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Optimal Motion Planning for Flexible Space Robots*

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Abstract

This paper is concerned with optimal motion planning of a flexible space robot. The robot is assumed to consist of two flexible links which are attached to a rigid space station floating in space. The optimal motion planning is first formulated as a two-stage functional optimization problem, which is further simplified into an optimal trajectory planning problem using recently developed stable inversion theory. The motion planning is optimal in the sense that the system performance measured by the maneuvering time together with control and structural vibration energy is minimized. Besides, The controller also keeps the interference from the arm to the space station satisfactorily small. A suboptimal solution to the corresponding trajectory planning problem is obtained via two decoupling on the linearized zero dynamics. One is of the hyperbolic and the nonhyperbolic parts, and another is of the stable and unstable parts. Numerical examples finally demonstrate the effectiveness of this approach.

1 Introduction

The flexibility of space robot manipulators and limited solar energy supplied by space station impose great challenges to the precise and satisfactory manipulator motion control. First, any control strategy has to result in a minimum energy consumption because of limited resource. Secondly, any movement of the manipulator arm would transmit an undesired interference force from arm to space station. Finally, any control forces or disturbances applied to the manipulator are very likely to excite structural vibrations in the arm as well as in the space station. Therefore, a good motion control design for a space manipulator should have the following properties: 1) Achieving a desired motion with the shortest possible time; 2) Not exciting structural vibrations; 3) Using a minimum amount of energy; and 4) Producing the minimum interference on the space station.

Researches concerning space robots have been mostly carried out by considering the rigid links assumption [5, 6]. By assuming relatively small elastic vibrations, a perturbation approach is utilized to design separate motion controllers for the rigid and the flexible parts [4]. Optimal control techniques have also been used in an effort to reduce energy consumption. By using reaction wheels or attitude control jets [7], the effect of interference from manipulator motion to space station can be compensated. Another method to reduce the interference is to include the space station in the trajectory planning or to use kinematic redundancy to optimize the robot trajectory [3]. It is noticed that all the methods mentioned above either lead to slow motion in order to keep down energy consumption and vibration excitation, or neglect the transient impact on the space station. Limitation on fuel consumption has also minimized the usage of attitude control jets. Motion control without requiring reaction wheels would be of great advantages because of the wheel's significant mass introduced to the system and their limited capability to correct for the interference.

By using stable inversion approach [2, 1], a completely different approach newly developed for designing tracking controllers, this paper investigates a motion control strategy for space manipulators with all flexible links and no control jets or reaction wheels. The remainder of the paper is organized as follows. Section 2 briefly describes the equations of motion of flexible space robots for purpose of fixing notations. It also introduces the formulation of a two-stage functional optimization problem which characterizes the optimal motion planning. For more details, readers are refered to [9]. Section 3 develops a suboptimal solution to the optimal trajectory planning. Analysis of simulation results are presented in section 4. A conclusion is finally given in section 5.

2 Dynamics and Optimal Motion Planning

Consider a flexible space robot system which consists of a rigid space station and a robot arm with two flexible links. Only planar maneuver is assumed so that we neglect the out-of-plane deformation. Any possible effect

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from the sun or the earth is also neglected which implies that there is no external force on the system. System's coordinate frames are defined in Figure 1.

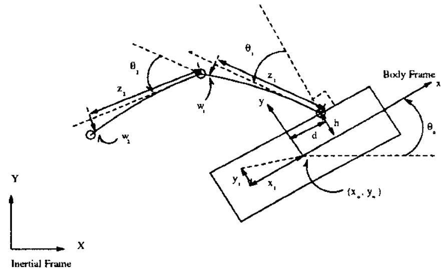

Figure 1: Flexible Space Robot

Assumed modes method is used to parameterize the continuous deformation of each flexible link. The admissible functions are chosen to be the ones for the clamped-free beams in this study and two flexible modes are taken for each link. The Lagrange's method is applied to form the system equations of motion. Denote the whole system's generalized coordinates as

$$\psi = (x_0, y_0, \theta_0, \theta_1, \theta_2, q_{11}, q_{12}, q_{21}, q_{22})^T,$$

which consists of the coordinates for the space station v, the rigid modes for links θ, and the flexible ones for the links q:

$$v \stackrel{def}{=} (x_0, y_0, \theta_0)^T, \quad \theta \stackrel{def}{=} (\theta_1, \theta_2)^T,$$

$$q \stackrel{def}{=} (q_{11}, q_{12}, q_{21}, q_{22})^T.$$

By the extended Hamilton's principle in the form of Lagrange's equation, the equations of motion can be written as

$$M(\psi)\ddot{\psi} + H(\psi, \dot{\psi}) + C\dot{\psi} + K\psi = Bu, \tag{1}$$

where $M(\psi)$ is the positive definite symmetric inertial matrix, $H(\psi, \dot{\psi})$ is the part containing the certrifigal and the coriolis terms, C the damping matrix, K the stiffness matrix, B the torque distribution matrix, and u is the vector of joint torque.

The angular coordinates for the tip positions of the links are chosen to be the system's output vector, which is given by

$$y = \begin{bmatrix} \theta_1 + \arctan(\frac{w_1(l_1,t)}{l_1}) \\ \theta_1 + \theta_2 + \arctan(\frac{w_2(l_2,t)}{l_2}) \end{bmatrix}. \tag{2}$$

For small elastic deformation

$$\arctan(\frac{w_i(l_i,t)}{l_i}) \approx (\frac{w_i(l_i,t)}{l_i}), \quad \forall i = 1, 2.$$

Thus, we have the output equation

$$y = D\psi, \tag{3}$$

where matrix D is defined accordingly. Thus, system's forward dynamics is given by equations (1) and (3).

Now consider the task usually performed by a robot arm attached to a space station. The task may consist of moving the manipulator from an initial configuration to a final desired configuration. Trajectory planning is usually first designed to provide a planned path to realize the change of configurations. The planned reference trajectory together with the system dynamics then determines the absolute best an ideal controller can do. This is measured in terms of the amount of control effort, the maneuvering time, the structural vibrations, and the interference from the arm to the space station. Consequently, optimizing the reference trajectory is expected to lead to overall better motion control performance.

Thus, the optimal motion planning can be characterized by the following optimization problem. The initial and the final configurations are specified by y_0 and y_f respectively. For convenience, we also define the energy part of the performance index as

$$J^e \stackrel{def}{=} \int_{-\infty}^{+\infty} W_q \|q(t)\|_2 \, dt + \int_{-\infty}^{+\infty} W_u \|u(t)\|_2 \, dt, \tag{4}$$

where W_q and W_u are the penalty weights put on the energy consumption. **Optimal Motion Planning:**

$$\min_u \{t_f - t_0 + J^e\}$$

subject to

$$y(t) = y_0, \quad \forall t \le t_0, \quad \text{and,} \quad y(t) = y_f, \quad \forall t \ge t_f,$$

$$\|q(t)\| \le \epsilon_q, \quad \|u(t)\| \le \epsilon_u,$$

$$\|v(t)\| \le \epsilon_v, \quad \|\dot{v}(t)\| \le \epsilon_{\dot{v}},$$

and system dynamics:

$$M(\psi)\ddot{\psi} + H(\psi, \dot{\psi}) + C\dot{\psi} + K\psi = Bu,$$

$$y = D\psi,$$

and t_0 is given.

Here $\|\cdot\|$ appeared in constraint equations is, by abuse of notation, taken to be the component-wise infinity norm, and t_0 and t_f are defined as the initial and the final time of maneuver respectively.

We attack this optimal motion planning problem in two steps. A trajectory planning is first carried out to find out a reference trajectory satisfying the hard constraints required on the initial and the final time. Then, a motion control strategy is designed to realize the reference trajectory. Thus, the above optimal motion planning problem is equivalent to the following two-stage optimization problem.

Two-Stage Optimal Motion Planning:

$$\min_{y_d} t_f - t_0 + \left\{ \begin{array}{cc} \min\limits_{u} & J^e \\ \text{s. t. } y(t) = y_d(t), \ \forall t & \end{array} \right\},$$

subject to

$$\|q(t)\| \le \epsilon_q, \quad \|u(t)\| \le \epsilon_u,$$

$$\|v(t)\| \le \epsilon_v, \quad \|\dot{v}(t)\| \le \epsilon_{\dot{v}},$$

and system dynamics:

$$M(\psi)\ddot{\psi} + H(\psi, \dot{\psi}) + C\dot{\psi} + K\psi = Bu,$$

$$y_d = D\psi,$$

and t_0, $y_d(t_0)$ *and* $y_d(t_f)$ *are given.*
See [9] for more details.

3 Optimal Trajectory Planning

Notice that the inner optimization in the above two-stage optimal motion planning problem is actually an unconstrained exact output tracking control problem with energy minimization. The newly developed stable inversion theory provides a solution that precisely addresses these issues. For any given reference trajectory, it has been shown that the stable inversion leads to a control strategy that guarantees exactly output tracking with internal stability. Furthermore, among all controls which generate the desired reference trajectory, the one being the solution of stable inversion has minimum energy, and so does the corresponding internal dynamics [8]. Thus, the inner optimization problem is automatically solved by using stable inversion, and the two-stage optimal motion control problem is therefore reduced to the outer stage optimal trajectory planning problem.

Optimal Trajectory Planning:

$$\min_{y_d} \{t_f - t_0 + J_*^e\}$$

subject to

$$\|q_*(t, y_d)\| \le \epsilon_q, \quad \|u_*(t, y_d)\| \le \epsilon_u,$$

$$\|v_*(t, y_d)\| \le \epsilon_v, \quad \|\dot{v}_*(t, y_d)\| \le \epsilon_{\dot{v}},$$

and t_0, $y_d(t_0)$, *and* $y_d(t_f)$ *are given.*
Here the subscript $*$ stands for the solutions of stable inversion which solve the inner-stage optimization problem.

In order to apply the stable inversion theory, it is required that the system should have a well-defined relative degree and its zero dynamics should have a hyperbolic equilibrium point at the origin. The first condition is well satisfied since the both components of the output vector chosen in this study uniformly have relative degree 2. This can be easily verified from the system's forward dynamics. However, the hyperbolicity condition is not satisfied in this flexible space robot case. Fortunately, this problem can be avoided when pursuing only suboptimal solutions as discussed later in this section.

It is clear that the above optimal trajectory planning problem is an infinite dimensional search. We simplify it now by taking the desired output reference trajectory $y_d(t)$, with specified initial and final values $y_d(t_0)$ and $y_d(t_f)$, as a linear combination of some base time functions. Hence, by search over a finite coefficient space, we can simplify the trajectory planing into a finite dimensional problem. It can be easily verified that the following parameterization is valid by choosing the sinusoidals as base functions:

$$\begin{aligned} & y_{d_k}(c_{1_k}, \ldots, c_{n_k}, t) \\ = \; & y_{d_k}(t_0) + [y_{d_k}(t_f) - y_{d_k}(t_0)] \cdot \\ & \cdot \left[\frac{t - t_0}{t_f - t_0} - \sum_{i=1}^{n} \frac{c_{i_k}}{2\pi i} \sin(2\pi i \frac{t - t_0}{t_f - t_0}) \right], \end{aligned} \tag{5}$$

with

$$\sum_{i=1}^{n} c_{i_k} = 1.$$

Now the stable inverses as functions of the desired output need to be found through the stable inversion approach. It is known from the stable inversion that these inverses are nonlinear functions of the solutions to a nonlinear two-point boundary value problem. In order to obtain the algebraic expressions for the constraint equations, the two-point boundary value problem is solved analytically by linearizing the zero dynamics at some equilibrium point followed by integrating the stable dynamics forwards in time and the unstable dynamics backwards in time. It is noticed that by doing this linearization, only a suboptimal solution will be achieved in the trajectory planning. As mentioned above, the stable inversion approach is not applicable when the zero dynamics is not hyperbolic. However, in this study, the hyperbolic part of the linearized zero dynamics is decoupled from the nonhyperbolic part. Thus, this part can be solved through stable inversion approach and then used to find the solution for the nonhyperbolic part. Therefore, we obtain the constraint equations in algebraic form only. See the following context for more details.

The derivation of zero dynamics from forward system dynamics is omitted here. The linearized zero dynamics can be written as

$$M_{v1}\ddot{v} + M_{q1}\ddot{q} + M_{y1}\ddot{y}_d = 0, \tag{6}$$

$$M_{v2}\ddot{v} + M_{q2}\ddot{q} + M_{q3}\dot{q} + M_{q4}q + M_{y2}\ddot{y}_d = 0. \tag{7}$$

Eliminating $\ddot{v}$ from the above two equations to obtain the hyperbolic part of the zero dynamics, we have

$$\dot{\bar{q}} = A\bar{q} + B\ddot{y}_d, \tag{8}$$

where $\bar{q} = (q^T, \dot{q}^T)^T$, and matrices A and B are defined accordingly. To solve for the stable inverses from the above hyperbolic part of the zero dynamics using stable inversion, we further decouple the dynamics (8) into stable and unstable parts by

$$\bar{q} \stackrel{def}{=} \begin{bmatrix} X_s & X_u \end{bmatrix} \tilde{q} \stackrel{def}{=} \begin{bmatrix} X_{sq} & X_{uq} \\ X_{s\dot{q}} & X_{u\dot{q}} \end{bmatrix} \tilde{q}, \tag{9}$$

which leads to

$$\begin{bmatrix} \dot{\tilde{q}}_1 \\ \dot{\tilde{q}}_2 \end{bmatrix} = \begin{bmatrix} J_s & 0 \\ 0 & J_u \end{bmatrix} \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{bmatrix} + \begin{bmatrix} B_s \\ B_u \end{bmatrix} \ddot{y}_d. \qquad (10)$$

We carry out a time-scaling at this moment to simplify the calculation by setting $t_0 = 0$ and

$$t_{\text{new}} \stackrel{def}{=} t_{\text{old}}/t_f.$$

Changes due to this scaling will be explained as they are inferred later. From the requirement of stable inversion we know that the hyperbolic part of the zero dynamics lies in unstable manifold at time t_0 while in stable manifold at time t_f, that is,

$$\tilde{q}_1(0) = 0, \quad and, \quad \tilde{q}_2(1) = 0.$$

Solving (10) with the above initial conditions, we obtain

$$\tilde{q}_1(t) = \int_0^t \exp\{J_{sf}(t-\tau)\} B_{sf} \ddot{y}_d(\tau)\, d\tau, \ \forall t \geq 0, \quad (11)$$

$$\tilde{q}_2(t) = \int_1^t \exp\{-J_{uf}(\tau - t)\} B_{uf} \ddot{y}_d(\tau)\, d\tau, \ \forall t \leq 1, \quad (12)$$

where by scaling

$$J_{sf} := t_f J_s, \quad and, \quad J_{uf} := t_f J_u,$$

and B_{sf} and B_{uf} are also defined accordingly. Straightforward integration on equations (11) and (12) provides

$$\begin{aligned} \tilde{q}_1(t) &= \sum_{i=1}^{n} (I + \frac{1}{\omega_i^2} J_{sf}^2)^{-1} \cdot \\ &\cdot \left[\frac{1}{\omega_i} \left(e^{J_{sf} t} - \cos(\omega_i t) I \right) B_{sf} - \right. \\ &\left. \frac{1}{\omega_i^2} J_{sf} B_{sf} \sin(\omega_i t) \right] C_i, \ \forall t \geq 0, \end{aligned}$$

$$\begin{aligned} \tilde{q}_2(t) &= \sum_{i=1}^{n} (I + \frac{1}{\omega_i^2} J_{uf}^2)^{-1} \cdot \\ &\cdot \left[\frac{1}{\omega_i} \left(e^{-J_{uf}(1-t)} - \cos(\omega_i t) I \right) B_{uf} - \right. \\ &\left. \frac{1}{\omega_i^2} J_{uf} B_{uf} \sin(\omega_i t) \right] C_i, \ \forall t \leq 1, \end{aligned}$$

where

$$\omega_i \stackrel{def}{=} 2\pi i,$$

$$C_i \stackrel{def}{=} \begin{bmatrix} (y_{d_1}(1) - y_{d_1}(0)) c_{i_1} 2\pi i \\ (y_{d_2}(1) - y_{d_2}(0)) c_{i_2} 2\pi i \end{bmatrix}.$$

Through inverse transformation of (9), we have constructed a mapping from C, set of coefficients C_i for all $i = 1, \ldots, n$, to q_*:

$$q_* := \mathcal{T}_q(C), \qquad (13)$$

which can be written as

$$q_*(t, y_d) = \begin{cases} X_{uq} e^{J_{uf} t} \tilde{q}_2(0), & \forall t \leq 0; \\ X_{sq} \tilde{q}_1(t) + X_{uq} \tilde{q}_2(t), & \forall 0 \leq t \leq 1; \\ X_{sq} e^{J_{sf}(t-1)} \tilde{q}_1(1), & \forall t \geq 1. \end{cases}$$

Using this mapping and integrations of equation (6) with time-scaling carefully involved will bring us mappings from C to v and $\dot{v}$:

$$v_* := \mathcal{T}_v(C), \quad \dot{v}_* := \mathcal{T}_{\dot{v}}(C). \qquad (14)$$

From System forward dynamics with θ expressed as a function of y_d and q_*, substituting (13) and (14) into it leads to the mapping from C to optimal control input

$$u_* := \mathcal{T}_u(C). \qquad (15)$$

The analytical expressions for $\mathcal{T}_v(C)$, $\mathcal{T}_{\dot{v}}$ and $\mathcal{T}_u(C)$ are omitted here.

With all these derivations, the corresponding optimal trajectory planning problem is further simplified, by solving for a suboptimal solution, into the following finite dimensional trajectory planning problem with only algebraic constraints. This problem can be solved by utilizing some software package available at present and is formulated as follows:

Suboptimal Trajectory Planning:

$$\min_C \{t_f - t_0 + J_*^e\}$$

subject to

$$\|\mathcal{T}_q(C)\| \leq \epsilon_q, \quad \|\mathcal{T}_u(C)\| \leq \epsilon_u,$$
$$\|\mathcal{T}_v(C)\| \leq \epsilon_v, \quad \|\mathcal{T}_{\dot{v}}(C)\| \leq \epsilon_{\dot{v}},$$

and t_0 is given.

4 Simulation Analysis

In this section, we present the simulation results to illustrate our optimal motion planning approach. The flexible space robot with properties listed in Table 1 is utilized as the physical model. We assume that the initial and final configurations of the arm are

$$y_d(t_0) = \begin{bmatrix} -45^o \\ 45^o \end{bmatrix}, \quad \text{and,} \quad y_d(t_f) = \begin{bmatrix} 40^o \\ 140^o \end{bmatrix},$$

which may be visualized from Figure 2.

The corresponding suboptimal trajectory planning problem is first solved to find the planned trajectory. We set both the energy weights W_q and W_u to zero for the convenience in comparison later. This leaves the only maneuvering time in the performance index. Thus, we are looking for a trajectory corresponding to a maneuver which requires the shortest time.

The constraint bounds are determined as follows. By requiring less than five degree's deformation in each link, we take the following as the bound on flexible modes q:

$$\epsilon_q = [0.06, 0.06, 0.06, 0.06]^T.$$

Links, k=1,2		Spacecraft	
$l_k(m)$	5.0	$h(m)$	1.0
$\rho_k(kg/m)$	0.2	$d(m)$	1.0
$EI_k(N/m^2)$	120.0	$m_0(kg)$	330.0
$m_{e_k}(kg)$	0.1	$I_0(kgm^2)$	3125.0
$I_{b_k}(kgm^2)$	20.0		

Table 1: Flexible Space Robot Properties

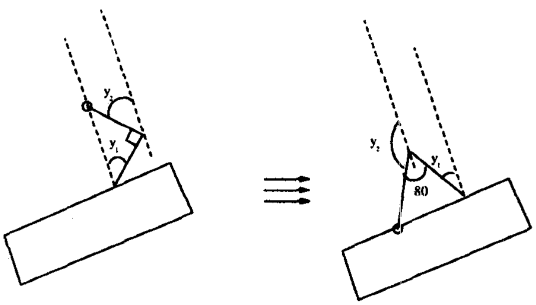

Figure 2: Initial and Final Configurations

Bound on the interference on the space station, which is characterized by generalized coordinates v and $\dot{v}$, is chosen by investigating the fixed position of the mass center of the whole system. Since there is no external forces acted on the system, the position of mass center should remain unchanged after the maneuver. Assume that links are moving, in an ideal case, extremely slowly to fulfill the change of configurations, the space station has also to move to compensate the motion in order to maintain the original position of the mass center. By

$$G_x(t_0) = G_x(t_f), \quad \text{and,} \quad G_y(t_f) = G_y(t_f), \tag{16}$$

and assuming $\theta_0(t_f) << 40^o$, we obtain

$$x_0(t_f) = 0.0339, \quad \text{and,} \quad y_0(t_f) = 0.0118.$$

Taking the bounds as 25% larger than these ideal values, and a reasonably small value for θ_0, we have

$$\epsilon_v = [0.042, 0.015, 0.046]^T.$$

Arbitrarily pick small bounds on $\dot{v}$ and u to represent the slow movement of the space station and allowable control torque size. Their numerical values are as follows:

$$\epsilon_{\dot{v}} = [0.01, 0.003, 0.008]^T,$$

$$\epsilon_u = [10.0, 6.0]^T.$$

The optimization is carried out by taking three different frequency components in output, that is, $n = 3$. Figure 3 shows the obtained suboptimal output trajectory. This trajectory corresponds to the fastest motion with all the vibrations, interferences and control satisfying the $\mathcal{L}_\infty$-norm bounds in the constraint equations. The minimum maneuvering time is found to be $t_f^* = 12.24$ seconds.

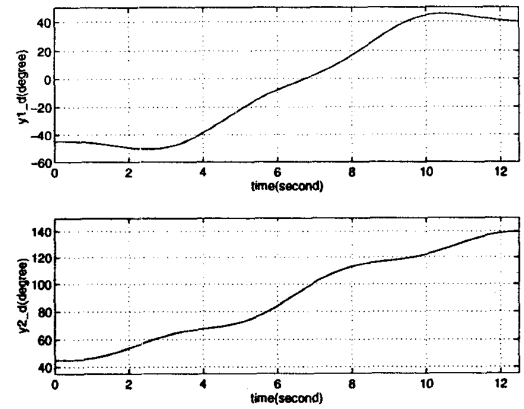

Figure 3: Suboptimal Trajectory Planned

Based on this planned trajectory, system's internal modes and control input can be computed through stable inversion approach to reproduce this output trajectory. The reason to use the stable inversion approach is that the so generated system's internal modes as well as the control possess the minimum energy property among all other modes and control which also produce the same output. This control design will be seen in our future work.

A comparison study is made at this point. A heuristic trajectory with sinusoidal acceleration profile is chosen as another planned motion trajectory. This trajectory is constructed such that it requires the same amount of time to fulfill the maneuver and is shown in Figure 4. Even though the heuristic trajectory has been chosen to be as smooth as possible, this trajectory requires control which will excites the system's internal modes above the bound set in the constraint equations. Figure 5 shows the vibrations modes q generated based on the two planned trajectories respectively as well as the bound on them. The violation is seen in the upper part of the figure which corresponds to the heuristic trajectory.

Another comparison is made by applying distinct controls to realized the same maneuvering trajectory. The heuristic sinusoidal trajectory is used as planned trajectory. Both the stable inversion approach and a PD controller are implemented to fulfill the required motion. The vibration energy generated and control energy consumed are summarized in table 2. It is seen that the minimum energy property still remains for the stable inversion approach. However, it is noticed that only the vibration energy is expected to be smaller by applying

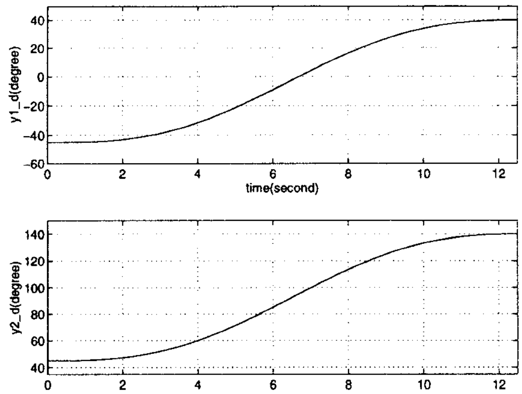

Figure 4: A Heuristic Trajectory Planned

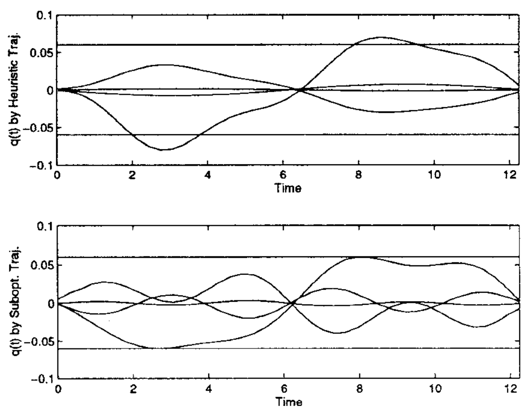

Figure 5: Vibration modes q and its bound

stable inversion since it is the only hyperbolic part in zero dynamics. It turns out in this simulation study that the control energy is also better in stable inversion than in PD controller which is not guaranteed because the system does not satisfy the hyperbolicity requirement.

5 Conclusion

This paper presents a new optimal motion planning problem for flexible space manipulators. First, a two-stage functional optimization problem is formulated to characterize the optimal motion, and it is then simplified into an optimal trajectory planning problem by utilizing the stable inversion theory. A suboptimal solution is obtained instead of the optimal one due to the nonhyperbolicity of the zero dynamics.

Simulation results illustrate the suboptimal trajectories planned for the optimal motion. A comparison study is made against the conventionally chosen sinusoidal trajectory. Both trajectories are of the same maneuvering time. Study shows that the sinusoidal trajectory requires the control which will excites the vibration modes above the restriction bound. Another study shows that for the same trajectory, stable inversion required control with less control effort and less vibrations. Future work will focus on the controller design due to the plant instability and uncertainty in parameters and truncated dynamics, and also the optimal solution approach to the problem.

Table 2: A Comparison on Energy

Control	$\|\|q(t)\|\|_2$ (10^{-3})	$\|\|u(t)\|\|_2$
Stable Inversion	184.9	14.73
PD Control	222.8	16.45

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