Transfer Characterization of CMOS Ring Voltage Controlled Oscillators

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Abstract—One of the most important properties of any voltage controlled oscillator (VCO) is the transfer characteristic, the relationship between the control voltage and the frequency of the output signal. In a voltage controlled ring oscillator, a number of identical delay stages are used. The most common delay stage is a delay cell with a pair of differential inputs and diode-connected loads paralleled with adjustable current sources. In this paper, a detailed analysis of the transfer characteristics based on the actual transistor operating regions is presented. It is shown that the relationship between the control voltage and the oscillation frequency is not monotonic. Simulation results verify the theoretical development. A design strategy is introduced that assumes that the VCO operates in the monotonic transfer characteristic region.

1. Introduction

Voltage controlled oscillators (VCO) are widely used in phase locked loop (PLL) and PLLs are widely used for frequency synthesis, data recovery in microprocessors and communication circuits. One of the most common CMOS VCOs used in high-speed microprocessors and communication circuits are ring oscillators. Monolithic CMOS ring voltage oscillators have advantages in terms of relative high spectral purity (compared to relaxation oscillator), high frequency capability, large tuning range, good matching and low fabrication cost. Fig. 1 shows the block diagram of a ring oscillator. If the delay stage is single-ended inverter, the number of the stages is odd. If the delay stage is differential structure, the number of the stages can be even. Fig. 2 shows a commonly used delay cells.

In Fig. 2, a differential input structure with current source paralleled with diode-connected load is used to reduce the supply and ground noise. Two control voltages $V_{\text{bias}}$ and $V_{\text{bias}}$ provide large operation frequency tuning range. Diode connected PMOS transistors provide fixed common mode and swing. One of the key properties of any VCO is the transfer characteristic, the relationship between the control voltage and the frequency of the output signal. Although the delay cell in Fig. 2 is widely used in the VCO design, the accurate transfer function has not been presented yet. Some designers thought the operation frequency is directly proportional to the control voltage $V_{\text{bias}}$, while others thought that the operation frequency is reversal proportional to the control voltage $V_{\text{bias}}$. In some applications, it is not required that the VCO has a linear relationship between operation frequency and control voltages. But for almost all the applications, the monotonic relationship between the frequency and the control voltage is necessary. This paper will provide a detail analysis about the transfer characteristic of the VCO. The results show that the transfer characteristic is not monotonic. A design strategy is introduced that assumes that the VCO operates in the monotonic transfer characteristic region.

![Fig. 1: The block diagram of a ring VCO](image1)

![Fig. 2: Commonly used delay cell](image2)
In Section 2, the detailed analysis of the transfer characterization will be presented. The simulation results will be presented in Section 3. Finally, conclusions are made in Section 4 regarding this work.

2. Transfer Characterization of the CMOS Ring Oscillators

Voltage control ring oscillator is a nonlinear oscillator. The design and analysis of a nonlinear oscillator are complicated tasks, because transform methods (s-plane) cannot be applied directly. Nevertheless, like sinusoidal oscillators, the ring oscillators can be analysis by two steps. The first step is a linear one, and frequency-domain methods of feedback circuit analysis can be readily employed. Subsequently, a nonlinear mechanism for amplitude control can be provided. The operation frequency of the oscillator is determined in the first step, while the swing of the output signal is determined in the second step.

Fig. 3 is the behavior model of a ring VCO. Although in an actual oscillator circuit, no input signal will be present, an input signal here is included to help us analyze. Let \(A_d(s)\) be the transfer function of one delay stage, and \(A_d(s)\) is the transfer function of the \(n\) delay stages. \(\beta(s)\) is the feedback factor. The closed loop transfer function \(A_f(s)\) is

\[
A_f(s) = \frac{A_d(s)}{1 - A_d(s)\beta(s)}
\]

The loop gain of the circuit is defined

\[
L(s) = A_d(s)\beta(s)
\]

The characteristic equation thus becomes

\[
1 - L(s) = 0
\]

According to Barkhausen criterion, at the oscillation frequency \(\omega_0\), the phase shift of the loop gain is \(2\pi\) and the magnitude of the loop gain is unity. Thus the condition for the circuit in Fig. 3 to provide a stable oscillation at frequency \(\omega_0\) is that

\[
L(j\omega_0) = A_d(j\omega_0)\beta(j\omega_0) = 1
\]

In the ring oscillator case, \(\beta(j\omega_0) = 1\). To satisfy equation (4), we have that

\[
A_d(j\omega_0) = \left|A_d(j\omega_0)\right|\angle\phi = 1
\]

where \(\phi = \angle A_d(j\omega_0)\), so that

\[
\left|A_d(j\omega_0)\right| = 1
\]

\[
\phi = 2\pi \quad or \quad \pi
\]

For the delay cell shown in the Fig. 2, the transfer function is that

\[
A_d(s) = \frac{V_o}{V_i} = \frac{-g_m}{sC + 1/R}
\]

where \(g_m\) is the transconductance of the differential input transistor, \(C\) is the total capacitance at the output node of the delay stage, \(R\) is the total resistance at the output node, so the total transfer function \(A_d(s)\) is that

\[
A_d(s) = \left(A_d(s)^n\right) \left(\left(-\frac{g_m}{sC + 1/R}\right)^n\right)
\]

from equation (8), the amplitude and the phase of the transfer function \(A_d(s)\) can be expressed as

\[
A_d(j\omega) = \left[\frac{g_m}{C\left(\omega^2 + \frac{1}{RC}\right)}\right]^{1/n} \times \theta
\]

\[
\theta = n \cdot \tan^{-1}\left(-\omega RC\right)
\]

At the oscillation frequency \(\omega_0\), the amplitude of the gain equals to unity and the phase shift \(\phi\) is \(2\pi\) or \(\pi\), so the VCO operation frequency is

\[
\omega_0 = \frac{1}{RC} \tan^{-1}\left(\frac{2\pi}{n}\right) \quad or \quad \frac{1}{RC} \tan^{-1}\left(\frac{\pi}{n}\right)
\]

Equation (10) shows that the operation frequency is determined by the number of delay stages in an oscillator, the total capacitance and resistance at the output node of the delay stage. The total capacitance and resistance at the output nodes depend on the operating regions of transistors in a delay stage. These transistors will operate in three regions as the control voltage \(V_{control}\) changes. We will analyze the circuit in three cases:

Case 1: M5 and M6 are in the saturation region
Case 2: M5 & M6 are in the triode region
Case 3: M5 and M6 are in the cutoff region

Case 1: M5 and M6 are in the saturation region.

If all the transistors are in the saturation region, the input and output swing must be less than threshold voltage \(V_{th}\). So small signal model can be used in the analysis. The small signal model of the
half circuit of the delay cell is shown in Fig. 4, and $g_m$ is given by

$$g_m = \frac{2\mu I_m W_{2x}}{l_t} I_1 = \frac{\mu I_m W_{2x}}{l_t} I_{init}$$  \hspace{1cm} (11)

where

$$I_{init} = \frac{\mu I_m W_{2x} (V_{bias} - V_m)^2}{2l_t}$$  \hspace{1cm} (12)

Substitute equation (12) into (11),

$$g_m = \frac{\mu I_m W_{2x}}{2l_t} (V_{bias} - V_m)$$  \hspace{1cm} (13)

Equation (13) shows that $g_m$ is proportional to the control voltage $V_{bias}$.

![Diagram showing a small signal of delay cell when all transistors operate at saturation region](image)

Fig. 4: small signal of delay cell when all transistors operator at saturation region

From Fig. 4, the resistance and capacitance at the output node are shown in equation (14) and (15), respectively.

$$R = \frac{1}{g_m} = \frac{1}{g_{m3}} = \frac{1}{g_{m5}}$$  \hspace{1cm} (14)

$$C = C_{out} + C_{in} + C_{out} + C_{in} + C_{out} + C_{in}$$  \hspace{1cm} (15)

where $g_{m3}$ is the transconductance of M3 and $g_{m5}$ is the conductance of M5.

$$g_m = \frac{2\mu I_m W_{2x}}{l_t} (I_{out} - I_{in}) = \frac{2\mu I_m W_{2x} (I_{out} - I_{in})}{2l_t}$$  \hspace{1cm} (16)

where $I_{out}$ is the current of M1 and $I_{in}$ is the current through M5

$$I_{out} = \frac{\mu I_m W_{2x} (V_{bias} - V_{out} - V_m)^2}{2l_t}$$  \hspace{1cm} (17)

Substitute equation (12) and (17) into (16),

$$g_m = \frac{\mu I_m W_{2x}}{4l_t} \left[ \frac{\mu I_m W_{2x} (V_{bias} - V_m)^2}{2l_t} \right]$$  \hspace{1cm} (18)

Finally substitute equation (19) into (10), the operation frequency of the oscillator is

$$\omega = \frac{1}{RC} \tan \left( \frac{2\pi}{n} \right) = \frac{1}{C} \tan \left( \frac{2\pi}{n} \right) \frac{\mu I_m W_{2x}}{L_5}$$  \hspace{1cm} (19)

Equation (19) shows that the oscillation frequency is directly proportional to control voltage $V_{bias}$ and $V_{bias}$ if all the transistors operate in the saturation region.

Case 2: M5 and M6 are in the triode region

The transconductance of the input transistors $g_{m1}$ is the same as that in Case 1, the total resistance at the output node is different. Consider the current through M1 (shown in Fig. 2) is

$$I = \frac{\mu I_m W_{1x} (V_{bias} - V_m)}{2L_1}$$

$$\left[ \frac{V_{bias} - V_{out} - V_m}{V_{bias} - V_{out}} \right] - \frac{1}{2} (V_v - V_{at})$$  \hspace{1cm} (20)

So the small signal impedance $r$ is

$$\frac{1}{r} = \frac{\partial I}{\partial V_v} = \frac{\mu I_m W_{1x}}{L_1} \left[ \frac{V_{bias} - V_{out} + V_m}{V_{bias} - V_{out}} \right] + \frac{\mu I_m W_{1x}}{L_3} \left[ \frac{V_{bias} - V_{out} + V_m}{V_{bias} - V_{out}} \right]$$  \hspace{1cm} (21)

let $W_3 = x \cdot W_3$ & $L_3 = L_3$

$$\frac{1}{r} = \frac{\mu I_m W_{1x}}{L_3} \left[ \frac{V_{bias} - V_{out} + V_m}{V_{bias} - V_{out}} \right] + (x-1) \left( V_{bias} - V_m \right) + (x+1) \left( V_v \right)$$  \hspace{1cm} (22)

Substitute equation (22) into (10), the operation frequency of the oscillator is

$$\omega = \frac{1}{RC} \tan \left( \frac{2\pi}{n} \right) = \frac{1}{C} \tan \left( \frac{2\pi}{n} \right) \frac{\mu I_m W_{1x}}{L_3}$$  \hspace{1cm} (23)

Here, there are three cases,

(i): $x=1$, i.e., the active load is a symmetric load

$$\omega = \frac{1}{RC} \tan \left( \frac{2\pi}{n} \right) = \frac{1}{C} \tan \left( \frac{2\pi}{n} \right) \frac{\mu I_m W_{1x}}{L_3}$$  \hspace{1cm} (24)

The frequency is proportional to the control voltage $V_{bias}$.

(ii): $x<1$, i.e., the size of the diode-connected transistor M3 is smaller than that of the PMOS current source transistor M5

Because $V_{diss}V_m$ is increased when $V_{bias}$ is increased, the frequency is also proportional to the control voltage $V_{bias}$.

(iii): $x>1$, i.e., the size of the diode connect transistor M3 is larger than that of the PMOS current source transistor M5

When $V_{bias}$ is increased, the first term in equation (24) is decreased, but the second term is
increased, so the relationship of the frequency and the control voltage is undetermined.

Case 3: M5 and M6 are in the cutoff region
In this case, equation (16) can be rewritten as

\[ g_{m3} = \sqrt{\frac{\mu_c C_{ox} W_3}{l_1}} I_{ref} \]  \hspace{1cm} (25)

Substitute equation (12) into (25), then

\[ g_{m3} = \sqrt{\frac{\mu_c C_{ox} W_3}{l_1} \left[ \frac{\mu_c C_{ox} W_3}{2l_1} \left( V_{bion} - V_n \right) \right]^2} \]  \hspace{1cm} (26)

The operation frequency of the VCO is given by

\[ f_o = \frac{\mu_c C_{ox} W_3}{2l_1} \left( \frac{V_{bion} - V_n}{V_{gs} - V_n} \right) \frac{1}{C} \tan \left( \frac{2\pi}{n} \right) \]  \hspace{1cm} (27)

where \( n \) is the stage number of the VCO.
So, the frequency of VCO is independent on the \( V_{bion} \).

3. Simulation Results

The analysis conclusions are verified by using Hspice simulator. The simulation results are based on 0.35 \( \mu \)m CMOS process.

Fig. 5 shows the relationship of the operation frequency of the VCO and the control voltage \( V_{bion} \).
The simulation results show that, if the size of M5 and M6 is larger or equal to that of M3 and M4, the whole voltage control range can be divided into three part. In part I, the transistor M5 & M6 are in the triode region, and the operation frequency is reversal proportional to the controlled Voltage \( V_{bion} \). In part II, M5 & M6 are in the saturation region, and the operation frequency is directly proportional to \( V_{bion} \). In part III, M5 & M6 are in the cutoff region, and frequency is independent on the \( V_{bion} \). If the size of M5 and M6 is smaller than that of M3 and M4, the operation frequency will be directly proportional to control voltage \( V_{bion} \) in the whole control voltage range. So in order to make the VCO have a monotonic relationship of the frequency and controlled voltage \( V_{bion} \), careful design is required.

Fig. 6 shows the relationship of the oscillation frequency of the VCO and the control voltage \( V_{bias} \) and \( V_{bion} \). This result shows that the oscillation frequency of the VCO is proportional to control voltage \( V_{bias} \). By Combining controlled voltage \( V_{bias} \) and \( V_{bion} \), the VCO operation frequency tuning range will be from 1.1 GHz to 2.5 GHz at normal temperature and typical process.

4. Conclusion

Transfer characterization of VCO is studied in detailed. Unlike most designers believe that the relationship of the frequency and the control voltage is monotonic relationship, we find, the relationship is based on the design size and the operation region of the transistors. The Hspice simulation results verified the analysis results.

References: