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Learning and H_∞ Control of an Overhead Crane for Obstacle Avoidance and Disturbance Rejection [1]

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Abstract

A control strategy is developed for a 3-dimensional overhead crane. Using the differential flatness of the crane and a parameterization method, we first calculate the optimal trajectory of the payload that results in minimum transfer time when there exist obstacles in the direct moving path. A learning algorithm is then used to generate the desired feed forward input signal that can drive the system output to track the optimal trajectory. A key feature of this strategy is that it is model free and thus is robust to uncertainties in modeling and parameters. When there exist external disturbances, an H_∞ optimal control method is used to reject the disturbances. Simulation results are given to verify the strategy and compare some performances.

1 Introduction

Cranes are important tools on many material handling sites, such as, construction sites, plants, harbors, airports, etc. In these places, large and heavy objects need to be transfered from one location to another. Such transportations are usually accomplished by some kinds of cranes. An overhead crane is one of the most common types of cranes found on working sites (See Figure 1). The fundamental movements of an overhead crane are: horizontal traverse of the bridge on the runway, trolley travel on the bridge, and vertical travel of the hoist below the trolley.

There are two major concerns in the operation of a crane system. First, the crane travel and traverse motions, especially during starting or stopping, induce the undesirable vibrations of the suspended object. In addition, external disturbances, such as gust wind, also can produce the vibrations of the payload. Second, there are often obstacles on the working sites, and the payload trajectory must be carefully designed to avoid collisions.

Much research has been done on crane control. Auernig and Troger (1987) developed time-optimal control for a system which had trolley travel and hoist movements. Meta and Unbehanen (1987) considered industrial application of time-optimal ore unloader control. Path optimization was also utilized to maintain a safety distance from the grab to the ship and other parts of the crane. Jones and Petterson (1988) programmed an acceleration profile and obtained an oscillation-damped transport with swing-free stops. Noakes and Jansen (1992) used programmed acceleration-profiles to implement oscillation-damped transports and swing-free stops for the suspended payload. Hämäläinen et al. (1995) developed the dynamical models of the crane mechanics and actuators. By dividing the path planning problem into five phases, they also obtained energy-optimal speed references for a given transfer time.

The above methods for the crane deal with one-dimensional movement (trolley travel or bridge traverse) or two-dimensional movement (trolley travel or bridge traverse with hoist movements). A more realistic case is a three-dimensional (3-D) movement of the overhead crane, that is, travel, traverse and hoist move concurrently. Moustafa (1994) gave a linearized model of the load swing dynamics of an overhead crane with simultaneous trolley, bridge and hoist movements. Time-varying coefficients were considered and a set of sufficient conditions on the parameters of feedback law were obtained to guarantee the asymptotic stability of the system.

All of the crane control systems above consider the swing motion produced by trolley or girder movement and do not deal with the case where vibrations are caused by external disturbances. Beilvean et al. (1993) proposed a control strategy to control payload oscillations regardless of the causes of excitation. The strategy was based on sensing the dynamic response at the load hoist cable and applying periodic balancing forces and moments to the cable to damp the oscillation of the load, whenever detected.

In this paper, a robust control strategy is developed for a 3-D overhead crane. First, we calculate the time-optimal trajectory for a payload to move from one place to another without colliding with obstacles. This planing does not involve any system parameters and is made possible by a set of special flatness relations. Next, we use a learning control algorithm to obtain the desired input signal which can drive the output to track the optimal trajectory. The learning control is necessary for a model free approach

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since the flatness equations relating input to payload trajectorie involve system parameters. Because of the model free nature, this method is robust to modeling errors as well as parameter uncertainties such as the weight of the payload. To be able to reject external disturbances, we use H_∞ method on a nominal model to design a stabilizing feedback law that is used in the learning controller. However, the issue of how to compromize between the learning convergence and the disturbance-rejection performance is not easily resolved. We will give some simulation results to illustrate this problem.

2 Model Description

Consider a 3-D overhead crane shown in Figure 1 with the following assumptions: 1. the payload is a point-mass. 2. the rope is massless and has no torsional stiffness. 3. the motions of the trolley and girder are frictionless. 4. the control inputs to the system are the mechanical forces. In

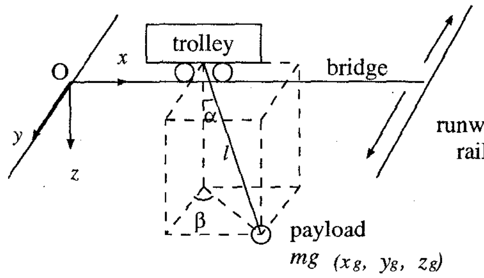

Figure 1: Overhead crane system

this 3-D crane system, there are five degrees of freedom. The dynamics can be separated into two parts. The upper part is related to the motor drives for trolley, girder and hoist movements. We use the coordinate system (x, y, l) to represent their positions. The lower part corresponds to the load. For the load dynamics, we can either use the Cartesian coordinates (x_g, y_g, z_g) or use the spherical coordinates (α, β, l) to indicate the load position. These two coordinate systems have the following relationship

$$x_g = x + l \sin\alpha \sin\beta \tag{1}$$
$$y_g = y + l \sin\alpha \cos\beta \tag{2}$$
$$z_g = l \cos\alpha \tag{3}$$

In this paper, we choose the Cartesian coordinates (x_g, y_g, z_g) to represent the load dynamics since the spherical coordinates exhibit singularity at all possible equilibrium points (when $\alpha = 0$).

The kinetic energy of the model in Figure 1 is

$$\begin{aligned} T &= \frac{1}{2}J_1(\frac{\dot{x}}{b_1})^2 + \frac{1}{2}J_2(\frac{\dot{y}}{b_2})^2 + \frac{1}{2}J_3(\frac{\dot{l}}{b_3})^2 + \frac{1}{2}m_2\dot{y}^2 \\ &+ \frac{1}{2}m_1(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}m(\dot{x_g}^2 + \dot{y_g}^2 + \dot{z_g}^2) \end{aligned} \tag{4}$$

and the potential energy is

$$U = -mgz_g \tag{5}$$

where g is the gravitational acceleration. m_1 and m_2 are the masses of the trolley and girder, respectively. J_1, J_2, J_3 are the mass moments of inertia of the trolley, girder and hoist motors respectively, while b_1, b_2, b_3 are the radii of the respective motor drums. m is the mass of the load.

In equation (4), there are six variables and only five of them are independent. If we choose x, y, l, x_g, y_g as independent variables, then z_g can be obtained by

$$z_g = \sqrt{l^2 - (x_g - x)^2 - (y_g - y)^2} \tag{6}$$

Suppose that F_1, F_2, F_3 are the mechanical forces applied to the trolley, girder and hoist, respectively. Let $q = [x\ y\ l\ x_g\ y_g]^T$, $\bar{u} = [F_1\ F_2\ F_3\ 0\ 0]^T$, then using the Lagrangian method, the equations of motion are given by

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}) - \frac{\partial L}{\partial q} = \bar{u} \tag{7}$$

where $L = T - U$. Let the state variable $\bar{x} = [q^T\ \dot{q}^T]^T$ and the output $\bar{y} = [x_g, y_g, z_g]^T$, then the state-space model of (7) can be written as

$$\begin{aligned} \dot{\bar{x}} &= \begin{bmatrix} \dot{q} \\ H^{-1}\{\bar{u} - [\frac{1}{2}\dot{H} + S(q,\dot{q})]\dot{q} + \frac{\partial G}{\partial q}\} \end{bmatrix} & (8) \\ &= f(\bar{x}, \bar{u}) & (9) \end{aligned}$$

where H denotes the inertia matrix of the system and is a function of $\bar{x}$ and the system parameters, and

$$G(q) = mg\sqrt{l^2 - (x_g - x)^2 - (y_g - y)^2} \tag{10}$$

$$S(q,\dot{q})\dot{q} = \frac{1}{2}\dot{H}\dot{q} - \frac{1}{2}\frac{\partial(\dot{q}^T H \dot{q})}{\partial q} \tag{11}$$

The above model involves $H, \dot{H}, S(q,\dot{q}), \frac{\partial G}{\partial q}$. Among them, $H, G(q)$ are explicitly expressed as functions of $\bar{x}$ and the system parameters. $H, S(q,\dot{q}), \frac{\partial G}{\partial q}$ can be obtained by symbolic MAPLE when we do simulation on computers.

3 Optimal Load Trajectory Using Flatness

An overhead crane usually transfers a load from one location to another repetitively. In case there exists an obstacle in the direct load moving path, a feasible load path must be carefully designed to ensure that the load will maintain a minimum safe distance from the obstacle as well as a small swing angle during transition.

In general, the trajectory planing needs to be performed for all state variables of $\bar{x}$ in order to meet system dynamic constraints. However, since the overhead crane is a differentially flat system which has the property that the state and input variables can be directly expressed in terms of the output and a finite number of its derivatives (Fliess et al. 1990). Therefore, if we develop trajectory planing for the output $\bar{y}$, then the state variable $\bar{x}$ can be calculated immediately. In section 3.1, we discuss the flatness of the crane. In section 3.2, we develop a method to obtain time-optimal trajectories for $\bar{y}$.

3.1 The flatness of the overhead crane

According to Fliess et al. (1990), an overhead crane is a flat system with following equations

$$(\ddot{z}_g - g)(x_g - x) = \ddot{x}_g z_g \tag{12}$$
$$(\ddot{z}_g - g)(y_g - x) = \ddot{y}_g z_g \tag{13}$$
$$(x_g - x)^2 + (y_g - y)^2 + z_g^2 = l^2 \tag{14}$$

These equations can be rearranged to yield

$$x = x_g - \frac{\ddot{x}_g z_g}{\ddot{z}_g - g} \tag{15}$$
$$y = y_g - \frac{\ddot{y}_g z_g}{\ddot{z}_g - g} \tag{16}$$
$$l = \sqrt{(x_g - x)^2 + (y_g - y)^2 + z_g^2} \tag{17}$$

Thus, x, y, l are the functions of x_g, y_g, z_g and their derivatives. By differentiating equations (15) to (17), the derivatives of x, y, l can also be represented as the functions of the derivatives of x_g, y_g, z_g. Thus, if $\bar{y}$ is known, then $\bar{x}$ can be obtained.

The load swing angles α, β can also be represented as the functions of the derivatives of $\bar{y}$. We can simply write down the following differential equations for the load dynamics

$$m\ddot{x}_g = -T \sin\alpha \sin\beta \tag{18}$$
$$m\ddot{y}_g = -T \sin\alpha \cos\beta \tag{19}$$
$$m\ddot{z}_g = -T \cos\alpha + mg \tag{20}$$

where T is the tension of the rope. ¿From these equations, α, β can be represented as

$$\alpha = \arctan\left(\sqrt{\ddot{x}_g^2 + \ddot{y}_g^2} / (\ddot{z}_g - g)\right) \tag{21}$$
$$\beta = \arctan(\ddot{x}_g / \ddot{y}_g) \tag{22}$$

That is, α, β are the functions of derivatives of $\bar{y}$.

3.2 Optimal trajectories for x_g, y_g, z_g

Let the desired output $\bar{y}_d = [x_{gd}, y_{gd}, z_{gd}]^T$ (subscript(d) indicates the desired trajectory), then x_{gd}, y_{gd}, z_{gd} should satisfy some geometric constraints and swing angle constraints.

Let $g_k(\bar{y}_d) \leq r_k, k = 1, \cdots, n$ represent geometric constraints needed for obstacle avoidance and $|\alpha(\bar{y}_d)| \leq \alpha_0$, $|\beta(\bar{y}_d)| \leq \beta_0$ are the constraints for the swing angles (α_0, β_0 are maximum swing angles allowed in the moving process), then the optimal trajectory planning problem for the crane system is to find $\bar{y}_d$ such that

$$\min_{\bar{y}_d} t_f$$

subject to

$$g_k(\bar{y}_d) \leq r_k \qquad k = 1, \cdots, n$$
$$|\alpha(\bar{y}_d)| \leq \alpha_0$$
$$|\beta(\bar{y}_d)| \leq \beta_0$$

where t_f is final time. The problem can be solved using a parameterization method.

For simplicity, the desired trajectories of x_{gd}, y_{gd}, z_{gd} are parameterized as linear combinations of sinusoidal functions

$$x_{gd} = x_{dg0} + x_{gdf}\frac{t}{t_f} + \sum_{i=1}^{n} c_{1i} \sin i\pi \frac{t}{t_f} \tag{23}$$
$$y_{gd} = y_{dg0} + y_{gdf}\frac{t}{t_f} + \sum_{i=1}^{n} c_{2i} \sin i\pi \frac{t}{t_f} \tag{24}$$
$$z_{gd} = z_{dg0} + z_{gdf} + \sum_{i=1}^{n} c_{3i} \sin (2i-1)\pi \frac{t}{t_f} \tag{25}$$

Some extra conditions at the initial and final time also need to be satisfied in order to transfer the load from one rest position to another rest position.

Thus, given the load initial position and final position, the time optimal trajectory planning problem is reduced to find c_{ji} such that

$$\min_{c_{ji}} t_f$$

subject to

$$g_k(c_{ji}) \leq r_k \qquad k = 1, 2, ...n$$
$$|\alpha(c_{ji})| \leq \alpha_0$$
$$|\beta(c_{ji})| \leq \beta_0$$

The problem can be solved using MATLAB. Once $\bar{y}_d$ is obtained , we can calculate $\bar{x}_d$. Notice that no system parameters are needed in this method.

4 Learning Control of the Overhead Crane

Given the desired trajectory, a control input needs to be designed such that this input can drive the system output to track the desired trajectory. Learning control is very useful to solve this problem. There are many learning control algorithms for various systems (Moore 1993). For the crane system which has a special dynamics, we use a learning algorithm that is based on Bondi et al. (1988). Their paper assumes that the number of actuators is the same as the number of degrees of freedom. However, for our 3-D crane system, the number of degrees of freedom are more than the number of actuators. Fortunately, we can show that their method is still useful in our system. The learning control scheme is shown in Figure 2. The system is operated as follows. Given the desired trajectory $\bar{x}_d$, the trajectory error at the k^{th} trial is calculated

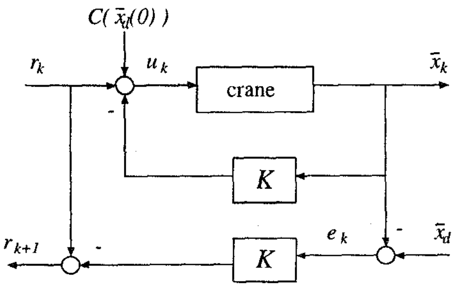

Figure 2: Learning control scheme

by $e_k = \bar{x}_d - \bar{x}_k$. Then e_k and r_k are used to update the reference input r_{k+1} for the next trial. The update law is given by the following algorithm

$$r_{k+1} = r_k - Ke_k \tag{26}$$

where K is the learning control gain which is the same as the feedback control gain.

For this learning control scheme, K can be chosen such that

$$\lim_{k\to\infty} \|e_k\| = 0$$

That is, the system output converges to the desired one as k increases.

Consequently, the learning control problem is reduced to find a "good" feedback controller to have the output converge to the desired one at a fast convergence rate. If the system is in "best" circumstances, that is, no disturbances, then we can use pole placement to obtain the feedback controller to satisfy the above requirements. If there exist external disturbances in the system, the controller from pole placement may not reject the disturbances effectively. Thus, we need to find another controller. However, in this case, it seems that we must sacrifice some learning performances in order to reject disturbances effectively.

In order to discuss more easily, we define good learning control performance and disturbance rejection performance as follows.

Good learning control performance is to let the y_k converge to y_d in a minimum number of trials.

Good disturbance rejection performance is to have: (1) the vibration is small due to the disturbances; (2) the trolley movement is not large in order to reject the disturbances; (3) the input energy is not large in order to withdraw the disturbances.

In the following section, we use two different methods, pole placement and H_∞ optimal control, to design the feedback controller.

4.1 Pole placement

Suppose the linearized crane system is given by

$$\begin{aligned} \dot{\bar{x}} &= A\bar{x} + Bu \\ y &= \bar{x} \end{aligned} \tag{27}$$

The feedback controller K is obtained by pole placement where the poles are chosen to have good learning performances. Then, the feedback control law is give by

$$u = r - K\bar{x} \tag{28}$$

4.2 H_∞ optimal control

If there exist external disturbances in the crane system, the feedback controller K may not have good disturbance rejection performances. Therefore, an H_∞ optimal control method is used to find a feedback controller such that the learning control algorithm can converge and the system can reject the external disturbances effectively. We form the H_∞ optimal problem as shown in Figure 3.

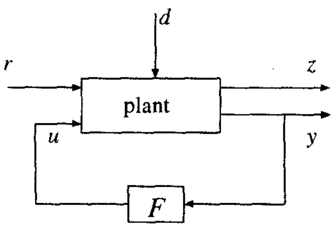

Figure 3: H-infinity problem setup

In Figure 3, d is the external disturbance that the system must be able to reject. r is the reference input that the system must be able to follow. z is the regulated output that we add to the system. This is the signal we are interested in controlling or regulating. y is the measured output which is the same as the state variable.

For the linearized system (27), the H_∞ problem can be represented as

$$\dot{\bar{x}} = A\bar{x} + B_1 w + Bu \tag{29}$$
$$z = C_1\bar{x} + D_{12}u \tag{30}$$
$$y = \bar{x} \tag{31}$$

where $w = \left[\begin{array}{c} r \\ \hline d \end{array}\right]$, $D_{12} = \left[\begin{array}{c} 0_{5\times 3} \\ \hline I_{3\times 3} \end{array}\right]$, $B_1 = \left[\begin{array}{c|c} B & B_d \end{array}\right]$.

It is important that the control signal u be included in the regulated output z so that we can bound the control magnitude. This also ensures that the H_∞ condition $D_{12}'\left[\begin{array}{c|c} C_1 & D_{12} \end{array}\right] = \left[\begin{array}{c|c} 0 & I \end{array}\right]$ is satisfied.

For the above system, C_1 and B_d still have to be chosen. We consider a disturbance which is directly applied to the load in the x-direction. Thus C_1 and B_d are chosen such that

$$z = C_1\bar{x} + D_{12}u = \begin{bmatrix} p_1 \begin{pmatrix} x - x_d \\ y - y_d \end{pmatrix} \\ p_2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \\ F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

where p_1, p_2 are penalizing parameters. They are selected via trial and error iterations to make the system have good disturbance rejection performances.

The H_∞ controller design is to minimize the effect of the disturbance w on the output z by finding an appropriate control input u. More precisely, we seek a static compensator F such that after applying the feedback law $u = F\bar{x}$ to the system, the resulting closed-loop system is internally stable and its transfer matrix has the minimum H_∞ norm. This is a standard H_∞ problem (Zhou 1996) and MATLAB functions can be used to obtain the solution.

5 Simulation

The control methods developed here are simulated on SGI Indy 5000 using MATLAB. The data used for simulation are given as follows: $m_1 = 100kg$, $m_2 = 1000kg$, $m = 15.8kg$, $J_1 = J_2 = J_3 = 0$, $x_0 = x_{g0} = 0, y_0 = y_{g0} = 0, z_{g0} = l_0 = 1.2m$, $x_f = x_{gf} = 1.7m$, $y_f = y_{gf} = 1.8m$.

5.1 Optimal trajectories

We consider two cylindrical obstacles whose surfaces are described by following equations

$$(x - .85)^2 + (y - .9)^2 = 0.2^2 \tag{32}$$

$$(z - 1.2)^2 + (x - .85)^2 = 0.4^2 \tag{33}$$

Let the obstacle geometric ans swing angle constraints be

$$(x_{gd} - .85)^2 + (y_{gd} - .9)^2 \geq 0.2^2$$

$$(z_{gd} - 1.2)^2 + (x_{gd} - .85)^2 \geq 0.4^2$$

$$|\alpha| \leq \frac{\pi}{8}$$

Figure 4 shows the optimal trajectories for x_g, y_g, z_g. The minimum transfer time is $t_f = 2.3s$. We can see swing angle does not exceed $\frac{\pi}{8}$ in the moving process and there is no oscillation at the end of the movement.

5.2 Disturbance rejection

We compare the disturbance rejection performance in the feedback control system when we use the feedback controller K from the pole placement and the static compensator F from the H_∞ optimal control.

First, we need to choose the penalizing parameters p_1, p_2 in the H_∞ optimal control problem setup. The values of

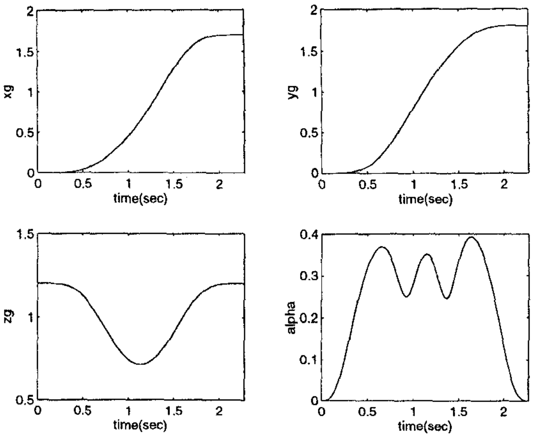

Figure 4: Optimal trajectories

p_1, p_2 affect the existence of the optimal control solution, swing angle, total input energy, gain from the disturbance to the regulation error $\frac{||z||_2}{||w||_2}$, magnitude and response time of the output. Table 1 and Table 2 show the variations of these values when p_1 and p_2 change, respectively. When

Table 1: Performance index vs. p_1 ($p_2 = 200$)

p_1	input energy	max $\lvert\alpha\rvert$	$\frac{\lvert\lvert z\rvert\rvert_2}{\lvert\lvert w\rvert\rvert_2}$	max $\lvert x\rvert$	response time of x
500	86	0.22	151	0.24	2.8
1000	112	0.18	193	0.34	3.5
1500	127	0.16	279	0.41	4.2
2000	135	0.15	357	0.44	4.9
2500	138	0.15	420	0.45	5.8
3000	140	0.16	468	0.45	7.2

Table 2: Performance index vs. p_2 ($p_1 = 2000$)

p_2	input energy	max $\lvert\alpha\rvert$	$\frac{\lvert\lvert z\rvert\rvert_2}{\lvert\lvert w\rvert\rvert_2}$	max $\lvert x\rvert$	response time of x
100	150	0.13	343	0.57	7.1
200	135	0.15	356	0.44	4.9
300	127	0.17	369	0.38	4.0
400	124	0.18	376	0.35	3.4
500	124	0.19	382	0.32	3.1
600	127	0.20	390	0.20	2.8

$p_2 = 200$ and p_1 increases, the maximum swing angle $|\alpha|$ will decrease first and then increase. $\frac{||z||_2}{||w||_2}$ and input energy will increase. The magnitude and the response time of x will also increase. When $p_1 = 2000$ and p_2 increases, the maximum swing angle $|\alpha|$ and the norm $\frac{||z||_2}{||w||_2}$ will increase. The input energy will decrease first and then increase while the magnitude and the response time of x will decrease.

Based on these comparisons, we choose $p_1 = 2000$ and $p_2 = 200$ that correspond to the "optimal choice" with relatively small α, fast response of x and small input energy.

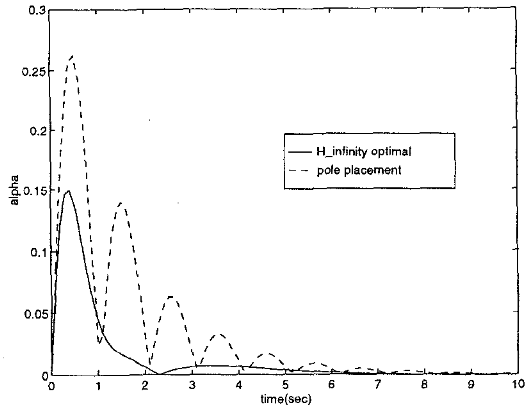

Figure 5: Swing angle of close-loop system

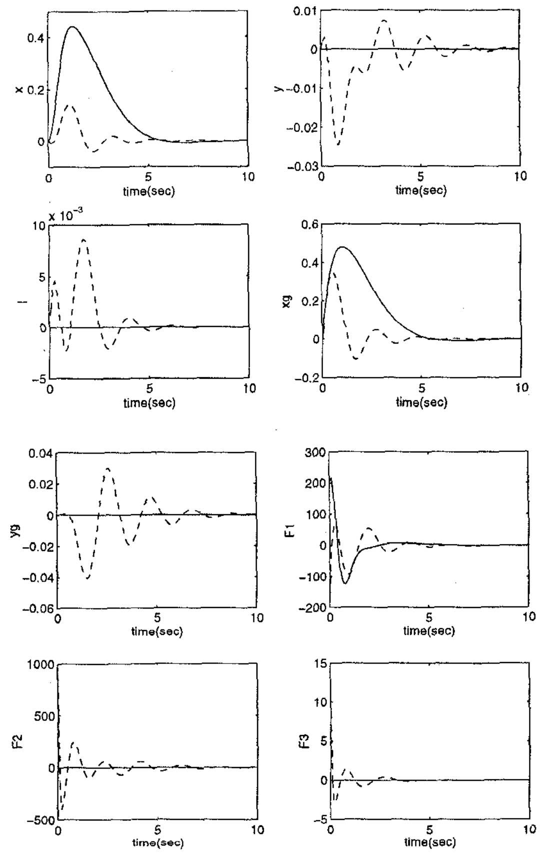

Figure 6: The inputs and outputs of close-loop system

Figure 5 shows the impulse response of the swing angle α corresponding to the disturbance. Figure 6 show the impulse responses of x, y, l, x_g, y_g, z_g and input forces F_1, F_2, F_3. In the H_∞ optimal control system, we can see that the swing motion can die out very quickly . Also, only x and x_g change when the x-direction disturbance exists and only $F1$ is needed to reject the disturbance. The input energy is 135 and norm $\frac{\|z\|_2}{\|w\|_2}$ is 357. For the pole placement method, there is a large load vibration and all three inputs F_1, F_2, F_3 are needed to reject the x-direction disturbance. The input energy is 328 and norm $\frac{\|z\|_2}{\|w\|_2}$ is 651.

Consequently, the H_∞ optimal control method has much better disturbance rejection performance than the pole placement method.

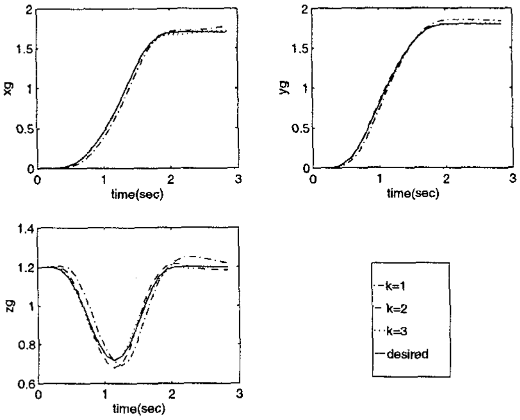

Figure 7: Learning results

5.3 Learning control

In the learning control scheme, we use K and F to do simulations. When we use K in the learning control scheme, x_g, y_g, z_g converge to the optimal trajectories at 3^{th} trial. However, if F is used in the learning control scheme, it needs 28 trials for x_g, y_g, z_g to converge to the optimal trajectories. Therefore, the pole placement method has much better learning performance than the H_∞ optimal control method. Figure 7 shows the learning results for the trajectories of x_g, y_g, z_g when the feedback controller K is used. We can see x_g, y_g, z_g almost track the optimal trajectories at 3^{th} trial.

6 Conclusions

The dynamic model of a 3-D overhead crane is developed. In case there exist obstacles in the load moving path, the time-optimal load trajectory is obtained under the obstacles and swing angle constraints. No system parameter knowledge is needed for trajectory planning. Using a learning algorithm, a desired reference input signal is calculated which ensures the system output to track the optimal trajectory exactly. For the crane system with disturbances, we setup an H_∞ optimal control system which can reject the disturbance effectively. However, it seems that some learning performances must be sacrificed. This is an area that needs further study.

References are available upon request.