Use of the Newton-Raphson Iteration to Eliminate Low Frequency Dipoles

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ABSTRACT

Amplifiers with closely-spaced low-frequency pole-zero pairs (dipoles) are normally avoided for applications that require fast settling because they have slow-settling components in the transient response. In this work, an algorithm that involves a Newton-Raphson iteration is utilized to tune an amplifier with multiple low-frequency dipoles and facilitate pole-zero cancellation. The tuned structure is more suitable for fast settling applications.

1. INTRODUCTION

As fabrication technology progresses into deep-submicron feature sizes, achieving an adequate DC gain is becoming increasingly difficult. The origin of this problem is twofold. First, reductions in supply voltages are making it difficult to employ cascoding and still maintain adequate signal swings. Second, the degradation in device output conductance is making it difficult to achieve an adequate gain in two or fewer non-cascoded stages. As a result, non-traditional amplifier topologies are being investigated with increased urgency.

Amplifiers with more than two stages of gain are a potential solution to the problem. However, to ensure their stability with negative feedback, multistage amplifiers need to be *compensated.* Since each additional stage introduces poles into the system transfer function, the task of compensation becomes more difficult as the number of stages is increased. Several multistage amplifier compensation strategies have appeared in the literature [1-4]. Unfortunately, as a side-effect of the compensation process, most techniques sacrifice the gain-bandwidth product of the amplifier in exchange for stability. As a result, amplifiers with three or more stages are typically too slow for applications that require fast settling.

There is at least one multistage amplifier compensation technique [5,6] that does not sacrifice the gain bandwidth product of an amplifier in exchange for achieving stability. Although the technique results in an amplifier that has a gain-bandwidth product that is as large as can be achieved with a single stage amplifier, the resultant structure is still not suitable for fast settling applications because it has a poor transient response. The limitation of this technique is the fact that it relies on the cancellation of low-frequency pole-zero pairs. Inexact cancellations result in the appearance of slow-settling components in the transient response [7,8] making these amplifiers unsuitable for applications that require fast accurate settling.

In an effort to overcome these limitations and extend the applicability of multistage multipath compensated amplifiers to the high-speed realm, a calibration technique to eliminate the dipole mismatch of a two-stage structure was proposed [9]. The viability of the technique was demonstrated in a 0.25μ CMOS process. The results are awaiting publication elsewhere.

The applicability of the method proposed in [9] is limited to amplifiers with two gain stages and one low-frequency dipole. In this paper we present a generalization of the technique to cover amplifiers composed of an arbitrary number of stages. Although we focus specifically on the multistage multipath compensated amplifier architecture proposed in [5], the technique is generally applicable to other architectures that suffer from low-frequency dipoles as well.

The problem and the assumptions required for its solution are briefly described in section 2. The new calibration technique is outlined in section 3 and an example and short discussion appear in section 4.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

An n-stage multipath-compensated amplifier has a system transfer function that has n poles and $n-1$ zeros. The details can be found in [5,6,9]. If the components of the system are prudently dimensioned, the zeros can be used to cancel all but one of the poles.

Fig. 1 shows a typical example of the closed-loop pole and zero locations in the complex s-plane for an n'th order multipath amplifier used in a standard feedback configuration. For notational purposes, the poles are numbered in the order of increasing magnitude. If perfect cancellation were possible, the system would be exactly first-order. Practically, mismatch always exists and the system only approximates a first-order response.

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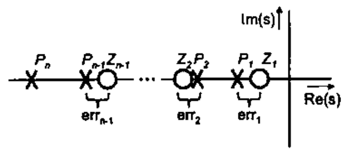

Fig. 1 Closed-loop pole and zero locations for an *n*'th order multipath amplifier

Imperfect cancellation of the pole-zero pairs (dipoles) results in the appearance of extra decaying exponential components in the transient response. These additional components will decay more slowly than the desired component because they lie at lower frequencies than the *uncovered* pole.

In this work it was assumed that the amplifier architecture allows the pole locations to be individually tuned. Fig. 2 illustrates the concept. The amplifier has a transfer function *H(s)* whose pole locations are adjustable via several control signals. The *k*'th control signal, b_k, is assumed to control the location of pole, p_k.

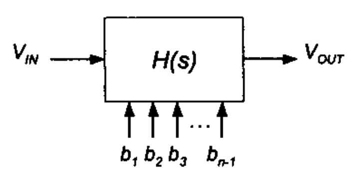

Fig. 2 Amplifier block with programmable pole locations

In a real physical implementation the *k*'th control signal is a voltage or current used to bias the *k*'th stage of the amplifier. In general, the relationship between a control signal and the corresponding pole location can be highly nonlinear. To ensure a reasonable model of the amplifier, a nonlinear relationship between the control signal and its corresponding pole location was assumed. The assumed relationship is shown Fig. 3. It is a hyperbolic tangent relationship scaled to allow tuning of ±25% of the pole's nominal value.

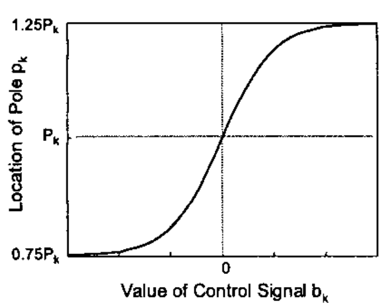

Fig. 3 Nonlinear relationship assumed between the control signals and their associated pole locations

Throughout the work we assume that the closed-loop poles of the system transfer function are widely separated on the real axis in the left half-plane. We also assume that each low frequency pole is located in close proximity (±15% their nominal values) to a zero. Although, the open-loop pole locations are highly sensitive to variations, feedback desensitizes these quantities and stabilizes their values. Therefore, these are reasonable assumptions for the *closed-loop* pole locations.

3. PROPOSED CALIBRATION TECHNIQUE

An n'th order system with a pole-zero map like the one shown in Fig. 1 has a transient step response given by:

$$y(t) = A_0\left(1 + \sum_{i=1}^{n} k_i \exp(p_i t)\right) \qquad (1)$$

where A_0 is the asymptotic steady-state gain, p_i is the location of the *i*'th pole, and k_i is a constant defined by:

$$k_i = -\frac{\prod_{j=1}^{n-1}\left(\frac{p_i}{z_j} - 1\right)}{\prod_{\substack{j=1 \\ j \neq i}}^{n}\left(\frac{p_i}{p_j} - 1\right)} \qquad i = 1,2,\cdots n \quad (2)$$

The first term in (1) is the asymptotic steady-state response. The remaining terms all decay with time and form the transient component of the response. Thus the transient step response of an *n*'th order system with widely separated real left half-plane poles can be decomposed into a sum of *n* decaying exponentials with differing time-constants. This relationship is illustrated for a third-order system in Fig. 4. Observe that the total response shown in Fig. 4(e) is simply the sum of the components shown in Figs. 4(a)-(d).

From (2) you can see that that adjusting the *i*'th control signal b_i such that p_i and z_i are coincident forces k_i to zero. Thus, by careful adjustment of the *n-1* control signals, the corresponding slow-settling terms in the transient response can be eliminated.

The calibration technique involves forcing the derivative of the transient step response to zero at specific instances in time. Judiciously choosing the points in time where the derivative is nulled ensures that each of the unwanted transient terms is eliminated.

The slowest settling component of the transient response is the term associated with p_1. Since the poles are widely separated, the other transient components decay significantly faster. Forcing the derivative of the step response to zero at a point in time after all the other transient components have decayed away ensures that the slowest settling component's coefficient, k_1, is forced to zero.

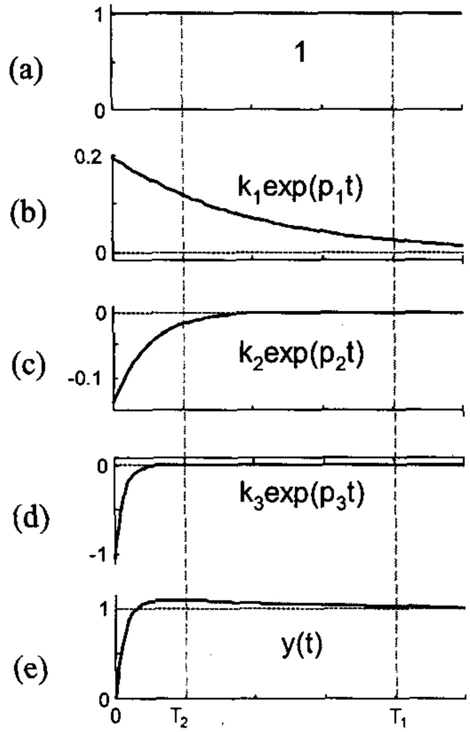

Fig. 4 Total transient step response of a third-order system (e) can be decomposed into a summation of (a) thru (d)

The next slowest settling component of the transient response is associated with p_2. Since the poles are widely separated, the other transient components, disregarding the one associated with p_1, decay significantly faster.

Forcing the derivative of the step response to zero at a point in time, T_2, after all the other transient components, except the p_1 term, have decayed away ensures that the joint variation due to the action of both the p_1 and p_2 terms sums to zero. However, since we forced the p_1 term to zero by choosing a time point at T_1, the joint effect of zeroing the derivative at times T_1 and T_2 is that the p_2 term must also be forced to zero.

The same arguments can be used justify repeated application of the concept until all n-1 slow-settling components are eliminated.

For example, forcing the derivative of the step response shown in Fig. 4 to zero at time T_1 requires that $k_1 \approx 0$. Simultaneously requiring that the derivative of the response is zero at time T_2 ensures that $k_2 \approx 0$ as well.

Reasonably good performance is obtained by choosing the time points equal to two time constants.

$$T_i = 2\tau_i = -\frac{2}{p_i} \quad i = 1,2,\cdots n-1 \tag{3}$$

In mathematical terms, minimization of the derivatives at the suggested time points can be written as:

$$f_1(b_1, b_2 \cdots b_{n-1}) = \left.\frac{\partial y}{\partial t}\right|_{t=T_1} = 0 \tag{4}$$

$$f_2(b_1, b_2 \cdots b_{n-1}) = \left.\frac{\partial y}{\partial t}\right|_{t=T_2} = 0 \tag{5}$$

$$\vdots$$

$$f_{n-1}(b_1, b_2 \cdots b_{n-1}) = \left.\frac{\partial y}{\partial t}\right|_{t=T_{n-1}} = 0 \tag{6}$$

where the f_i's are nonlinear functions of the control signals. In vector notation, these equations can be written as:

$$\mathbf{f}(\mathbf{b}) = \mathbf{0} \tag{7}$$

Equation (7) is a system of *n-1* nonlinear equations in *n-1* unknowns. The Newton-Raphson algorithm is one technique that is commonly used to solve these types of problem and is described elsewhere [10,11]. One drawback of this approach is that the derivatives of each of the nonlinear functions with respect to each of the control signals is required at each iteration. The required derivatives can be obtained using a finite difference method on samples of the transient *step* response. An easier method to obtain the same information involves sampling the *impulse* response.

4. EXAMPLE

A fourth-order linear system with poles spaced at an interval of a decade was assumed. Using a normal distribution with a standard deviation equal to 15% of the magnitude of the associated pole, three zeros were randomly generated near the low-frequency poles.

Nonlinear relationships similar to the one shown in Fig. 3 were assumed to relate the control signals to the pole locations.

Table 1 shows the locations of the poles and zeros prior to and after calibration. The algorithm converged in 6 iterations. Note that there were significant dipole mismatches prior to calibration, but after calibration, the results agree to better than 6 significant digits.

Table 1 Pole and zero locations before and after the calibration routine was performed.

Poles (pre-cal)	Zeros	Poles (post-cal)
-1e3	-1.134345e3	-1.134345e3
-1e4	-1.109643e4	-1.109643e4
-1e5	-1.086678e5	-1.086678e5
-1e6		-1.000000e6

The algorithm has been used to successfully tune systems as large as 9'th order containing 8 dipoles. The technique has also been used to tune structures with poles spaced as close as an octave apart. Convergence problems may arise if the magnitude of the pole-zero mismatch is on the same order as the spacing between the poles.

Before a practical implementation of this algorithm can be implemented, the effects of noise and quantization need to be considered.

5. SUMMARY

Amplifiers with low-frequency pole-zero pairs are not suitable for applications that require fast, accurate settling because dipole mismatches result in slow settling components in the transient response. In an effort to overcome this limitation, a technique to tune the responses of amplifiers to eliminate the dipole mismatch is proposed.

The procedure requires sampling the transient step response and uses the Newton-Raphson algorithm to determine the values of the bias currents and voltages required to achieve accurate cancellations.

To demonstrate the technique a 4'th order system with 3 mismatched low-frequency dipoles was calibrated. After six iterations, the dipoles matched to better than 6 significant digits.

6. ACKNOWLEDGMENT

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