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Minimum-Energy Approach to Stable Inversion of Nonminimum Phase Systems

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Abstract

This paper presents a new approach to solve the stable inversion problem of nonlinear nonminimum phase systems. Based on the resent results [1], the minimum-energy property is first established for the solutions of the stable inversion problem. Then, an iterative algorithm via linearization, discretization and Moore-Penrose generalized inverse is constructed. This is followed by simulation results to illustrate that the exact output tracking without any transients has been achieved.

1 Introduction

The problem of output tracking control for nonlinear systems is of great importance from both the theoretical and the practical viewpoints and it has been studied extensively for many years. There are two basic approaches to attack this problem. Using state feedback is one way which involves stabilizing the closed-loop system so as to achieve asymptotic tracking of a class of reference input. The second approach is to implement the tracking controller with feed-forward signals generated by an inverse system coupled with a stabilizing feedback loop.

For the linear multivariable cases, the asymptotic tracking problem was solved by [3, 4] and subsequently crystallized as the internal model principle[5]. The matrix equations defining an asymptotic tracking controller for linear systems were translated to nonlinear partial differential equations in the nonlinear cases[6]. Although nonlinear partial differential equations are only numerically tractable for systems of low order, solutions for tracking periodic trajectories have been developed based on Fourier series[7, 8]. However, the transient error phenomenon is still a fundamental limitation of the regulation approach.

The transient behavior can be precisely controlled by using stabilizing feedback together with feed-forward signals generated by an inverse system. For linear multivariable systems the inversion problem has been solved to a large degree by Brockett and Mesarovic[9] and Silverman[10]. However, these inverses are all causal. The linear inversion results were extended to nonlinear real-analytic systems and conditions for the invertibility of these systems have been derived by Hirschorn[11, 12]. Singh[13], for example, had similar results on nonlinear inversion with some modified conditions. All these inversion algorithms produce causal inverses for a given desired output $y_d(t)$ and a fixed initial condition $x(t_0)$. Unbounded $u(t)$ and $x(t)$ were produced for nonminimum phase systems. This fundamental difficulty has been noted for a long time.

Motivated by the success of the noncausal inverse dynamics approach [14], the notion of stable inversion has recently been developed [1] in an effort to find feedforward signals for the tracking controller. The problem has been solved for a class of nonlinear nonminimum phase systems with well defined relative degree and hyperbolic zero dynamics. In this paper, we derive a numerical procedure for the constructing of the stable inverses. The remainder of this paper is organized as follows. In section 2 we state the problem of stable inversion of nonlinear systems and review previous results. Section 3 shows that the system states and control input obtained by the solutions of the corresponding equivalent two-point boundary value problem are of minimum 2-norm. Then, a numerical procedure aimed at finding the minimum-energy control input via the Moore-Penrose generalized inverse is constructed at the beginning of section 4. This is followed by the simulation results of an example which is a fourth order nonminimum phase nonlinear system. Finally, some remarks are given in section 5.

2 Stable Inversion Problem

Consider the multivariable nonlinear control systems of the form

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

$$y = h(x), \tag{2}$$

where the system state x is defined on a neighborhood X of the origin of $\mathbf{R}^n$ and input $u \in \mathbf{R}^m$ and output $y \in \mathbf{R}^m$. $f(x)$ and $g_i(x)$ (the ith column of $g(x)$) for $i = 1, 2, \ldots, m$ are smooth vector fields and $h_i(x)$ for $i = 1, 2, \ldots, m$ are smooth functions on X, with $f(0) = 0$ and $h(0) = 0$. For such systems, the stable inversion problem is stated as follows[1]:

Stable Inversion Problem: *Given a smooth reference* *output trajectory $y_d(t)$ with compact support, find a control input $u_d(t)$ and a state trajectory $x_d(t)$ such that*

1) $u_d(t)$ and $x_d(t)$ satisfy the differential equation

$$\dot{x}_d(t) = f(x_d(t)) + g(x_d(t))u_d(t);$$

2) exact output tracking is achieved

$$h(x_d(t)) = y_d(t);$$

3) $u_d(t)$ and $x_d(t)$ are bounded and

$$u_d(t) \to 0, \quad x_d(t) \to 0 \quad \text{as} \quad t \to \pm\infty.$$

We call $x_d(t)$ the desired state trajectory and $u_d(t)$ the nominal control input. The following notation will be used throughout this paper.

With $\mathbf{N} \stackrel{def}{=} \{1, 2, \ldots\}$, $r \stackrel{def}{=} (r_1, r_2, \ldots, r_m)^T \in \mathbf{N}^m$ and $y : \mathbf{R}^n \to \mathbf{R}^m$, we define $|r| \stackrel{def}{=} \sum_{i=1}^m r_i$,

$$y^{(r)} \stackrel{def}{=} (\frac{d^{r_1}y_1}{dt^{r_1}}, \frac{d^{r_2}y_2}{dt^{r_2}}, \ldots, \frac{d^{r_m}y_m}{dt^{r_m}})^T,$$

$$L_f^r y \stackrel{def}{=} (L_f^{r_1}y_1, L_f^{r_2}y_2, \ldots, L_f^{r_m}y_m)^T.$$

Assume that the system has a well-defined relative degree $r \in \mathbf{N}^m$ at the equilibrium point at the origin. To partially linearize the system, we define $\xi_k^i \stackrel{def}{=} y_i^{(k-1)}$ for all $i = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, r_i$; and denote

$$\xi \stackrel{def}{=} (\xi_1^1, \xi_2^1, \ldots, \xi_{r_1}^1, \xi_1^2, \ldots\ldots, \xi_{r_m}^m)^T$$

$$\stackrel{def}{=} (y_1, \dot{y}_1, \ldots, y_1^{(r_1-1)}, y_2, \ldots\ldots, y_m^{(r_m-1)})^T. \quad (3)$$

Choose η, an $n - |r|$ dimensional function on $\mathbf{R}^n$ such that $(\xi^T, \eta^T)^T = \psi(x)$ forms a change of coordinate with $\psi(0) = 0$[15]. In this new coordinate system, the system dynamics of equations (1)–(2) becomes

$$y^{(r)} = \alpha(\xi, \eta) + \beta(\xi, \eta)u, \quad (4)$$

$$\dot{\eta} = q_1(\xi, \eta) + q_2(\xi, \eta)u, \quad (5)$$

where

$$\alpha(\xi, \eta) = L_f^r h(\psi^{-1}(\xi, \eta)),$$

$$\beta(\xi, \eta) = L_g^1 L_f^{r-1} h(\psi^{-1}(\xi, \eta)),$$

and $\alpha(0, 0) = 0$ since $f(0) = 0$. Define

$$u \stackrel{def}{=} [\beta(\xi_d, \eta)]^{-1}(y_d^{(r)} - \alpha(\xi_d, \eta)), \quad (6)$$

where the subscript d refers to the desired output trajectory. Then, equation (5) becomes the so-called reference dynamics,

$$\dot{\eta} = p(y_d^{(r)}, \xi_d, \eta), \quad \eta \in \mathbf{R}^{n-|r|} \quad (7)$$

where $p(y_d^{(r)}, \xi_d, \eta)$ is obviously defined.

It is now clear that an integration of the reference dynamics gives rise to a trajectory of the original states through the inverse coordinate transformation $x = \psi^{-1}(\xi, \eta)$ and an input trajectory by equation (6). Now the problem is how to integrate the reference dynamics to generate bounded solutions to the stable inversion problem since the reference dynamics may be unstable in both positive and negative time directions in general.

For reference trajectories with compact support, the reference dynamics become autonomous zero dynamics for t outside the compact interval $[t_0, t_f]$. Assume that the zero dynamics has a hyperbolic equilibrium point at the origin. It has been shown that the stable inversion problem is equivalent to the following two-point boundary value problem[1]:

$$\dot{\eta} = p(y_d^{(r)}, \xi_d, \eta), \quad (8)$$

subject to

$$\begin{cases} B^s(\eta(t_0)) = 0 \\ B^u(\eta(t_f)) = 0, \end{cases} \quad (9)$$

where $B^s(\eta) = 0$ characterizes the unstable manifold denoted W^u, and $B^u(\eta) = 0$ the stable manifold W^s. It has also been shown that this two-point boundary value problem locally has a unique solution[2]. Once the two-point boundary value problem is solved, $x_d(t)$ and $u_d(t)$ can be constructed as follows:

$$x_d = \psi^{-1}(\xi_d, \eta), \quad (10)$$

$$u_d = [\beta(\xi_d, \eta)]^{-1}(y_d^{(r)} - \alpha(\xi_d, \eta)). \quad (11)$$

The main purpose of this paper is to construct a numerical procedure to solve this two-point boundary value problem. Now we end this section by recalling the following theorems from them our main results of the next section are benefited.

Theorem 1 *[16] Let W^s and W^u be the local stable and unstable manifolds of the equilibrium point at the origin of the zero dynamics. Then the solutions of the zero dynamics with initial conditions in W^s (respectively W^u) approach the origin at an exponential rate asymptotically as $t \to +\infty$ (respectively $t \to -\infty$).*

Theorem 2 *[17] Let $\eta(t)$ be the solution of the zero dynamics. Then there is a $\delta_1 > 0$ (resp. $\delta_2 > 0$) such that if $(\tau, \eta(\tau)) \in \mathbf{R} \times B(\delta_1)$ (resp. $\in \mathbf{R} \times B(\delta_2)$) for some solution η but $(\tau, \eta(\tau)) \notin W^s$ (resp. $\notin W^u$), then $\eta(t)$ must leave the ball $B(\delta_1)$ (resp. $B(\delta_2)$) at some finite time $t_1 > \tau$ (resp. $t_2 < \tau$).*

3 Minimum Energy Solution

This section establishes the following statement: The solution of the two-point boundary value problem equivalent to the stable inversion problem is of minimum 2-norm, thus, the corresponding desired state trajectory and nominal control input that map to the desired output trajectory are also of minimum 2-norm.

For the convenience of notation, we use $\eta(t)$ to denote the solutions of both reference dynamics and zero dynamics in the following context since the reference dynamics becomes the zero dynamics for t outside the compact interval $[t_0, t_f]$. First about the solution $\eta(t)$, we have the following result.

Theorem 3 *Among all the solutions $\eta(t)$ of the two-point boundary value problem (8), the one which satisfies the boundary condition (9), $\eta_d(t)$, is of minimum 2-norm.*

Due to limitation of space, the proof is omitted here. Interested readers can get the report [18] from the author by a mail or email request.

The following two theorems claim that the 2-norms of the state trajectory and of the control input obtained via $\eta_d(t)$ are minimum.

Theorem 4 *Among all the state trajectories $x(t)$ which produce the exact output tracking of the reference trajectory, the $x_d(t)$ computed by $x_d = \psi^{-1}(\xi_d, \eta_d)$, where η_d is the solution of (8) subjected to (9), is of minimum 2-norm.*

Again, the proof is omitted and can be found in [18].

For nonlinear control systems of the form (1)–(2), and equivalently of the form (4)–(5) in the new coordinate system, its corresponding unforced dynamics expressed in the new coordinate system is described as follows.

$$\left\{ \begin{array}{rcl} y^{(r)} & = & \alpha(\xi, \eta) \\ \dot{\eta} & = & q_1(\xi, \eta), \end{array} \right. \tag{12}$$

where the definition of ξ follows equation (3). Assume that the nonlinear systems under consideration satisfy the following condition:

Condition 1 *The unforced dynamics of the nonlinear system has a local property that its states are all zero if and only if its output is identically zero.*

Thus, for t outside the compact interval $[t_0, t_f]$, the output and all its derivatives of any orders are identically zero, that is, ξ is identically zero. The above condition together with $\alpha(0,0) = 0$ implies that $\alpha(0, \eta) = 0$ if and only if $\eta = 0$. With this in mind, we prove the following theorem regarding control input.

Theorem 5 *Assume that the nonlinear system (1)–(2) satisfies Condition 1. Then, among all the control inputs which produce the exact output tracking of the reference trajectory, the u_d computed by $u_d = [\beta(\xi_d, \eta_d)]^{-1}(y_d^{(r)} - \alpha(\xi_d, \eta_d))$, where η_d is the solution of (8) subjected to (9), is of minimum 2-norm.*

The proof is given in [18]

The results of these theorems allow us to construct a new numerical procedure which will be described in the next section to solve the equivalent two-point boundary value problem, thus solving the stable inversion problem of the nonlinear systems.

4 Algorithm and Simulation

An iterative numerical procedure is constructed here to find the control input producing the desired output trajectory with minimum energy. In each iteration, we linearize the system dynamics (1)–(2) along the solutions obtained in the previous step. Then the approximated linear time varying system is discretized to give rise to a linear algebraic system which can be solved by the Moore-Penrose generalized inverse to obtain the minimum energy solution, thus, the state trajectory and control input with minimum energy are obtained. This process which will be described in some detail as follows is continued until a convergent criterion is met.

Let x^{i-1} and u^{i-1} be the solutions obtained in the (i-1)th step. Let x^i and u^i be the new solutions to be solved in the current ith step. Linearizing the system equations (1)–(2) along x^{i-1} and u^{i-1} we get

$$\dot{x}^i = f(x^{i-1}) + J_x(f)(x^{i-1})(x^i - x^{i-1})$$
$$+J_x(gu)(x^{i-1}, u^{i-1})(x^i - x^{i-1}) + g(x^{i-1})u^i,$$
$$y^i = h(x^{i-1}) + J_x(h)(x^{i-1})(x^i - x^{i-1}),$$

where $J_x(f)(x0)$ denotes the Jacobian matrix of f with respect to x evaluated at $x0$. Thus, we have

$$\dot{x}^i = A^i(t)x^i + B^i(t)u^i + E^i(t), \tag{13}$$
$$y^i = C^i(t)x^i + D^i(t), \tag{14}$$

where

$$A^i(t) = J_x(f)(x^{i-1}(t)) + J_x(gu)(x^{i-1}(t), u^{i-1}(t));$$
$$B^i(t) = g(x^{i-1}(t));$$
$$C^i(t) = J_x(h)(x^{i-1}(t));$$
$$D^i(t) = h(x^{i-1}(t)) - J_x(h)(x^{i-1}(t))x^{i-1}(t);$$
$$E^i(t) = f(x^{i-1}(t)) - J_x(f)(x^{i-1}(t))x^{i-1}(t) -$$
$$J_x(gu)(x^{i-1}(t), u^{i-1}(t))x^{i-1}(t).$$

The discretization in time of the above linear time varying system via the standard approach will give us a sampled data system. For the sake of convenience, we drop the superscript i in the followings. Using the Variation of Constant Formula, we immediately get the sampled data system:

$$x_{k-1} = F_k x_k + G_k u_k + e_k, \tag{15}$$
$$y_k = H_k x_k + d_k, \tag{16}$$

where

$$F_k = \Phi((k+1)T, kT);$$
$$G_k = \int_{kT}^{(k+1)T} \Phi((k+1)T, t)B(t)\, dt;$$
$$e_k = \int_{kT}^{(k+1)T} \Phi((k+1)T, t)E(t)\, dt;$$
$$H_k = C(kT); \quad d_k = D(kT).$$

Notice that the above matrices can be precomputed for all integer k once the original continuous time linear system is known. Now, take an envelope interval $[t_1, t_2]$ such that $t_1 << t_0$ and $t_2 >> t_f$, and perform the sampling over the entire enlarged interval. Output y_k at each sampling time kT can then be evaluated as a linear combination of $u_{\bar{k}}$'s and $e_{\bar{k}}$'s with $\bar{k} < k$ via the sampled data system (15)–(16), that is,

$$y_k = H_k(G_{k-1}u_{k-1} + F_{k-1}G_{k-2}u_{k-2} + \cdots + F_{k-1}F_{k-2}\ldots F_1G_0u_0) + H_k(e_{k-1} + F_{k-1}e_{k-2} + \cdots + F_{k-1}F_{k-2}\ldots F_1e_0) + d_k + y_{k_0}, \quad (17)$$

where y_{k_0} is the effect of initial condition $x_0 = x(t_1)$.

To simplify the calculation, we try to get rid of the involvement of x_0 as follows. Assuming that the forward system is controllable, the any desired initial condition x_0 can be set up by using appropriate control input $u_{-1}, u_{-2}, \ldots, u_{-n}$ starting from zero initial state, where n is the order of the system. To avoid the potential use of large such control inputs, we allow more time than required to set up the correct initial condition, that is, we will use $u_{-1}, u_{-2}, \ldots, u_{-M}$ with $M >> n$. Then, equation (17) becomes

$$y_k = H_k(G_{k-1}u_{k-1} + F_{k-1}G_{k-2}u_{k-2} + \cdots + F_{k-1}F_{k-2}\ldots F_{-M+1}G_{-M}u_{-M}) + H_k(e_{k-1} + F_{k-1}e_{k-2} + \cdots + F_{k-1}F_{k-2}\ldots F_{-M+1}e_{-M}) + d_k. \quad (18)$$

There will be N such equations, where N is the number of total samples in $[t_1, t_2]$. Let Y be the column vector formed by stacking y_k's together, that is, y_k is the kth block row of Y. Similarly, let U be the column vector by stacking u_k's together. Then, the set of N equations of the form (18) can be written as a compact linear algebraic matrix equation

$$Y = M_\alpha U + M_\beta, \quad (19)$$

where M_α, the coefficient matrix, and M_β are defined in an obvious fashion. The above equation (19) defines U as a function of Y in a loose sense. Once Y is given, there are infinitely many U which will solve the equation since there are more unknowns in $U(= (N+M)m)$ than the number of equations$(= Nm)$. Invoking the Moore-Penrose generalized inverse, we get

$$U = M_\alpha^\dagger(Y - M_\beta). \quad (20)$$

This U will have minimum 2-norm among all the solutions of equation (19), hence the corresponding $u(t)$ will have minimum energy. Then, forward time simulation on the linearized time varying system using the computed input U, equivalently u as a function of time, will give us the approximated state x of the current step. The simulation stops when the states computed in the adjacent two steps are sufficiently close. By the theorems proved in the previous section, the control and state trajectories calculated this way are solutions to the stable inversion problem since both have minimum energy.

Notice that since the stable inversion method to implement the output tracking is always accompanied by the stabilizing feedback, we may assume that the linearized time varying system is asymptotically stable at every step. This is to guarantee the convergence of using of Variation of Constant Formula in forming the linear algebraic system.

Besides, when the sampling period T is taken to be sufficiently small, the linear time varying system can then be viewed as a time invariant system within any one sampling period. Thus, the computation of the transition matrices Φ will be much easier, leading to large reduction of time needed for discretization in each step.

The algorithm described above is illustrated by the following example of a slightly nonlinear single-input single-output system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ -3x_2 + x_1^2 \\ x_1 - 2x_3 \\ -x_4 + x_3^2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} u, \quad (21)$$

$$y = x_1 - 3x_3. \quad (22)$$

The reference output trajectory is given by:

$$y_d = \begin{cases} 2(1 - \cos(t)) & t \in [0, 2\pi], \\ 0 & \text{otherwise}. \end{cases} \quad (23)$$

It can be easily verified that this system has a well-defined relative degree $r = 2$ and its zero dynamics has one dimensional stable manifold and one dimensional unstable manifold. The corresponding unforced system satisfies the Condition 1 in a certain neighborhood of the origin. Here only x_1, x_2 and x_3 need to be considered since x_4 never appears in the output.

The equivalent two-point boundary value problem can be found as follows[1]

$$\begin{cases} \dot{\eta}_1 = \eta_1 + y_d \\ \dot{\eta}_2 = -\eta_2 + \eta_1^2, \end{cases} \quad (24)$$

subject to

$$\begin{cases} \eta_1(t_f) = 0 \\ \eta_2(t_0) = \frac{1}{3}\eta_1^2(t_0). \end{cases} \quad (25)$$

Since the reference dynamics are in a triangular form, the exact solution of the two point boundary value problem can be solved analytically. The simulated results via the iterative algorithm constructed above are shown in Figure 1, Figure 2 and Figure 4. Figure 1 indicates that almost perfect output tracking has been obtained except that the simulated trajectory is half sampling period behind the desired one. This error will be reduced if the sampling time is reduced. It can also be verified that the simulated sate trajectories in Figure 2 and control trajectory in Figure 3 are almost identical to the analytic solution. Hence, the algorithm has converged

5 Conclusion

For nonlinear systems of the form (1)–(2) with a well-defined relative degree and their zero dynamics has a hyperbolic equilibrium point at the origin, the stable inversion problem is equivalent to a two-point boundary value problem (8)–(9). In this paper, we have presented an iterative algorithm for the construction of desired state trajectory and nominal control input under the results that the solution of the two-point boundary value problem is of minimum 2-norm when the nonlinear system satisfies the Condition 1. The solutions produced by our method are bounded and causal. Tracking controllers using the signals generated by the stable inversion together with the stabilizing feedback will offer an exact output tracking without any transients. The key assumptions here on the nonlinear systems, when using the minimum-energy approach, are the well-defined relative degree, hyperbolicity of the fixed point of the zero dynamics and Condition 1 mentioned above. Even though this already covers a large number of physical systems in many engineering applications, systems without these assumptions or with any other forms will still be of great interest.

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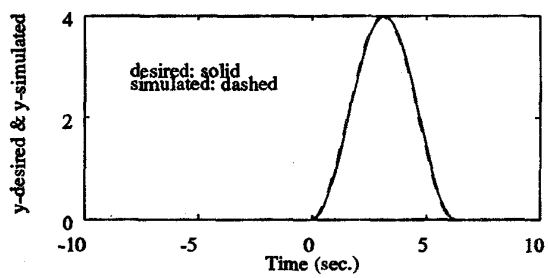

Figure 1. Desired and simulated output trajectories

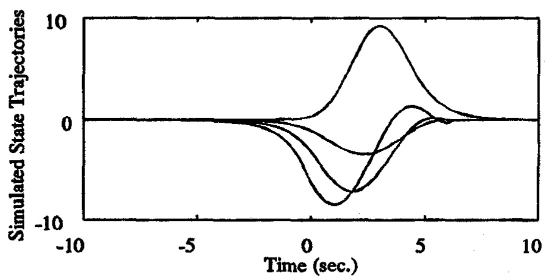

Figure 2. Simulated state trajectories

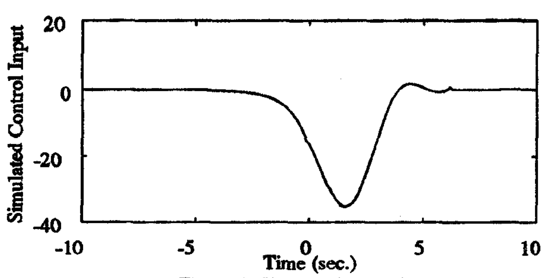

Figure 3. Simulated control input