OUTPUT TRACKING CONTROL OF NONMINIMUM PHASE SYSTEMS VIA CAUSAL INVERSION

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ABSTRACT

This paper introduces a new design procedure for output tracking control of nonminimum phase systems. This new controller achieves stable ϵ–tracking of a reference profile given in real time via a causal inversion approach. In contrast to stable inversion, the causal inversion approach does not require precalculation. In contrast to nonlinear regulation, the causal inversion approach avoids the numerical intractability of solving nonlinear PDEs and transient errors are significantly smaller. As an example, a causal inversion-based controller is designed for tip trajectory tracking of a one-link flexible manipulator. Simulation results demonstrate its effectiveness in output tracking.

1. INTRODUCTION

Output tracking control of nonminimum phase systems is a highly challenging problem encountered in many practical engineering applications. The classical inversion approach for output tracking control uses stabilizing feedback together with feed-forward signals generated by an inverse system. The classical inverse problem was first studied by Brockett and Mesarovic [1]. The linear results were extended to nonlinear systems by Hirschorn [3]. These inversion algorithms produce causal inverses but unbounded solutions for nonminimum phase systems.

The nonlinear regulation technique was first developed by Isidori and Byrnes [4]. The solution of the nonlinear regulator involves solving nonlinear PDEs; however, the transient errors could not be controlled precisely for nonminimum phase systems. Later, the stable inversion approach was successfully applied to output tracking control of nonminimum phase systems [2]. Both bounded state and input trajectories were obtained. The downside is that stable inversion is noncausal.

This paper introduces a new procedure for designing an output tracking controller for nonminimum phase systems using a causal reference trajectory. This new controller will achieve stable ϵ–tracking. This is obtained by using a novel approach derived from causal inversion. As an example, a causal inversion-based controller is designed for a one-link flexible manipulator system, in which preloading the links is not required.

The remainder of this paper is organized as follows. The next section defines the basic framework and the problem to be solved. Section 3 defines the causal inversion problem and presents the solution. Section 4 studies the controller design problem. Section 5 applies the causal inversion approach to designing a controller for a one-link flexible manipulator. Simulation results are discussed. Finally, concluding remarks are given in Section 6.

2. BASIC FRAMEWORK

First, consider a nonlinear system of the form

$$\dot{x} = f(x) + g(x)u \qquad (1)$$

$$y = h(x) \qquad (2)$$

defined on a neighborhood X of the origin in $\Re^n$, with input $u \in \Re^m$ and output $y \in \Re^p$. $f(x)$ and $g_i(x)$ (the i^{th} column of $g(x)$) for $i = 1, 2, \cdots, m$ are smooth vector fields. And $h_i(x)$ for $i = 1, 2, \cdots, p$ are smooth functions on X, with $f(0) = 0$ and $h(0) = 0$. In such a context, these assumptions are made:

A1 : The above system (1, 2) is stabilizable and observable.

A2 : The reference output trajectory $y_d(t)$ and its certain derivatives are continuous. $y_d(t) \in L_1 \cap L_\infty$, with $y_d(t) \equiv 0$ for $t \leq 0$.

Remark : The condition $y_d(t) \in L_1 \cap L_\infty$ can be replaced by a condition that requires $\dot{y}_d(t) \in L_1 \cap L_\infty$ so that y_d can approach the same constant values, as frequently happens in practical situations. This change requires little modification of the general method. There is also a simple way to ensure that $y_d(t) \in L_1 \cap L_\infty$, avoiding this modification. In practice, any trajectory y_d will be bounded and nonzero with a finite duration. One can simply modify y_d after a sufficient settling time t_f so that it comes back to 0.

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For a given reference $y_d(t)$ satisfying Assumption A2, the following tracking problem is defined:

Definition 1 : Given an $\epsilon > 0$, the system is said to achieve *stable ϵ-tracking*, if $\|y_d(\cdot) - y(\cdot)\|_{L_2} < \epsilon$ with bounded $x(t)$ and $u(t)$.

3. CAUSAL INVERSION PROBLEM

For system (1, 2), the following problem is posed [6]:

Causal Inversion Problem: Given $y_d(t)$ satisfying Assumption A2, find a nominal control input $\bar{u}_d(t)$ and a desired state trajectory $\bar{x}_d(t)$ such that

(1) $\bar{u}_d$ and $\bar{x}_d$ are bounded, and

$$\bar{u}_d(t) \to 0, \;\; \bar{x}_d(t) \to 0, \;\; \text{as} \;\; t \to \infty$$

(2) $\bar{x}_d(t)$ and $\bar{u}_d(t)$ are causal; that is, $\bar{x}_d(t) \equiv 0, \bar{u}_d(t) \equiv 0$ for $t \leq 0$, where $\bar{x}_d$ is the desired state trajectory and $\bar{u}_d$ is the nominal control input.

(3) Exact output matching is achieved:

$$h(\bar{x}_d(t)) = y_d(t), \;\; \text{for all} \;\; t \qquad (3)$$

(4) $\dot{x}_d - f(\bar{x}_d) - g(\bar{x}_d)\bar{u}_d \to 0$ as $t \to \infty$.

Remark : Note that condition (1) ensures stability of the inverse solution. Condition (2) gives rise to the term "causal" inversion. Strictly speaking, $\bar{u}_d$ is not an inverse solution since applying $\bar{u}_d$ to the input of the system (1, 2) will not lead to $y = y_d$ in general. The solution $\bar{u}_d$ and $\bar{x}_d$ are termed a causal "inverse" solution because of condition (3). Condition (4) further ensures that $\bar{u}_d$ is indeed the asymptotically inverse solution to the dynamics (1, 2).

This paper only provides a guideline of the causal inversion solutions; more details can be found in [6].

3.1. Causal Inversion for Nonlinear Systems

Consider a nonlinear system of the form (1, 2) with the same number m of inputs and outputs.

A3 : The system (1, 2) has a well-defined relative degree r at the equilibrium point $x = 0$.

The system can be partially linearized using the standard approach. Thus the zero dynamics driven by the reference output trajectory is obtained $\dot{\eta} = p(y_d^{(r)}, \xi_d, \eta)$. From Assumption A2, "certain" is made clear here, which means y_d should be r times differentiable and $(r-1)$ times continuously differentiable.

A4 : $\eta = 0$ is a hyperbolic equilibrium point of the autonomous zero dynamics.

Linearizing the zero dynamics at the equilibrium point and introducing a controller $\bar{v}$ to stabilize the unstable reference zero dynamics, two dynamic equations are defined as follows:

$$\dot{\bar{\eta}}_s = A_s\bar{\eta}_s + B_s y_d^{(r)} + d_s(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u), \bar{\eta}_s(0) = 0 \quad (4)$$

$$\dot{\bar{\eta}}_u = A_u\bar{\eta}_u + B_u y_d^{(r)} + d_u(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u) + \bar{v}, \bar{\eta}_u(0) = 0 \quad (5)$$

where $\bar{\xi}_d = \xi_d$. By selecting $\bar{v} = -2A_u\bar{\eta}_u - 2B_u y_d^{(r)} - 2d_u(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u)$, Equation (5) becomes

$$\dot{\bar{\eta}}_u = -A_u\bar{\eta}_u - B_u y_d^{(r)} - d_u(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u), \bar{\eta}_u(0) = 0 \quad (6)$$

By a similar argument mentioned in [6], this yields $\bar{\eta}_u(t)$, $\bar{\eta}_s(t) \to 0$ as $t \to \infty$. Also, $\bar{\xi}_d = 0$ for $t \leq 0$ and for $t \geq t_f$. Furthermore, it can be shown that $\bar{x}_d$ and $\bar{u}_d$ are bounded, and $\bar{x}_d(t), \bar{u}_d(t) \to 0$ as $t \to \infty$. By the definition of $\bar{\xi}, h(\bar{x}_d) = y_d$ and $L_f^i h(\bar{x}_d) = y_d^{(i)}$ for $i \leq r$ are obtained. Thus a casual inversion solution to the nonlinear system has been provided.

3.2. Causal Inversion for Linear Systems

Consider a linear system of the form

$$\dot{x} = Ax + Bu \quad (7)$$

$$y = Cx \quad (8)$$

with a well-defined vector relative degree.

By properly picking the transformation matrix, it yields the unstable subsystem

$$\dot{\eta}_u = A_u\eta_u + B_u y_d^{(r)}, t \leq t_f; \eta_u(t) = 0, \;\; \forall\, t \geq t_f \quad (9)$$

Here, a controller $\bar{v}$ is introduced to stabilize the unstable subsystem $\dot{\bar{\eta}}_u = A_u\bar{\eta}_u + B_u y_d^{(r)} + \bar{v}$, $\bar{\eta}_u(0) = 0$. By choosing $\bar{v} = -2A_u\bar{\eta}_u - 2B_u y_d^{(r)}$, this step then yields $\dot{\bar{\eta}}_u = -A_u\bar{\eta}_u - B_u y_d^{(r)}$, $\bar{\eta}_u(0) = 0$. By a similar approach, the inverse solution for the linear system can be obtained.

Theorem 1 : *Consider a linear system described by (7, 8). Given a low pass signal $g(t) \in L_1 \cap L_\infty$ and $y_d^{(r)}(t) = g(\alpha t)$, for any $\epsilon_\eta, \epsilon_u, \epsilon_v > 0$, there exist a time scaling factor α such that*

(1) $\|\tilde{\eta}_u\|_{L_2} < \epsilon_\eta$

(2) $\|\bar{u}_d - u_d\|_{L_2} < \epsilon_u$

(3) $\|\bar{v}\|_{L_2} < \epsilon_v$

where $\tilde{\eta} = \bar{\eta}_u - \eta_u$.

Proof : To simplify the proof, the dimension of the unstable subsystem is assumed to be $n_u = 1$. In this case, both A_u and B_u become scalars. Let scalar a and b represent A_u and B_u respectively.

For the noncausal signal $\eta_u(t)$, since $\dot{\tilde{\eta}}_u = -A_u\tilde{\eta}_u - 2\dot{\eta}_u$, the bilateral Laplace transform is given by $\eta_u(s) = \frac{bs}{s-a}y_d^{(r)}(s)$, where the region of convergence is $Re(s) < a$ with $a > 0$.

The Laplace transform of $\tilde{\eta}_u$ is given by $\tilde{\eta}_u(s) = \frac{-2bs}{s^2-a^2}y_d^{(r)}(s)$, where the region of convergence is $-a < Re(s) < a$ with $a > 0$. Let $H_1(s) = \frac{-2bs}{s^2-a^2}$, then $\tilde{\eta}_u(s) = H_1(s)y_d^{(r)}(s)$.

Given a low pass signal $g(t), \exists$ a time scaling factor $\alpha_1 > 0$ such that $y_d^{(r)}(t)$ can be chosen as $y_d^{(r)}(t) = g(\alpha_1 t)$.

Then $g(\omega) = \mathcal{F}(g(t))$ and $H_1(\omega) = H_1(s)|_{s=j\omega}$. Furthermore, by the scaling property of the Fourier Transform, it follows that $y_d^{(r)}(\omega) = \mathcal{F}(g(\alpha_1 t)) = \frac{1}{\alpha_1} g(\frac{\omega}{\alpha_1})$. Since both $H_1(\omega)$ and $y_d^{(r)}(\omega)$ are bounded, set $\|y_d^{(r)}(\omega)\|_\infty^2 + \|H_1(\omega)\|_\infty^2 = K_1$, where K_1 is finite and $K_1 > 0$.

Notice that $\int_{\omega_1}^{\infty} |g(\omega)|^2 d\omega \to 0$ as $\omega_1 \to \infty$. Thus, $\forall \epsilon_\eta > 0, \exists\ \omega_1(\epsilon_\eta)$ such that $\int_{\omega_1}^{\infty} |g(\omega)|^2 d\omega < \frac{\pi\epsilon_\eta^2}{K_1}$. Similarly, $\forall \epsilon_\eta > 0$, $\exists\ \omega_2(\epsilon_\eta)$ such that $\int_0^{\omega_2} |H_1(\omega)|^2 d\omega < \frac{\pi\epsilon_\eta^2}{K_1}$. Given ω_2, $\forall \epsilon_\eta > 0, \exists\ \alpha_1$ such that when $\alpha < \alpha_1$ $\int_{\omega_2}^{\infty} |y_d^{(r)}(\omega)|^2 d\omega < \frac{\pi\epsilon_\eta^2}{K_1}$. The spectral separation is shown in Figure 1. Consequently, this yields

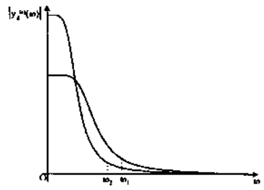

Figure 1: Spectral separation

$$\begin{aligned}&\int_0^{\infty} |\tilde{\eta}(\omega)|^2 d\omega \\ &= \int_0^{\omega_2} |H_1(\omega) y_d^{(r)}(\omega)|^2 d\omega + \int_{\omega_2}^{\infty} |H_1(\omega) y_d^{(r)}(\omega)|^2 d\omega \\ &< \frac{\pi\epsilon_\eta^2}{K_1}(\|y_d^{(r)}(\omega)\|_\infty^2 + \|H_1(\omega)\|_\infty^2) = \pi\epsilon_\eta^2\end{aligned}$$

By Parseval's theorem, it then follows that $\|\tilde{\eta}_u(t)\|_{L_2} < \epsilon_\eta$. Furthermore, by a similar argument, $\|\bar{u}_d - u_d\|_{L_2} < \epsilon_u$ and $\|\bar{v}\|_{L_2} < \epsilon_v$ easily follow. **Q.E.D.**

Note that the proposed proof easily extends to the $n_u > 1$ case.

4. OUTPUT TRACKING CONTROL

Define $\tilde{x}(t) = \bar{x}_d(t) - x(t)$, $\tilde{u}(t) = \bar{u}_d(t) - u(t)$, and $\tilde{y}(t) = y_d(t) - y(t)$. Then the error dynamics for (1, 2) is as follows:

$$\dot{\tilde{x}} = f(\tilde{x}) + g(\tilde{x})u + g(\bar{x}_d)\tilde{u} + P_{\bar{v}}\bar{v} \quad (10)$$
$$\tilde{y} = h(\tilde{x}) \quad (11)$$

where $f(\tilde{x}) = f(\bar{x}_d) - f(x), g(\tilde{x}) = g(\bar{x}_d) - g(x)$, and $h(\tilde{x}) = h(\bar{x}_d) - h(x)$.

Suppose the causal inversion of nonlinear systems can be solved. Furthermore, given an $\epsilon_v > 0$, suppose the following inequality is also satisfied: $\|\bar{v}\|_{L_2} < \epsilon_v$. Since the design goal is to achieve *stable ϵ−tracking* so that $\|\tilde{y}\|_{L_2} < \epsilon$ as $t \to \infty$ with bounded $x(t)$, an H_∞ controller is a natural choice. Suppose the closed-loop nonlinear mapping from $\bar{v}$ to $\tilde{y}$ is given by $\tilde{y} = \Phi(\cdot)\bar{v}$. If an H_∞ controller $K(\cdot)$ could be found, it would then yield $\|\tilde{y}\|_{L_2} \le \|\Phi(\cdot)\|_\infty \|\bar{v}\|_{L2}$. Since $\|\Phi(\cdot)\|_\infty \|\bar{v}\|_{L2}$ is bounded, this implies $\|\tilde{y}\|_{L_2} < \gamma\epsilon_v = \epsilon$ with $\epsilon > 0$. From the definition of causal inversion, $\bar{x}_d$ is bounded, and so $\tilde{x}$ is bounded by the property of the H_∞ controller. As a result, $x(t)$ must be bounded. Thus *stable ϵ−tracking* is achieved. The block diagram of the closed-loop system is shown in Figure 2. Similarly, the error dynamics for linear system (7, 8) is addressed as follows:

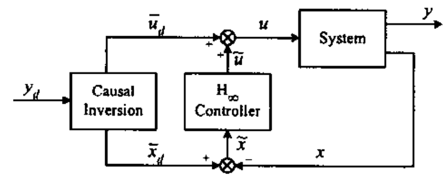

Figure 2: Block diagram of closed-loop system

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} + P_{\bar{v}}\bar{v} \quad (12)$$
$$\tilde{y} = C\tilde{x} \quad (13)$$

Theorem 1 in Section 3.2 has provided the solution of the *Causal Inversion Problem* for linear systems. Moreover, given an $\epsilon_v > 0$, $\|\bar{v}\|_{L2} < \epsilon_v$ can be made. An H_∞ controller is required for these linear systems. Suppose the closed-loop transfer function from $\bar{v}$ to $\tilde{y}$ is given by the linear fractional transformation $\tilde{y} = F_l(G, K)\bar{v}$. Let $K(s)$ be the H_∞ controller that minimizes the gain from $\bar{v}$ to $\tilde{y}$; it would then yield $\|\tilde{y}\|_{L_2} \le \|F_l(G,K)\|_\infty \|\bar{v}\|_{L2} < \gamma\epsilon_v = \epsilon$, where $\epsilon > 0$. This implies *stable ϵ−tracking* has been achieved, which indicates that the total energy in the transient tracking error can be controlled within any given requirement.

5. AN EXAMPLE: A ONE-LINK FLEXIBLE MANIPULATOR

A nonlinear dynamic model (shown in Figure 3) and the parameters of the one-link flexible manipulator are chosen from [5]. $l = 1$ m, $m = 0.2$ kg, $k = 5$ Nm/rad, and $d_1 = d_2 = 0.01$ Nm · sec/rad. It is easy to verify that

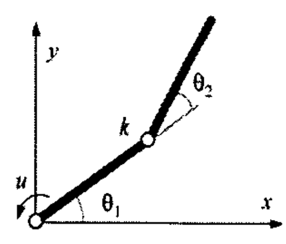

Figure 3: A simple one-link flexible manipulator

the linearized zero dynamics is unstable, which means the system is a nonminimum phase system.

After the nominal input is calculated, the controller is composed by the following structure $u = \bar{u}_d + K(\bar{x}_d - x)$, where $\bar{x}_d$ denotes the state variables of the forward dynamics, $\bar{x}_d = (\theta_{1d}, \theta_{2d}, \dot{\theta}_{1d}, \dot{\theta}_{2d})$. Then a standard H_∞ optimal controller is designed to find the controller K for stabilizing the forward dynamics. Let the desired output trajectory be defined as follows:

$$y_d = \begin{cases} \pi^2(\frac{1}{2\pi t} - \frac{1}{(2\pi)^2 \sin(2\pi t)}), & 0 \leq t \leq t_f \\ \frac{\pi}{2}, & t > t_f \end{cases}$$

as shown by the solid curve in Figure 4. For the given trajectory, the following data were used: $y_0 = 0^o, y_f = 90^o$. The initial conditions are $\theta_1 = \theta_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0$. Two cases are simulated below.

Case 1 : $t_f = 1$ second and **Case 2** : $t_f = 0.5$ seconds.

The desired and actual trajectories of the output for Cases 1 and 2 are shown in Figures 4 and 5 respectively. The output tracking errors for Cases 1 and 2 are shown in Figures 6 and 7 respectively. The maximum error during transients for Case 1 is relatively small (around 0.7^o); whereas, the maximum error during transients for Case 2 is around 3.5^o, which is significantly larger than in Case 1.

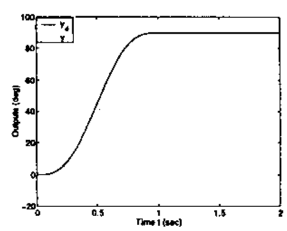

Figure 4: Desired (solid) and actual (dotted) output trajectories for Case 1

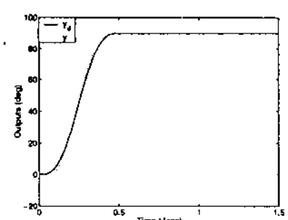

Figure 5: Desired (solid) and actual (dotted) output trajectories for Case 2

Furthermore, when applied to a one-link flexible manipulator, causal inversion has the advantages of not requiring the preloading for the links (as does stable inversion), as well as eliminating the need to solve the nontrivial solution of a set of partial differential algebraic equations (as required by the nonlinear regulation approach).

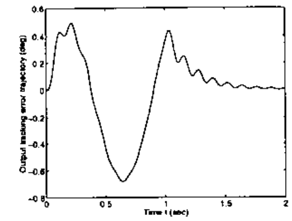

Figure 6: Output tracking error trajectory for Case 1

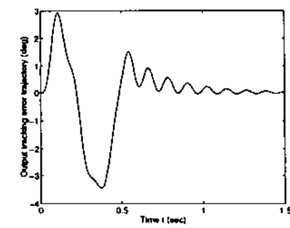

Figure 7: Output tracking error trajectory for Case 2

6. CONCLUSIONS

This paper has introduced a new procedure for designing a nonminimum phase output tracking controller driven by a causal reference profile. This new controller achieves stable ϵ−tracking. Simulation results demonstrate that the causal inversion approach is very effective for obtaining output tracking for flexible manipulators.

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