ .

**Theorem A.3: Exponential Stability Theorem [30].** Consider the adaptive control system:

$$\dot{\phi} = -\gamma wH(w'\phi) \quad (\text{A.3})$$

where  $\gamma > 0$  is the adaptation gain. If  $H(s)$  is SPR and  $w$  is PE, then  $\phi$  and the internal state of  $H(s)$  converge to zero exponentially with convergence rate  $\gamma k + O(\gamma^2)$  for some  $k > 0$ .

**Lemma A.4: Positive Real Lemma [31].** Let  $H(s) = c'(sI - A)^{-1}b + d$  be SPR and proper with  $H(\infty) < \infty$ . Then there exists  $P > 0$  such that

$$PA + A'P = -2\rho_x I - QQ' \quad (\text{A.4})$$

$$Pb = c - Q\xi$$

$$\xi\xi' = 2d$$

for some nonzero  $Q$  and  $\rho_x > 0$ .

**Theorem A.5: Converse Lyapunov Theorem.** Assume that  $f(t, x): \mathbb{R}_+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  has continuous and bounded first partial derivative in  $x$  and is piecewise continuous in  $t$  for all  $x$  in a neighborhood of zero and  $t \geq 0$ . If the solution of

$$\dot{x} = f(t, x) \quad x(t_0) = x_0 \quad (\text{A.5})$$

converges to zero exponentially with convergence rate  $\alpha$ , then there exist a function  $v$  and strictly positive constants  $\alpha_1, \alpha_2$  and  $\alpha_3$  with  $\alpha_2 = 2k\alpha$  for some  $1 \geq k > 0$  such that for all  $t \geq 0$  and  $x$  in a neighborhood

$$\|x\|^2 \geq v(t, x) > \alpha_1 \|x\|^2,$$

$$\dot{v}^{(A.5)}(t, x) \leq -\alpha_2 \|x\|^2,$$

$$\left\| \frac{\partial v(t, x)}{\partial x} \right\| \leq \alpha_3 \|x\|.$$

This is a modified version of the theorem as in Sastry and Bodson [27]. (Normalize their  $v$  by their  $\alpha_2$ , and take their  $(\alpha_3/\alpha_2)$  to get our  $\alpha_3$ . Finally take  $T = (1/2\alpha) \ln(2m^2)$  to get the form of our  $\alpha_2$ ).

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