EFFECTS OF OPEN-LOOP NONLINEARITY ON LINEARITY OF FEEDBACK AMPLIFIERS*

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ABSTRACT

A quantitative analysis of the effects of nonlinearity in an open loop amplifier on the linearity of negative feedback amplifiers is presented. Results show that the ratio of the open-loop nonlinearity (OLN) to the closed-loop nonlinearity (CLN) in the presence of varying amounts of second- and third-order nonlinearities in the open loop amplifier is quantified. Results show that for the same amount of nonlinearities in the open loop amplifier, the second-order nonlinearity is suppressed more through feedback than the third-order nonlinearity. The effect of high open loop gain on reducing the nonlinearies through negative feedback is limited.

I. INTRODUCTION

Nonlinearity is a major nonideality of an amplifier circuit. It can be depicted as a nonlinear input/output characteristic as shown in figure 1. Usually when the input signal is small, the output has a reasonable linear relationship to the input. But with an increase of the input level, the output typically exhibits an increase in nonlinearity as depicted in the figure.

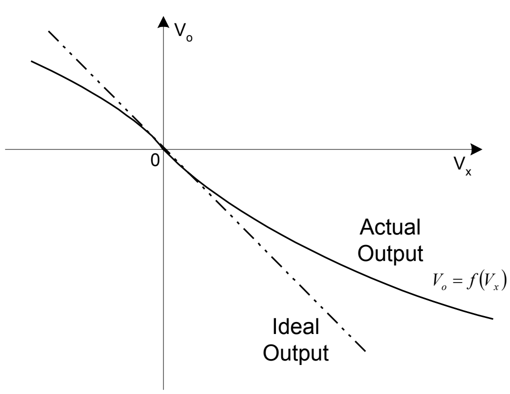

Figure 1. Nonlinearity in the amplifier

The nonlinearity of a circuit can also be considered as the "variation" of the slope (gain) in the input/output characteristics as a function of operating point. It means that a given incremental change at the input results in different incremental changes at the output depending on the quiescent input level.

Several techniques have been used to improve linearity. One of the most widely used linearization strategies is using negative feedback and the linearization properties associated with negative feedback were one of the major reason feedback concepts were developed. It is well known that another property of feedback circuits is gain desensitization. Since nonlinearity can be viewed as a variation of the small-signal gain with the input level, negative feedback techniques also decrease the variation.

While the general effect of negative feedback on linearity is well known, little research has been done from a quantitative viewpoint on how much nonlinearity can be reduced through feedback. The issue of what effect feedback will have on different order harmonics that contribute to the nonlinearity in the open loop amplifier has also not received much attention. The work presented in this paper provides a quantitative assessment of how several nonlinearity properties of the open loop amplifier affect feedback amplifiers.

A unified definition of the nonlinearity is given in Section II that can be used to evaluate the amount of nonlinearity under different situations. Several analyses are given in Section III and IV.

II. DEFINITION OF THE NONLINEARITY

In order to make a meaningful and fair comparison of the nonlinearities under different circumstances, a rigorous definition of the nonlinearity that is suitable for both open loop and feedback structures are needed.

Consider the open loop amplifier shown in figure 2(a). The input-output relationship is $V_o = f(V_x)$, where it will be assumed that $f(V_x)$ can be approximated by a desired

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first-order term and two undesired nonlinear terms. Thus, $f(V_x)$ can be expressed as

$$f(V_x) = -AV_x + BV_x^2 + CV_x^3, \quad A, B, C > 0, V_x > 0 \quad (1)$$

This equation characterizes the open loop transfer characteristics of the amplifier in the fourth quadrant. Its characteristic in the second quadrant is similar as depicted in figure 1. This expression includes the second and the third harmonic distortions that generally dominate the nonlinearity in most open loop amplifier.

Assume that the transfer characteristic of the open loop amplifier is as shown in figure 1 with the solid line that shows an increase in nonlinearity as the input amplitude increases. When negative feedback is applied, the gain of the feedback amplifier is usually decreased and considerably less distortion is experienced.

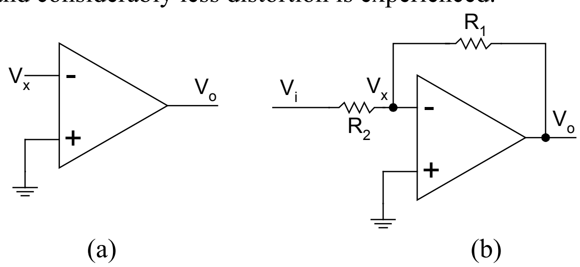

Figure 2. Open loop and feedback amplifiers

Ideally, the amplifier should have a linear input-output relationship of $V_o = -AV_x$. This ideal linear relationship corresponds to the tangent line through the origin with a slope of $k = f'(V_x)\big|_{V_x=0} = -A$. This ideal output is shown in figure 1 as the dotted line.

For the feedback amplifier shown in figure 2(b), it follows that:

$$V_x = \frac{R_1}{R_1 + R_2} V_o + \frac{R_2}{R_1 + R_2} V_{in} \quad (2)$$

The feedback gain (amount of feedback applied) is usually defined as:

$$\beta = \frac{R_1}{R_1 + R_2} \quad (3)$$

It follows that $V_x = \beta V_o + (1 - \beta) V_{in}$ (4)

Combining equation (4) with (1), we can obtain an exact input output relationship $V_o = g(V_{in})$ for the feedback amplifier. The closed-loop form of the expression for $g(V_{in})$ is unwieldy, even in the presence of only second-order and third-order nonlinearities. This function can be solved with the help of the MATLAB symbolic toolbox. Even though the solution is too complicated to show here because of the existence of the third order harmonics in the solution.

Again, the ideal output of the feedback amplifier is defined as the tangent line that passes through the origin with a slope of $k = g'(V_{in})\big|_{V_{in}=0} = -\frac{1-\beta}{\beta + 1/A}$ (5)

The nonlinearity for any specific input is defined to be the deviation of the actual output from the ideal output at the given input. With this definition, each input to an amplifier has its own nonlinearity value. What we are interested here is to see the effect of feedback on linearity. We need to choose a reference point where nonlinearities are investigated.

The nonliearity of an amplifier is usually closely associated with the output level. In what follows nonlinearity will be compared not at a certain input level but at a fixed ideal output level that is within our range of interest. We will base our comparisons on the output level rather than the input level because the gain of the feedback amplifier varies a lot with β. The quantization of the nonlinearity is shown in figure 3. For comparison purposes, the nonlinearies of the feedback amplifier will be compared at the ideal output level of $V_o = -1$ which corresponds to the input level of V_{-1}. The actual output for input V_{-1} is V_{ao} because of the nonlinearity. The nonlinearity, expressed in percentage, can be expressed as:

$$Nonlinearity(\%) = 100 \times (1 + V_{ao}) \quad (6)$$

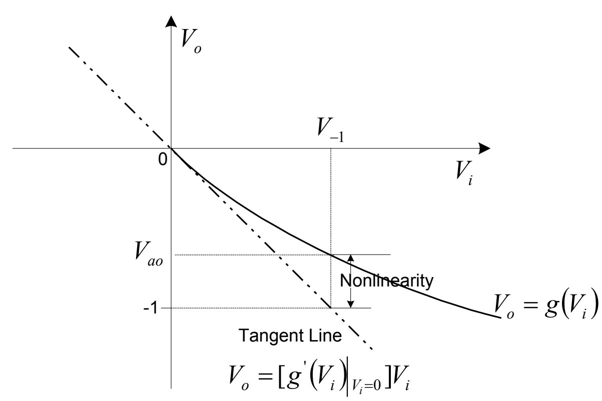

Figure 3. Quantization of the nonlinearity

It should be mentioned here that more simulations based on different reference points yield similar results and the same conclusions.

III. EFFECTS OF THE FEEDBACK ON NONLINEARITY

3.1 Effects of the Feedback Factor on CLN

It is well known that with deeper negative feedback (larger β), more nonlinearity can be reduced. But no quantitative analysis has been done to resolve the relationship between feedback factor and the nonlinearity. The following investigation will look at how the amount of nonlinearity is related to the feedback factor β.

A typical feedback system is shown in figure 4. The gain of the amplifier can be expressed as:

$$A_f = \frac{V_o}{V_{in}} = \frac{A}{1 + A\beta}. \qquad (7)$$

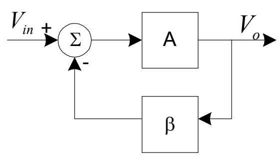

Figure 4. Negative feedback system

For the original open loop amplifier shown in figure 2(b), we assume DC gain A=1000; total nonlinearity at the ideal output level $V_o = -1$ is $OLN = 10\%$. Depending on the percentage combination of the second and the third harmonics that constitute the nonlinearity, the coefficients B and C in equation (1) can be determined accordingly.

Special care must be take to guarantee the monotonically of the input-output relationship of the original amplifier within the range of our interest so that the solutions of the feedback amplifier equation are real. This was done by limiting the amount of the nonlinearity in the open loop amplifier in the calculations.

The nonlinearities measured at the ideal output level of $V_o = -1$ in several feedback amplifiers are shown in figure 5. X axis shows the inversion of the feedback factor, i.e. $1/\beta$. Y axis shows the percentage of the nonlinearities in the feedback amplifier. Two cases are shown in figure 5. One is that 100% of the OLN is due to the 2nd order harmonic, the other is that 100% of the OLN is due to the 3rd harmonic.

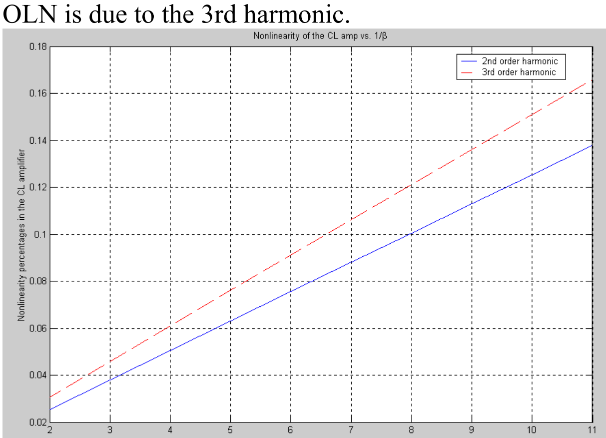

Figure 5. Closed-Loop Nonlinearity vs. $1/\beta$

In both cases, the amount of nonlinearity is linearly proportional to the inverse of the feedback factor. We conclude that the amount of Closed-Loop Nonlinearity $CLN = \frac{k}{\beta} + C$ where k and C are constants and only determined by open loop amplifier characteristics.

In our calculations, the amount of OLN was fixed. Therefore, another conclusion can also be drawn, $\frac{CLN}{OLN} = \frac{m}{\beta} + D$, where m and D are constants and only determined by open loop amplifier characteristics.

3.2 Effects of the Open Loop Gain on CLN

Under that same assumption (except the open loop gain) as in previous section, the effects of open loop gain on nonlinearities in the feedback amplifier were investigated. As shown in figure 6, open loop gain was swept from 1000 to 10000. Their corresponding CLN were calculated. The relationship between open loop gain and the amount of CLN is not linear, either for 2nd or 3rd order harmonics, or for different amount of OLN.

CLN drops dramatically when open loop gain start to increase. After open loop gain becomes larger than 3000-4000, the decrease of the CLN is much less. This property suggests that the effect of high open loop gain on reducing CLN is limited.

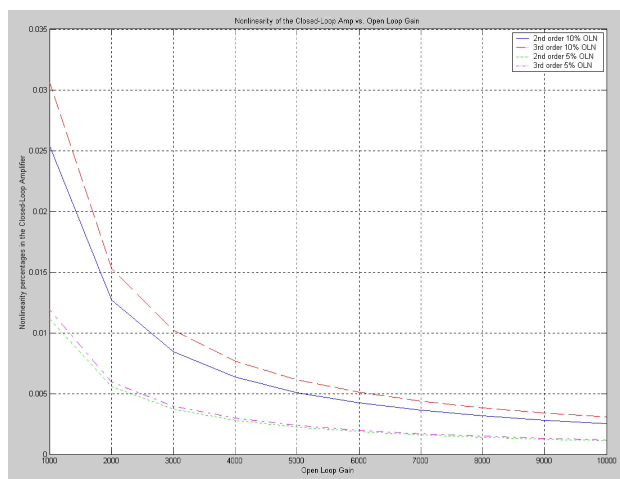

Figure 6. Closed-Loop Nonlinearity vs. Open-Loop Gain

3.3 Effects of the amount of OLN on CLN

Another interesting topic would be whether different amount of OLN would be suppressed linearly upon feedback. As shown in figure 7, different amount of OLN were tested with feedback factor of 0.5 and open loop gain A=1000. For both 2nd and 3rd order harmonics, the suppressions of the nonlinearity through feedback were not linear.

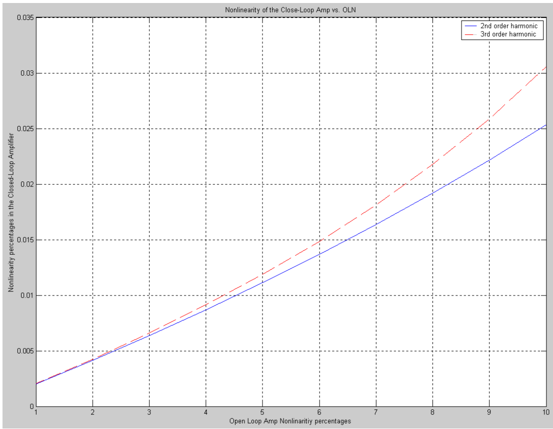

Figure 7. Closed-Loop Nonlinearity vs. amount of Open-Loop Nonlinearity

The increase of the CLN becomes faster with the increase of the OLN. This trend is more obvious for the 3rd harmonic. This property suggests the importance of limiting the OLN in design a low-distortion amplifier.

3.4 Effects of different harmonics on CLN

Modern integrated circuits design is often based on fully differential structure in order to eliminate even order harmonics. It would an interest to investigate if different order harmonic behaves differently in the feedback amplifier.

It is already shown in figure 5 that for the second and the third order harmonics, the same amount of nonlinearity in open loop amplifier will result in different amount of nonlinearity in the feedback amplifier. The third order harmonic will result a higher amount of nonlinearity in the feedback amplifier.

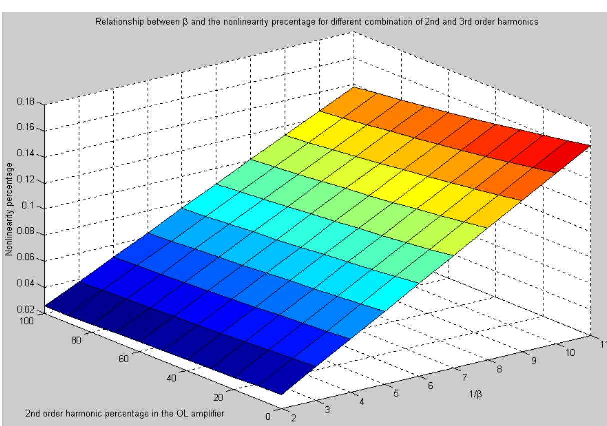

Figure 8. Nonlinearity vs. $1/\beta$ vs. different percentages of the second and the third harmonics

More investigations were done to see how different combination of the second and the third harmonics in open loop amplifiers would affect the nonlinearity in the feedback amplifier. As the surface plot shown in figure 8, CLN were calculated with different percentages of 2nd and 3rd harmonics in the open loop amplifier.

It is very clear that for any percentage combination of the second and the third order harmonics, the amount of CLN is still linearly proportional to the inverse of the feedback factor. For a certain feedback factor, the amount of the CLN changes linearly with the percentage of the 2nd and the 3rd harmonics.

IV. SUMMERY

This paper presented an analysis of the nonlinearity in feedback amplifiers. A new way to quantize the amount of nonlinearity was proposed. Using this method, a general-purpose negative feedback amplifier was analyzed for its nonlinearity under several different situations. We observed that the nonlinearity in the feedback amplifier is linearly proportional to $1/\beta$ and lower order harmonic nonlinearity will be reduced more through feedback. Results also show that the effect of high open loop gain on reducing nonlinearities through feedback is limited.

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