# Yield Enhanced Layout Strategies for Ratio-Critical Analog Circuits

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## Abstract

A yield enhanced layout method for maximizing yield of integrated resistor networks with a fixed total area is discussed. This approach incorporates both random variations in the sheet resistance and random variations in the contact resistances. The concept of contact/sheet resistance crossover which gives the crossover between contact-resistance dominance and sheet resistance dominance is developed.

# 1. Introduction

Layout plays a key role in the yield of ratio-critical circuits. Techniques such as segmentation and commoncentroid layout can minimize the first and higher-order gradient effects and consequently improve the ratio accuracy [1-3]. With gradient effects reduced, the local random variations become the dominant contibutor to mismatch. To date, most of the work has concentrated on the matching of two nominally identical devices and little has appeared in literature on the matching of devices with ratios not equal to 1. Previous work shows that the standard deviation of the relative resistance or capacitance of two closely placed resistors and capacitors varies inversely proportional to the square root of the area of the devices [4-5]. In an earlier work [6], study of the random effects on the resistors of feedback amplifier showed that that the yield of the feedback amplifier for any gain can be maximized if the area of the input and feedback resistors is equal. However, this conclusion did not shed light on how to layout each resistor. Section 2 will address this issue.

In addition to the variation of sheet resistance, the contact resistance variations may also become significant. Therefore, the condition of optimal yield needs to be reformulated. The resistors in applications can be implemented by unit cells with appropriate segmentation. There are a lot of ways to implement the resistors even with the total resistor area fixed. In different approaches, the area of the unit resistor cell may be very different which may or may not result in very different yield.

Although the techniques described in this paper are applicable to both resistors and capacitors, resistors will be used for all formulations. In what follows, we will limit our discussion to resistors of feedback amplifier. Considering the process feature size tends to decrease, the variation of the contact resistance can not be neglected any longer. A model that combines the random effects of the sheet resistance and contact resistance will be developed. Additionally, the condition when the contact resistance variation becomes dominant will also be determined.

#### 2. Resistor layout method

It has been shown [4,7,8] that the standard deviation of normalized resistance of any rectangular resistor due to random variation of sheet resistance is given by

$$\sigma \frac{R_{ran}}{R_{N}} = \frac{A_{\rho}}{\rho_{N}\sqrt{WL}} = \frac{A_{\rho}}{\rho_{N}\sqrt{A_{R}}} = \frac{K_{\rho}}{\sqrt{A_{R}}} \qquad (1)$$

Where  $A_{\rho}$  is process parameter characterizing the local variations in the sheet resistance,  $\rho_N$  is the nominal sheet resistance, and W and L are the width and length of the resistor. The product of W and L is area, which is represented by  $A_R$ . In this expression, the contact resistance and edge roughness have been neglected.

In practical applications, it is often difficult to size only one rectangular resistor to get the required ratio. Often, the resistors are made of unit resistor cells with segmentations. In this situation, the standard deviation may be different even with the same total component area. For example, for the three different approaches shown in Fig.1, the standard deviations are given by

a) 
$$\sigma \frac{R_{ran}}{R_{N}} = \frac{A_{\rho}}{\sqrt{A_{RT}}}$$
  
b) 
$$\sigma \frac{R_{ran}}{R_{N}} = \frac{A_{\rho}}{\sqrt{A_{RT}}}$$

c)

$$\sigma \frac{R_{ran}}{R_N} = \frac{A_{\rho}}{\frac{4}{5}\sqrt{A_{RT}}}$$



Fig. 1: Three different approaches to implement the same resistor

where the total resistor area of these three layouts is A<sub>RT</sub> and the nominal value is also the same. It can be shown that the relative standard deviation is the same for the (a) and (b) approaches but is different for (c). Analysis shows that in approach (c), the single series unit cell has 80% contribution to the overall standard deviation and each of the other four cells has equal contribution. Therefore, for this configuration, the contribution of the series resistor towards total standard deviation is 16 times that of one unit parallel resistors. From another point of view, the current flow through this series resistor is twice that through the parallel resistors. For similar cases, it can be proved that the contribution ratio from each resistor is the 4<sup>th</sup> power of the corresponding current ratio. If every unit cell carries the same current, the overall relative standard deviation is independent of the specific layout and the value is minimized. Therefore, for the feedback amplifier, there are two requirements: First, the areas of the input and feedback resistors must be the same. Second, the current flowing through the unit cells within a resistor must also be the same.

#### 3. Layout strategy with contact resistance

A basic feedback amplifier is shown in Fig.2. The standard deviation [6] is given by

$$\sigma_{\theta} = \theta_{N} \sqrt{\sigma^{2} \frac{R_{Aran}}{R_{AN}} + \sigma^{2} \frac{R_{Bran}}{R_{BN}}}$$
(2)



Fig.2: A symbolic feedback amplifier

where  $\theta_N$  is the nominal absolute gain of the amplifier given by the ratio of  $R_B$  and  $R_A$ . In previous work [6], the contact resistance is neglected in order to get insight on the role sheet resistance area plays in the yield. However, the variation of contact resistance may be the major contributor of mismatch effects in some cases. A comprehensive consideration of contact resistance and sheet resistance is therefore necessary. A symbolic layout of rectangular resistor is shown in Fig.3 where W and L are the width and length of a resistor and n is the number of contacts on each side of one unit component cell. If the layout of contacts is homogeneous, then

$$W = nt$$
 (3)

where t is the pitch of the contact. We also assume the area of each unit cell to be  $A_{RC}$  and the total area for one resistor component to be  $A_{R}$ . As mentioned earlier, the unit cell should be exactly same, therefore

$$A_{\rm R} = NA_{\rm RC} \qquad (4)$$

where N is the number of unit cells.



Fig. 3: A symbolic layout of rectangular resistor

A tedious but straightforward analysis results in the standard deviation of a resistor component as shown below:

$$\sigma^{2} \frac{R_{ran}}{R_{N}} = \frac{1}{N} \sigma^{2} \frac{R_{Uran}}{R_{UN}}$$

$$= \frac{1}{N} \left\{ \frac{2R_{CN}^{2}}{n^{3} \left(\frac{2R_{CN}}{n} + R_{SHN}\right)^{2}} \sigma^{2} \frac{R_{Cr}}{R_{CN}} + \left[\frac{R_{SHN}^{2}}{\left(\frac{2R_{CN}}{n} + R_{SHN}\right)^{2}} \sigma^{2} \frac{R_{SHr}}{R_{SHN}} \right\}^{(5)}$$

where  $R_{UN}$  is the nominal resistance value of unit resistor cell,  $R_{Uran}$  is the corresponding resistance variation,  $R_{CN}$  is the nominal value of contact resistance,  $R_{Cr}$  is the variation of the resistance,  $R_{SHN}$  is the nominal value of resistance from sheet resistor film, and  $R_{SHr}$  is the resistance variation of the sheet resistor.

Combining Equation (1), (3), (4), and (5) results in

$$\sigma^{2} \frac{\mathbf{R}_{ran}}{\mathbf{R}_{N}} = \frac{1}{\mathbf{A}_{R}} \left[ \frac{2 \cdot \mathbf{R}_{CN}^{2} \cdot \sigma^{2} \frac{\mathbf{R}_{Cr}}{\mathbf{R}_{CN}} \cdot \mathbf{t}^{3} \cdot \mathbf{L} + \mathbf{L}^{2} \cdot \mathbf{K}_{p}^{2}}{\left(2\mathbf{R}_{CN} \cdot \mathbf{t} + \boldsymbol{\rho}_{N} \cdot \mathbf{L}\right)^{2}} \right]$$
(6)

Referring to Fig. 1, the total area  $A_T$  can be expressed by

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$$A_{\rm T} = A_{\rm RA} + A_{\rm RB} \tag{7}$$

where  $A_{RA}$  and  $A_{RB}$  are the area of resistors  $R_A$  and  $R_B$ . If the area ratio  $\gamma$  is defined as  $A_{RA}/A_T$ , then combining equations (2), (6), and (7), we can obtain the standard deviation of the gain of the amplifier as

$$\mathbf{q}_{\theta} = \theta_{N} \left\{ \frac{1}{\gamma A_{r}} \left[ \frac{2 \cdot R_{CN}^{2} \cdot \sigma_{R_{CN}}^{2R_{Cr}} \cdot \mathbf{t}_{1}^{3} \cdot \mathbf{L}_{1} + \mathbf{L}_{1}^{2} \cdot \mathbf{K}_{p}^{2}}{\left( 2 \cdot R_{CN} \cdot \mathbf{t}_{1}^{3} \cdot \mathbf{L}_{1} + \mathbf{L}_{1}^{2} \cdot \mathbf{K}_{p}^{2} \right] + \frac{1}{(1 - \gamma) \cdot A_{r}} \left[ \frac{2 \cdot R_{CN}^{2} \cdot \sigma_{R_{CN}}^{2R_{Cr}} \cdot \mathbf{t}_{2}^{3} \cdot \mathbf{L}_{2} + \mathbf{L}_{2}^{2} \cdot \mathbf{K}_{p}^{2}}{\left( 2 \cdot R_{CN} \cdot \mathbf{t}_{1} + \mathbf{p}_{N} \cdot \mathbf{L}_{p}^{2} \right)^{2}} \right] (8)$$

Typically, same unit cell and contact pitch for each unit cell are used, i.e.  $L_1 = L_2$  and  $t_1 = t_2$ . In addition,  $R_{CN}$ ,  $\sigma^2 \frac{R_{Cr}}{R_{CN}}$ ,  $K_\rho$ ,

 $\rho_N$  are process parameters. For the same process, these are same. Therefore, equation (8) can be simplified as

$$\sigma_{\theta} = \theta_{N_{v}} \left[ \frac{1}{\gamma \cdot A_{\Gamma}} + \frac{1}{(1 - \gamma) \cdot A_{\Gamma}} \right] \cdot \left[ \frac{2 \cdot R_{CN}^{2} \cdot \sigma_{R_{CN}}^{2} \cdot t^{3} \cdot L + L^{2} \cdot K_{\rho}^{2}}{(2R_{CN} \cdot t + \rho_{N} \cdot L)^{2}} \right] (9)$$

If the contact pitch and length are determined for one application, the second term will be known. The first derivation shows that only when  $\gamma = 0.5$ , i.e. the area is split equally between the input and feedback resistors, the standard deviation of ratio will be minimized. This also indicates that the number of unit cells and contacts should be exactly the same for the input and feedback resistors.

It can be shown from (9) that getting the minimum value of the area term is not enough. In order to minimize the standard deviation of gain and the consequent optimal yield, the second term, which is a function of process parameters, L and t, should also be minimized.

#### 4. Contact Resistance modeling

For a single unit resistor cell, the dominant contribution of the standard deviation of gain can be from random effects of the contact resistance or from the random effects of the sheet resistance. From Equation (5) and (6), we know:

$$\sigma^{2} \frac{R_{U_{ran}}}{R_{U_{N}}} = \left\{ \frac{2R_{CN}^{2}}{n^{3} \left(\frac{2R_{CN}}{n} + R_{SHN}\right)^{2}} \right] \sigma^{2} \frac{R_{Cr}}{R_{CN}} + \left[ \frac{R_{SHN}^{2}}{\left(\frac{2R_{CN}}{n} + R_{SHN}\right)^{2}} \right] \sigma^{2} \frac{R_{SH}}{R_{SHN}} \right\} (10)$$

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The first term is the contribution from contact resistance and the second term is the contribution from the sheet resistance. The crossover will occur when

$$\frac{2 \cdot \mathbf{R}_{CN}^2 \cdot \sigma^2 \frac{\mathbf{R}_{Cr}}{\mathbf{R}_{CN}}}{n^3 \left(\frac{2 \cdot \mathbf{R}_{CN}}{n} + \mathbf{R}_{SHN}\right)^2} = \frac{\mathbf{R}_{SHN}^2 \cdot \sigma^2 \frac{\mathbf{R}_{SHr}}{\mathbf{R}_{SHN}}}{\left(\frac{2 \cdot \mathbf{R}_{CN}}{n} + \mathbf{R}_{SHN}\right)^2}$$
(11)

Inserting equation (1) into equation (11) yields

$$L_{cri} = \frac{2 \cdot R_{CN}^2 \cdot \sigma^2 \frac{R_{cr}}{R_{CN}}}{A_{\rho}^2} \cdot \frac{W^3}{n^3}$$
(12)

If we assume the pitch of the contact is  $4\lambda$  and W is sufficiently large, then W=  $4\lambda n$ , and equation (12) can be rewritten as

$$L_{cri} = \frac{2^7 \cdot \lambda^3 \cdot R_{CN}^2 \cdot \sigma^2 \frac{R_{cr}}{R_{CN}}}{A_{\rho}^2}$$
(13)

As can be seen from (13), the critical length is a process parameter. When the length of the resistor is larger than  $L_{cri}$ , the variation of contact resistance will dominate the contribution of the sheet resistance variation and vice versa.

#### 5. Conclusions

It has been shown that optimal area allocation for two ratio-matched resistors is to have equal area for each resistor. These two resistors should be implemented with the same unit resistor cell and each resistor cell should have the same contribution to the overall standard deviation, i.e. the current following through each unit cell should be the same. With contact resistance included, if the same unit resistor cell is used for the two resistors, the statement above is still valid indicating that the total number of contacts for each component is the same. However, when the contact resistance is not negligible, optimal yield may not be achieved if we only allocate area equally to these two resistors. The length of the resistor and the number of contacts on each resistor sheet film should also be considered to minimize the standard deviation of the ratio and consequently optimize the yield. A model which combines the random effects of the contact resistance and the sheet resistance was developed. The model provides us the contact/sheet critical length which can be used to determine whether the sheet resistance or the contact resistance variation contribution is dominant.

## 6. References

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