Impact of Model Errors on Predicting Performance of Matching-Critical Circuits

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***Abstract*-Existing approaches to modeling mismatch effects in matching-critical circuits are based upon models derived under the widely accepted premise that distributed parameter devices can be modeled with lumped parameter models. It is shown in this paper that the lumped parameter models do not consistently reflect device performance and introduce substantial errors in matching-critical circuits if either systematic or random parameter variations occur in the channel.**

I. Introduction

It is well recognized that the performance of many linear and mixed-signal integrated circuits such as current mirrors and differential amplifiers is limited by how well the I/V characteristics of transistors can be matched. The shrinkage of feature sizes and the reduction of supply voltages generally worsen the matching performance. In most matching research [1-3], the matching characteristics are attributed to systematic and random variations in both geometric parameters and process parameters. It has been reported and is widely accepted that the mismatch due to random parameter variations is inversely proportional to the active area of the matching critical transistors and thus, tradeoffs can be made between area and performance to compensate for random variations in these parameters. It is also widely accepted that the systematic variations can be modeled as a stochastic process with a long correlation distance and can be reduced or eliminated by placing the matching-critical transistors closely to one another using segmented common centroid layout techniques. With the existing approaches to predicting matching characteristics, considerable discrepancies between predicted performance and actual measured performance exist. These discrepancies are inherently attributable to limitations of the models used to predict the matching performance. In this paper, consistency and limitations of existing models and their impact on predicting the performances of analog circuits will be discussed. A new stochastic model is presented that offers improvement in predicting the effects of random parameter variations on device matching.

II. Parameter Variation Modeling

Process parameters vary from batch to batch, wafer to wafer, die to die and from transistor location to transistor location on a die. To characterize the location-dependant parameters, a parameter P_i can be represented by the expression

$$P_i(x,y) = P_{iNOM} + P_{iPROC} + P_{iWAFER} + P_{iDIE} + P_{iSYS}(x,y) + P_{iRAN}(x,y) \quad (1)$$

where x and y represent the position on the die. In (1), P_{iNOM} is the nominal value of the parameter P_i and the five remaining terms are themselves random variables that some authors choose to combine together into a single random variable. For notational convenience, the subscript "i" will be suppressed in the following discussions. The variable P_{PROC} characterizes the variation of the parameter P from one lot of wafers to another. The parameter P_{WAFER} characterizes the variation of P from one wafer to another wafer in a "lot" of wafers and the parameter, P_{DIE}, characterizes the variation of the parameter from die location to die location. The parameter P_{SYS} characterizes the systematic variation of the parameter from one location to another on the die and is position dependent. The variable P_{RAN} characterizes the random part of the parameter at the position (x,y). When considering devices in close proximity to each other on a die, the values of the random variables P_{PROC}, P_{WAFER} and P_{DIE} are nearly constant throughout the region. Thus, almost all the matching-related research focus only on the effects of the two rightmost terms in (1).

For using a SPICE-type simulator, the performance of a device is characterized by a set of model parameters. The model parameters for a MOSFET include the threshold voltage (V_{T0}), the mobility (μ), the gate oxide capacitance density (Cox), etc. Some of these model parameters are determined from well-known relationships between the process parameters and others are more empirical in nature. Since the process parameters are position dependent, the model parameters are position dependent as well and thus since the underlying process parameters are stochastic, the model parameters are stochastic. Although many of the process parameters are uncorrelated or weakly correlated, a single process parameter often affects more than one model parameter causing correlation between the model parameters. This correlation is often neglected when characterizing the matching characteristics of linear circuits. Following this standard approach for characterizing the process parameters, the device model parameters can be expressed in the form

$$g(x,y) = g_{NOM} + g_{PROC} + g_{WAFER} + g_{DIE} + g_{SYS}(x,y) + g_{RAN}(x,y) \quad (2)$$

where g represents the model parameters.

Implicit in the functional form of (2) is the distributed nature of the model parameter. Essentially all

device models and, in particular, the device models used in Spice-type simulators are based upon lumped parameter models. In most works [1-2], it is assumed that the actual values of the lumped model parameters can be obtained by integrating the position-dependent distributed model parameters over the area of the channel region of the device as given by the equation.

$$\gamma(x_A, y_A) = \frac{1}{\text{Area}} \iint_{\text{Area}} \gamma(x, y)\,dxdy \qquad (3)$$

where (x_A,y_A) is a point representation of the location of the device on the die. Although not critical in what follows, it is convenient to define (x_A,y_A) to be the geometrical centroid of the device. We will refer to this lumped parameter extraction from a distributed parameter domain as the integral model through this paper. This approach of mapping from a distributed stochastic parameter to a single lumped model parameter has been used almost exclusively for well over a decade and the issue of validity of this mapping is generally not questioned. Since both systematic effects and random fluctuations in device parameters are known to play key roles in matching performance, it is particularly important that the lumped device models effectively incorporate these fluctuations. Unfortunately, it has been recently shown that the integral model leads to errors when used to predict the actual performance of distributed devices [4]. In the next section, we will focus on the inconsistencies of the integral model and show that the lumped integral model can result in substantial modeling errors.

III. Model Consistency

The invalidity of the integral model can be demonstrated by considering the non-conventional transistor depicted in Fig 1 by looking at the affects of a single positionally dependent model parameter, the threshold voltage. In this figure, it will be assumed that d is very small compared to L so that the channel region is decomposed into two parts, the left part of area A_1 and termed the A_1 region and the right part of area A_2 and termed the A_2 region. If it is assumed that the threshold voltage in the A_2 region is V_{T2} and in the A_1 region it is V_{T1}, it follows from the integral model that the equivalent threshold voltage of the device is

$$V_{TEQ} \approx \frac{A_1 V_{T1} + A_2 V_{T2}}{A_1 + A_2} \qquad (4)$$

If $A_2 >> A_1$ it follows from (4) that the threshold voltage can be expressed as $V_T \approx V_{T2}$. However, since the distance d is assumed to be small compared to the length L, almost no current flows in the A2 region and thus the actual device will have a threshold voltage of $V_T \approx V_{T1}$. It is apparent from this example that the integral model can result in large model errors. The structure of Fig 1 is admittedly an impractical transistor layout but does demonstrate that the integral model can lead to substantially erroneous results.

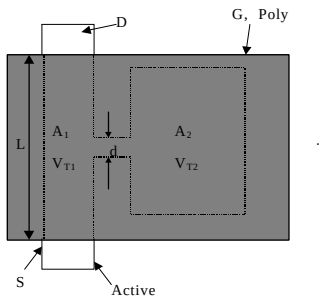

Fig 1. Non-conventional transistor

Validity of a model is often difficult to ascertain. The concept of model consistency or equivalently inconsistency is often useful for identifying problems with a model.

We define a device model to be consistent if alternate but equivalent representations of a device by the model result in equivalent predicted device performance. If equivalent representations of a device by a model predict different performance, we say the model is inconsistent.

An accurate or correct device model must be consistent. Consistency is not sufficient to guarantee validity of a model but inconsistency of a device model does flag significant problems with a model. We will now show that the integral model of (2) is inconsistent if systematic parameter variations, $\gamma_{SYS}(x,y)$ are present. This inconsistency limits the applicability of the integral model in predicting the effects of systematic parameter variations. To show its inconsistency, consider the parallel connection of two geometrically identical MOS transistors shown in Fig 2. This kind of segmented structure is commonly used in layouts. The transistor on the left is assumed to have threshold voltage V_{T1} and the transistor on the right assumed to have threshold voltage V_{T2}. If we treat the devices as a single transistor operating in the saturation region and apply the integral model, it follows from (2) that the equivalent threshold voltage is

$$V_{TEQ} = \frac{V_{T1} + V_{T2}}{2} \qquad (5)$$

With the integral model and the standard square-law model which can be shown to be consistent, the current of parallel connection of Fig 2 is given by

$$I_{D\ \text{Integral}\ \text{Model}} = \frac{m\ C_{OX}\ W}{2L}\left(V_{GS} - \left(\frac{V_{T1} + V_{T2}}{2}\right)\right)^2 \qquad (6)$$

If, however, we treat the parallel connection as two separate devices and use the integral model to obtain the equivalent threshold voltage of each of the devices, it follows trivially that $V_{T1EQ}=V_{T1}$ and $V_{T2EQ}=V_{T2}$. Thus, using the same square law model we obtain the three equations

$$I_{D1} = \frac{m\ C_{OX}\ W}{2L}(V_{GS} - V_{T1})^2 \qquad (7)$$

$$I_{D2} = \frac{m\ C_{OX}\ W}{2L}(V_{GS} - V_{T2})^2 \qquad (8)$$

(9)

These equations can be solved to obtain

$$I_D = \frac{\mu C_{OX} W}{L}\left[\left(V_{GS} - \frac{V_{T1}+V_{T2}}{2}\right)^2 + \left(\frac{V_{T1}-V_{T2}}{2}\right)^2\right] \quad (10)$$

Since I_D as modeled by (6) differs from I_D as modeled by (10) if $V_{T1} \neq V_{T2}$, it follows that the integral model is inconsistent.

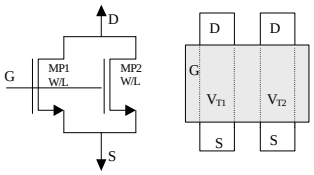

Fig 2. Parallel connection of two transistors

We will also show that the integral model is not consistent with experimental results. From the integral model, it is apparent that the effects of random parameter variation on parameter extraction are independent of the current or device orientation. Experimental data relating to device orientation and matching was presented in [2]. Although these results show no significant shift in the averages of the current factor, $\beta=(\mu C_{OX}W/L)$, between parallel and 90° rotated transistors, the standard deviation of β is significantly affected by the orientation as shown in the experimental data which is repeated here in Fig 3. This data indicates the mismatch in β is affected by device orientation in contrast to the independence predicted by the integral model. The authors of [2] suggested that the effect observed in Fig 3 was due to local mobility variations presumably the $\gamma_{SYS}(x,y)$ dependence of (2). One can argue that the local variation noted in [2] is typical of the variation of other parameters across the active region of a distributed device. By using the integral model in extracting lumped model parameters, the information about the local variations will be suppressed or skewed.

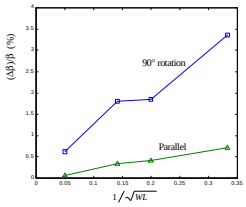

Fig 3. Standard deviation of β with parallel and rotated placement (from Fig. 4(b) of [2])

IV. Model Inconsistency with Random Parameter Variations

The integral model does not only suppress or skew information about local systematic variations, it also skews prediction of the effects of the random mismatch, the $\gamma_{RAN}(x,y)$ term in (2). Applying the integral model to model the variance of a parameter $\gamma(x_A,y_A)$ between two rectangular devices of length L and width W it follows that the variance can be expressed as [2]

$$s^2(g) = \frac{A_g^2}{WL} + S_g^2 D_X^2 \quad (11)$$

where A_γ and S_γ are the area and spacing proportionality constants for the parameter γ and D_X is the distance between the centroids of the two devices. We will now show that the integral model is inconsistent if used to model the effects of random parameter variations.

Fig 4 shows a simple current mirror comprised of two transistors of length L and width W. The mismatch in the mirror is defined as $(I_{D2}-I_{D1})/I_{D1}$ where it is assured that $V_{D1}=V_{D2}$. Alternate and equivalent representations of a transistor are shown in Fig. 5. In these equivalent representations, the transistor is decomposed into the series connection of N transistors each of length L/N. To test the consistency of the integral model with random parameter variations, all model parameters except the threshold voltage were assumed to be equal to their nominal value and only the random part of the threshold voltage was assumed to be non-ideal, i.e. S_γ in (11) was assumed to be 0. M1 and M2 were each represented as a series connection of N transistors and the threshold voltage of each of the transistors was modeled by the integral equation, specially with the threshold variance for a transistor pair predicted by (11) with $D_X=0$ or equivalently, with the threshold variance of an individual transistor given by the equation,

$$s^2_{V_T}(N) = \frac{0.5 \cdot A^2_{V_{TO}}}{W(L/N)} \quad (12)$$

where W, L and A_{VTO} are assumed 100μm, 100μm, and 42.43 (mV)(μm) respectively. It should be noted that there is some possible confusion about the definition of the parameter A_{VTO}. In this work, we have assured A_{VTO} is defined by (11) which characterizes the difference in the threshold voltage of two devices, each of area W·L. With this definition, the scaling factor of 0.5 appears in the variance expression for a single transistor as indicated by (12). Alternatively, the parameter "A_{VTO}" could be defined to characterize the variance of the threshold voltage of a single transistor. In this case, the factor of 0.5 would have been absent from (12) but the parameter A_{VTO} itself would have been reduced by $\sqrt{2}$ factor. A Monte Carlo simulation of the current mirror mismatch, $\sigma(\Delta I_D)/I_D$, was done in Matlab with a Level 2 device model and the results have been verified by Hspice. Fig 6 shows the simulation results for $\sigma^2(\Delta I_D)/I_D^2$ as a function of the number of segments, N. In these simulations, it was assumed that V_G=2.5V, $V_{D1,2}$=1.7V, V_S=0V and V_{TN}=0.8V. Each value of N corresponds to an alternate and equivalent representation of the transistors in the current mirror. If the integral model were consistent at predicting the effects of random parameter variations, $\sigma^2(\Delta I_D)/I_D^2$ would be independent of N. The simulation results in Fig. 6 show a curve that starts from 4.9913e-4 when N=1 and then diverges when N becomes larger. A routine analysis [5] of the standard

deviation of the drain current mismatch based upon the integral equation gives the relationship

$$\frac{s^2(\Delta I_D)}{I_D^2} = \frac{4s^2(\Delta V_T)}{(V_{gs} - V_{TO})^2} = \frac{8s^2(V_T)}{(V_{gs} - V_{TO})^2} \qquad (13)$$

For the parameters used in the present simulation, it follows from (13) that $\sigma(\Delta I_D)/I_D$ =4.9913e-4 which is in considerable discrepancy from the simulation results even for relatively small N. These results show the inconsistency of the integral model in the presence of random parameter variations.

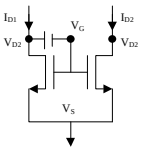

Fig 4. Simple Current Mirror

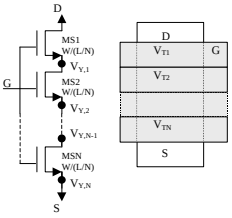

Fig 5. Partitioning of Current Mirror into N Segments

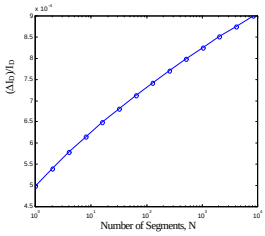

Fig 6. Standard deviation of drain current using the existing model (12)

V. A New Model for Random Parameter Variations

A new model for the effects of random parameter variations of the threshold voltage in rectangular transistors that is consistent has been introduced [6]. Details about how the model was developed can be found in [6]. In this model, the variance of the threshold voltage of an incremental segment of the channel of length dy at position y, 0≤y≤L, is given by

$$s^2_{V_T}(y) = \frac{0.5\,\tilde{A}^2_{VTO}\left(1 - \frac{V(ys)}{V_{DS,SAT}}\right)^2}{Wdy} \qquad (14)$$

where $\tilde{A}_{VTO}$ is a constant and V(ys) is the voltage at position y in the channel to the source of the transistor. The parameter $\tilde{A}_{VTO}$ was intentionally used to draw a direct relationship with the parameter A_{VTO} used in (12). If a transistor is decomposed into N segments, each of length Δy=L/N, it follows from (14) that the variance of the threshold voltage in segment n, 0≤n≤N-1, can be expressed as

$$s^2_{V_T}(n) = \frac{0.5\,\tilde{A}^2_{VTO}\left(1 - \frac{V((\frac{nL}{N})s)}{V_{DS,SAT}}\right)^2}{W(\frac{L}{N})} \qquad (15)$$

where V((nL/N)s) is the voltage between the channel and the source at a distance of n(L/N) from the source.

The simple current mirror of Fig. 4 in which both M1 and M2 were segmented into N segments as depicted in Fig. 5 was simulated for current gain mismatch using the model of (15) for different values of N. The simulation results are shown in Fig 7. From the simulation results, it is apparent that the curve converges very quickly. Of course, convergence does not necessarily imply a consistent model. To verify the consistency of the new model, we will consider the series connection of each of the two MOS transistors as shown in Fig. 8. In this representation, all four transistors were assumed to be of length L/2 and of width W. The model of (14) approximated by (15) was used for modeling each of the transistors and simulations were run for a large number of values of N. The simulation results are shown in Fig 9. The curves labeled MS and MS1+MS2 which represent a single equivalent transistor and the series connection of two transistors are essentially coincident for large N supporting our contention that the new model is consistent.

Although the modified $\sigma(V_T)$ model of (14) can make the model consistent, the converged value of 2.4968e-4 differs from the value of 4.9913e-4 obtained by using the equation (13) with N=1 by a fact of 2. The difference is because of limitations in the mapping from the distributed domain to the lumped domain.

Some comments on obtaining the parameter $\tilde{A}_{VTO}$ are in order. If measurements of V_{TO} are made directly or, if threshold voltage variations dominate current mirror mismatch so that current mirror mismatch can be measured and threshold voltage statistics can be inferred, the measurements will inherently be made on a distributed device and the asymptotic values depicted in Fig. 7 or Fig. 9 will be obtained. The parameter $\tilde{A}_{VTO}$ in (14) that gives an asymptotic value that agrees with the measured results is what is needed. We are not in a position at this time to give a simple closed-form expression for $\tilde{A}_{VTO}$. The simulation results, however, suggest that for rectangular devices there is a factor of 2 difference in $\sigma(\Delta I_D)/I_D$ between the value predicted using a single segment lumped model and the distributed model. Formally,

$$s_{V_T}(Measured) = s_{V_T}(n = \infty) = \frac{1}{2} s_{V_T}(n = 0) \qquad (16)$$

where σ_{VT} is the standard deviation of the threshold voltage for a single transistor. Substituting (15) into (16) with N=1 and n=0, it follows that

$$\tilde{A}_{V_{TO}} = (2\sqrt{W \cdot L})s_{V_T}(Measured) \quad (17)$$

or, equivalently, from a plot of σ_{VT} Vs $(1/\sqrt{W \cdot L})$, $\tilde{A}_{VTO}$ is 2 times the slope.

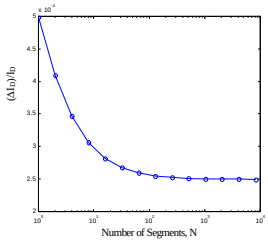

Fig 7 Standard deviation of current mirror mismatch using the modified model (14)

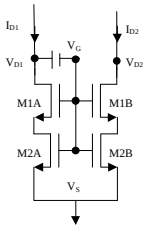

Fig 8 An Equivalent Representation of Basic Current Mirror

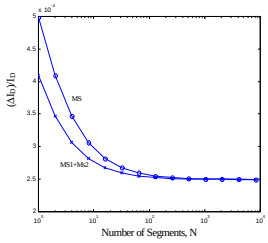

Fig 9 Verification of consistency of the modified model (14)

VI. Conclusion

It has been shown that existing approaches to modeling both systematic and stochastic mismatch based on using the integral model to predicting matching characteristics have considerable discrepancies between predicted results and actual measured performance. These discrepancies are inherently attributable to limitations of the models used to predict the matching performance. The commonly used mapping from the distributed parameter domain to a lumped parameter model is the cause of these discrepancies. A modified consistent model of random mismatch has been proposed that results in simulation convergence.

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