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Tip Trajectory Tracking for a One-Link Flexible Manipulator using Causal Inversion

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Abstract—**Tip trajectory tracking for a flexible link robotic manipulator is a challenging problem. Due to the nonminimum phase nature of the system, existing methods encounter difficulties achieving high level performance. The classical causal inverses proposed by Hirschorn result in unbounded solutions to the inverse problem when the zero dynamics are unstable. Stable inversion introduced by Chen and Paden obtains bounded but noncausal inverses for nonminimum phase systems. In this paper, a causal inversion-based controller is applied to a one-link flexible manipulator. Simulation results demonstrate the effectiveness of the new causal inversion approach in output tracking.**

Key Words: Causal Inversion, One-Link Flexible Manipulator, Nonlinear Systems

I. Introduction

Flexible manipulators have many advantages over rigid links: They are lighter in weight, consume less power, and respond faster [12]. Due to the flexible nature of the system, the dynamics are highly nonlinear and complex. The most elementary task in robot control is driving the end-effector of a manipulator to follow a given desired trajectory. Flexibility in a manipulator will degrade trajectory tracking control and manipulator tip positioning [7].

Early works on flexible manipulators were carried out by Cannon and Schmitz [4] and Siciliano and Book [8]. Besides the link flexibility, Yang and Donath also considered the flexibility of the joint [15]. Bayo [2] applied the Fourier transform to obtain stable but noncausal control input. De Luca *et al.* [14] applied nonlinear regulation to the control of nonlinear flexible arms and the asymptotic tracking of the periodic output trajectory was achieved. However, it requires the nontrivial solution of a set of PDEs.

Alternative approach to output tracking based on inversion was studied by Brockett and Mesarovic [3] and Silverman [9]. These linear results were extended to nonlinear real-analytic systems by Hirschorn [6] and Singh [10]. For a given desired output and a fixed initial condition, all these inversion algorithms produce causal inversions that are unbounded for nonminimum phase systems. To overcome these difficulties, Chen and Paden [11] developed a stable inversion approach to solve output tracking control of nonminimum phase systems. The down side is that stable inversion is noncausal.

In this paper, a causal inversion-based controller is designed for a one-link flexible manipulator system. Compared to stable inversion, causal inversion does not require preloading the links. Compared to the nonlinear regulation approach, causal inversion avoids the numerical intractability of nonlinear PDEs.

The remainder of this paper is organized as follows. In the next section, the class of reference trajectories under consideration is defined and the causal inversion problem is stated. Then a causal inversion solution for nonlinear nonminimum phase systems is presented. Section III describes the system dynamics of a one-link flexible manipulator, and the causal inversion approach is applied to design a tip trajectory tracking controller. Section IV contains the simulation results. Finally, concluding remarks are given in Section V.

II. Causal Inversion

Problem Statement First, consider a nonlinear system of the form

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

defined on a neighborhood X of the origin of $\Re^n$, with input $u \in \Re^m$ and output $y \in \Re^p$. $f(x)$ and $g_i(x)$ (the i^{th} column of $g(x)$) for $i = 1, 2, \cdots, m$ are smooth vector fields. And $h_i(x)$ for $i = 1, 2, \cdots, p$ are smooth functions on X, with $f(0) = 0$ and $h(0) = 0$. For such a system, the causal inversion problem is stated as follows [16]:

Causal Inversion Problem: Given a smooth reference output trajectory $y_d(t) \in L_1 \cap L_\infty$, with $y_d(t) \equiv 0$ for $t \leq 0$, find a control input $\bar{u}_d(t)$ and a state trajectory $\bar{x}_d(t)$ such that

(1) $\bar{u}_d$ and $\bar{x}_d$ are bounded, and

$$\bar{u}_d(t) \to 0, \quad \bar{x}_d(t) \to 0, \quad \text{as} \quad t \to \infty$$

(2) $\bar{x}_d(t)$ and $\bar{u}_d(t)$ are causal; that is, $\bar{x}_d(t) \equiv 0, \bar{u}_d(t) \equiv 0$ for $t \leq 0$, where $\bar{x}_d$ is the desired state trajectory, and $\bar{u}_d$ is the nominal control input.

(3) Exact output matching is achieved

$$h(\bar{x}_d(t)) = y_d(t) \tag{3}$$

(4) $\dot{\bar{x}}_d - f(\bar{x}_d) - g(\bar{x}_d)\bar{u}_d \to 0$ as $t \to \infty$.

Note that condition (1) ensures stability of the inverse solution. Condition (2) gives rise to the term "causal" inversion. In a strict sense, $\bar{u}_d$ is not a inverse solution, since applying $\bar{u}_d$ to the input of the system (1) and (2) will not lead to $y = y_d$ in general. The solution $\bar{u}_d$ and $\bar{x}_d$ are termed a causal "inverse" solution because of the condition (3). Condition (4) further ensures that $\bar{u}_d$ and $\bar{x}_d$ are indeed asymptotically inverse solutions to the dynamics (1) and (2).

The condition $y_d(t) \in L_1 \cap L_\infty$ can be replaced by a condition that requires $\dot{y}_d(t) \in L_1 \cap L_\infty$ so that y_d can approach same constant values as frequently happens in practical situations. This modification requires little modification of the general method. There is also a simple way to ensure that $y_d(t) \in L_1 \cap L_\infty$ and avoid modification of the method itself. Recognize that in practice any trajectory y_d will be bounded and has a finite duration for being nonzero. One can simply modify y_d after a sufficient settling time after t_f so that it comes back to 0.

Consider a nonlinear system of the form (1) and (2) with the same number m of inputs and outputs and

$$\begin{aligned} y &= (y_1, y_2, \cdots, y_m)^T \\ u &= (u_1, u_2, \cdots, u_m)^T \\ h(x) &= [h_1(x), h_2(x), \cdots, h_m(x)]^T \\ g(x) &= [g_1(x), g_2(x), \cdots, g_m(x)] \end{aligned}$$

Assume that the system has a well-defined relative degree $r = \{r_1, r_2, \cdots, r_m\}^T \in N^m$ at the equilibrium point 0, that is, in an open neighborhood of 0;

(i) for all $1 \le j \le m$, for all $1 \le i \le m$, for all $k < r_i - 1$ and $k \ge 0$, and for all x,

$$L_{g_j} L_f^k h_i(x) = 0 \tag{4}$$

(ii) the $m \times m$ matrix $\beta(x)$ with $\beta_{ij}(x) \triangleq L_{g_j} L_f^{(r_i-1)} h_i(x)$ is nonsingular.

Note that since the control u does not appear explicitly in Equation (2), we have $r_i \ge 1$ for all i, and $r_i - 1 \in N^m$ and the operation in the definition of β is well defined. Furthermore, we assume that $\sum r_i < n$ to avoid trial cases.

Under this assumption, the system can be partially linearized. To do this, we differentiate y_i until at least one u_i appears explicitly. This will happen at exactly the r_i^{th} derivative of y_i due to (4). Define $\xi_k^i = y_i^{(k-1)}$ for $i = 1, \cdots, m$ and $k = 1, \cdots, r_i$, and denote [13]

$$\begin{aligned} \xi &= (\xi_1^1, \xi_2^1, \cdots, \xi_{r_1}^1, \xi_1^2, \cdots, \xi_{r_2}^2, \cdots, \xi_{r_m}^m)^T \\ &= (y_1, \dot{y}_1, \cdots, y_1^{(r_1-1)}, y_2, \cdots, y_2^{(r_2-1)}, \cdots, y_m^{(r_m-1)})^T \end{aligned}$$

Choose η, an $n - \sum r_i$ dimensional function on $\Re^n$, such that $(\xi^T, \eta^T)^T = \psi(x)$ forms a change of coordinate with $\psi(0) = 0$ [1]. In this new coordinate system, the system dynamics of Equations (1) and (2) become

$$\begin{cases} \dot{\xi}_1^i = \xi_2^i \\ \vdots \\ \dot{\xi}_{r_i-1}^i = \xi_{r_i}^i \qquad \text{for } i = 1, \cdots, m \\ \dot{\xi}_{r_i}^i = \alpha_i(\xi, \eta) + \beta_i(\xi, \eta) u \end{cases}$$

$$\dot{\eta} = q_1(\xi, \eta) + q_2(\xi, \eta) u$$

which, in a more compact form, can be rewritten as:

$$y^{(r)} = \alpha(\xi, \eta) + \beta(\xi, \eta) u \tag{5}$$

$$\dot{\eta} = q_1(\xi, \eta) + q_2(\xi, \eta) u \tag{6}$$

where

$$\begin{aligned} \alpha(\xi, \eta) &= L_f^r h(\psi^{-1}(\xi, \eta)) \\ \beta(\xi, \eta) &= L_g^1 L_f^{r-1} h(\psi^{-1}(\xi, \eta)) \end{aligned}$$

where α and β are formed by using α_i and β_i as the i^{th} row of α and β respectively, and $\alpha(0,0) = 0$ since $f() = 0$. By the relative degree assumption, $\beta(\xi, \eta)$ is nonsingular, and the following feedback control law

$$u \triangleq \beta^{-1}(\xi, \eta)[v - \alpha(\xi, \eta)] \tag{7}$$

is well defined and partially linearizes the input-output dynamics relationship into a chain of integrators, $y^{(r)} = v$, where $v \in \Re^m$ is the new control input. For the inversion problem, we require $y(t) \equiv y_d(t)$ which leads to

$$v = y_d^{(r)}$$

$$\xi = \xi_d \triangleq (y_{d_1}, \dot{y}_{d_1}, \cdots, y_{d_1}^{(r_1-1)}, y_{d_2}, \cdots, y_{d_2}^{(r_2-1)}, \cdots, y_{d_m}^{(r_m-1)})^T \tag{8}$$

Equation (6) becomes the zero dynamics driven by the reference output trajectory,

$$\dot{\eta} = p(y_d^{(r)}, \xi_d, \eta) \tag{9}$$

where

$$\begin{aligned} p(y_d^{(r)}, \xi_d, \eta) \quad = \quad & q_1(\xi_d, \eta) + q_2(\xi_d, \eta) \beta^{-1}(\psi^{-1}(\xi_d, \eta)) \\ & [y_d^{(r)} - \alpha(\psi^{-1}(\xi_d, \eta))]. \end{aligned}$$

When the reference trajectories are zero, the reference dynamics become autonomous zero dynamics. Assume $\eta = 0$ is a hyperbolic equilibrium point of the autonomous zero dynamics. Linearizing the right hand side of Equation (9) at the equilibrium point $\eta = 0$ gives

$$\dot{\eta} = A\eta + b(t) \tag{10}$$

where

$$\begin{aligned} A &= \tfrac{\partial p}{\partial \eta}(y_d^{(r)}, \xi_d, \eta)|_{\eta=0,\ \xi_d=0,\ y_d^{(r)}=0} \\ b(t) &= p(y_d^{(r)}, \xi_d, \eta) - A\eta \end{aligned}$$

For a real matrix A, there exists an invertible $(n - \sum r_i) \times (n - \sum r_i)$ matrix P_1, such that $\bar{J} = P_1^{-1} A P_1$, where $\bar{J}$ is the real Jordan form of A. Therefore, with the coordinate transformation $\eta = P_1[\eta_s \ \ \eta_u]^T$, the reference dynamics in the new coordinate is also in real Jordan form. As a result, Equation (9) can be rewritten as

$$\begin{cases} \dot{\eta}_s = A_s \eta_s + B_s y_d^{(r)} + d_s(y_d^{(r)}, \xi_d, \eta_s, \eta_u) \\ \dot{\eta}_u = A_u \eta_u + B_u y_d^{(r)} + d_u(y_d^{(r)}, \xi_d, \eta_s, \eta_u) \end{cases} \tag{11}$$

where A_s has all eigenvalues in the open left-half plane with dimension n_s, A_u has all eigenvalues in the open right-half plane with dimension n_u, and $d_s(\cdot)$ and $d_u(\cdot)$ denote the higher-order-terms (H.O.T.) of the expression.

From (11), two dynamic equations are defined as follows:

$$\dot{\bar{\eta}}_s = A_s\bar{\eta}_s + B_s y_d^{(r)} + d_s(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u), \bar{\eta}_s(0) = 0 \quad (12)$$
$$\dot{\bar{\eta}}_u = A_u\bar{\eta}_u + B_u y_d^{(r)} + d_u(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u) + \bar{v}, \bar{\eta}_u(0) = 0 \quad (13)$$

where $\bar{\xi}_d = \xi_d$ and $\bar{v}$ is to be chosen to reach the asymptotic stability of (12) and (13). Here a causal inversion solution is provided, which is different from that in [16].

By selecting $\bar{v} = -2A_u\bar{\eta}_u - 2B_u y_d^{(r)} - 2d_u(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u)$, (13) becomes

$$\dot{\bar{\eta}}_u = -A_u\bar{\eta}_u - B_u y_d^{(r)} - d_u(y_d^{(r)}, \bar{\xi}_d, \bar{\eta}_s, \bar{\eta}_u), \;\; \bar{\eta}_u(0) = 0 \quad (14)$$

Thus both A_s and $-A_u$ are Hurwitz. The H.O.T. $d_s(\cdot)$ in (12) and $d_u(\cdot)$ in (14) are dominated by linear terms when restricting attention to a sufficiently small neighborhood of the equilibrium point. Then solving (12) and (14) yields solution $\bar{\eta}_s(t)$ and $\bar{\eta}_u(t)$. We assume $y_d^{(i)} \to 0$ for $i = 1, \cdots, r$ as $t \to \infty$. Furthermore, $\bar{\eta}_s(t) \to 0$ and $\bar{\eta}_u(t) \to 0$ as $t \to \infty$ are obtained. Also, $\bar{\xi}_d = 0$ for $t \leq 0$ and for $t \geq t_f$.

Since the system has a well-defined relative degree at the equilibrium point 0, $\psi(x) = [\xi^T \;\; \eta^T]^T = [\xi^T, \;\; [\eta_s \;\; \eta_u]P_1^T]^T$ defines a local diffeomorphism. Its inverse is $x = \phi(\xi, \eta)$. Define $\bar{\eta} = P_1[\bar{\eta}_s \;\; \bar{\eta}_u]^T$. Let

$$\bar{x}_d = \phi(\bar{\xi}_d, \bar{\eta}) \quad (15)$$
$$\bar{u}_d = \beta^{-1}(\bar{\xi}_d, \bar{\eta})[y_d^{(r)} - \alpha(\bar{\xi}_d, \bar{\eta})] \quad (16)$$

Then $\bar{x}_d$ and $\bar{u}_d$ are bounded, and $\bar{x}_d(t), \bar{u}_d(t) \to 0$ as $t \to \infty$. And by the definition of $\bar{\xi}, h(\bar{x}_d) = y_d$ and $L_f^i h(\bar{x}_d) = y_d^{(i)}$ for $i \leq r$ are obtained.

Note that in [16], the main focus is to seek a minimum $\bar{v}$, which is very difficult for nonlinear systems. However, in this paper, $\bar{v}$ is selected such that the mismatch between $\bar{\eta}_u(t)$ and $\eta_u(t)$ generated by stable inversion could be minimized.

Thus a casual inversion solution to nonlinear systems has been provided.

III. A One-Link Flexible Manipulator

A closed-loop controller for a one-link flexible manipulator is designed in this section using causal inversion.

A. Dynamics Model

A nonlinear one-link flexible manipulator model is obtained from [14]. A simple modeling technique divides the flexible link into rigid segments that are connected by elastic springs, where link deformation is concentrated. The following treatment will be limited to the case of two equal segments of uniform mass, moving along the horizontal plane. Let m and l denote the total link mass and length, k the spring elasticity, and u the input torque. With reference to Figure 1, θ_1 is the angular position of the link base, while θ_2 is the flexible variable. The dynamic equations are

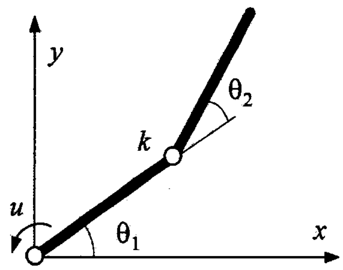

Fig. 1. A Simple one-link flexible manipulator

$$\begin{bmatrix} b_{11}(\theta_2) & b_{12}(\theta_2) \\ b_{12}(\theta_2) & b_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} c_1(\theta_2, \dot{\theta}_1, \dot{\theta}_1) \\ c_2(\theta_1, \dot{\theta}_1) + k\theta_2 + d_2\dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad (17)$$

with the elements of the inertia matrix $B(\theta_2)$ given by

$$b_{11}(\theta_2) = a + 2c\cos(\theta_2)$$
$$b_{12}(\theta_2) = b + c\cos(\theta_2)$$
$$b_{22} = b$$

and Coriolis and centrifugal terms

$$c_1(\theta_2, \dot{\theta}_1, \dot{\theta}_2) = -c(\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2)\sin\theta_2$$
$$c_2(\theta_2, \dot{\theta}_1) = c\dot{\theta}_1^2\sin\theta_2$$

where

$$a = 5m\ell^2/24, \;\; b = m\ell^2/24, \;\; c = m\ell^2/16$$

In (17), d_1 and d_2 are damping coefficients representing viscous friction at the joint and link structural (passive) dissipation, respectively. State equations can be obtained by setting $x = (\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2) \in \Re^4$.

The linearized expression of the end-effector angular position, as seen from the base,

$$y = \theta_1 + \frac{1}{2}\theta_2 \quad (18)$$

will be taken as controlled output for the system.

B. Controller Design

Compute the time-derivatives of the output until the input u appears explicitly. By setting $\ddot{y} = v$, solving for u yields

$$u = c_1(\theta_2, \dot{\theta}_1, \dot{\theta}_2) + d_1\dot{\theta}_1 + \frac{b_{11}(\theta_2) - 2b_{12}(\theta)_2}{2b_{22} - b_{12}(\theta)_2}$$
$$(c_2(\theta_2, \dot{\theta}_1) + k\theta_2 + d_2\dot{\theta}_2) + \frac{2detB(\theta_2)}{2b22 - b12(\theta_2)}v$$
$$= \alpha(x) + \beta(x)v$$

In the system after inversion, the input-output linearizing coordinates are $\tilde{x} = (y, \dot{y}, \theta_2, \dot{\theta}_2)$. $\Psi(x)$ is simply a linear transformation in state space, which could be expressed as follows:

$$\begin{bmatrix} y \\ \dot{y} \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \triangleq \Psi(x) \tag{19}$$

Then the zero dynamics driven by the reference output trajectory can be obtained by setting $y(t) \equiv y_d(t)$,

$$\begin{aligned} \ddot{\theta}_2 &= \frac{2(c_2(\dot{y}_d,\theta_2,\dot{\theta}_2)+k\theta_2+d_2\dot{\theta}_2)}{b_{12}(\theta_2)-2b_{22}} + \frac{2b_{12}(\theta_2)}{b_{12}(\theta_2)-2b_{22}}v \\ &= p(y_d, \dot{y}_d, \ddot{y}_d, \theta_2, \dot{\theta}_2) \end{aligned} \tag{20}$$

The zero-dynamics of the system is then obtained by setting $y(t) \equiv 0$

$$\ddot{\theta}_2 = -\frac{((c/2)\dot{\theta}_2^2 \sin\theta_2 + 2(k\theta_2 + d_2\dot{\theta}_2)}{b - c\cos\theta_2} \tag{21}$$

It is easy to see that this two-dimensional dynamics is unstable in the first approximation.

Rewriting the differential equation (20) in state-space form:

$$\begin{bmatrix} \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = A_\eta \begin{bmatrix} \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + B_\eta \ddot{y}_d + \bar{b}(t) \tag{22}$$

where

$$\begin{aligned} A_\eta &= \frac{\partial p}{\partial \eta}(y_d, \dot{y}_d, \ddot{y}_d, \theta_2, \dot{\theta}_2)|_{y_d=0,\dot{y}_d=0,\theta_2=0,\dot{\theta}_2=0} \\ &= \begin{bmatrix} 0 & 1 \\ \frac{2k}{c-b} & \frac{2d_2}{c-b} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B_\eta &= \frac{\partial p}{\partial \ddot{y}_d}(y_d, \dot{y}_d, \ddot{y}_d, \theta_2, \dot{\theta}_2)|_{y_d=0,\dot{y}_d=0,\theta_2=0,\dot{\theta}_2=0} \\ &= \begin{bmatrix} 0 \\ \frac{2(b+c)}{c-b} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \bar{b}(t) &= p(y_d, \dot{y}_d, \ddot{y}_d, \theta_2, \dot{\theta}_2) - A\eta[\theta_2, \dot{\theta}_2]^T - B_\eta \ddot{y}_d \\ &= \frac{2(c\dot{\theta}_1^2 \sin\theta_2(\dot{y}_d,\theta_2,\dot{\theta})+k\theta_2+d_2\dot{\theta}_2)}{ccos(\theta_2)-b} + \frac{2(b+ccos(\theta_2))}{ccos(\theta_2)-b}\ddot{y}_d \\ &\quad - \frac{2k\theta_2+2d_2\dot{\theta}_2+2(b+c)\ddot{y}_d}{c-b} \end{aligned}$$

With a linear transformation

$$\begin{bmatrix} \theta_2 \\ \dot{\theta}_2 \end{bmatrix} = P_1 \begin{bmatrix} \eta_s \\ \eta_u \end{bmatrix} \tag{23}$$

equation (22) is transformed into

$$\begin{cases} \dot{\eta}_s = A_s\eta_s + B_s\ddot{y}_d + d_s(y_d, \dot{y}_d, \ddot{y}_d, \eta_s, \eta_u) \\ \dot{\eta}_u = A_u\eta_u + B_u\ddot{y}_d + d_u(y_d, \dot{y}_d, \ddot{y}_d, \eta_s, \eta_u) \end{cases} \tag{24}$$

where

$$\begin{bmatrix} A_s & 0 \\ 0 & A_u \end{bmatrix} = P_1^{-1} A_\eta P_1$$

$$\begin{bmatrix} B_s \\ B_u \end{bmatrix} = P_1^{-1} B_\eta$$

$$\begin{bmatrix} d_s(\cdot) \\ d_u(\cdot) \end{bmatrix} = P_1^{-1}\bar{b}(t)$$

Two dynamic equations are defined as follows:

$$\begin{cases} \dot{\bar{\eta}}_s = A_s\bar{\eta}_s + B_s y_d^{(r)} + d_s(y_d, \dot{y}_d, \ddot{y}_d, \bar{\eta}_s, \bar{\eta}_u), \bar{\eta}_s(0) = 0 \\ \dot{\bar{\eta}}_u = A_u\bar{\eta}_u + B_u y_d^{(r)} + d_u(y_d, \dot{y}_d, \ddot{y}_d, \bar{\eta}_s, \bar{\eta}_u) + \bar{v}, \bar{\eta}_u(0) = 0 \end{cases} \tag{25}$$

By choosing $\bar{v} = -2A_u\bar{\eta}_u - 2B_u\ddot{y}_d - 2d_u(y_d, \dot{y}_d, \ddot{y}_d, \bar{\eta}_s, \bar{\eta}_u)$ and solving the following equations:

$$\begin{cases} \dot{\bar{\eta}}_s = A_s\bar{\eta}_s + B_s\ddot{y}_d + d_s(y_d, \dot{y}_d, \ddot{y}_d, \bar{\eta}_s, \bar{\eta}_u), \bar{\eta}_s(0) = 0 \\ \dot{\bar{\eta}}_u = -2A_u\bar{\eta}_u - 2B_u\ddot{y}_d - 2d_u(y_d, \dot{y}_d, \ddot{y}_d, \bar{\eta}_s, \bar{\eta}_u), \quad \bar{\eta}_u(0) = 0 \end{cases} \tag{26}$$

Thus the bounded $\bar{\eta}_s$ and $\bar{\eta}_u$ can be obtained. Furthermore, $[\theta_{2d} \;\; \dot{\theta}_{2d}]^T = P_1[\bar{\eta}_s \;\; \bar{\eta}_u]^T$. Then it follows:

$$\begin{bmatrix} \theta_{1d} \\ \theta_{2d} \\ \dot{\theta}_{1d} \\ \dot{\theta}_{2d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_d \\ \dot{y}_d \\ \theta_{2d} \\ \dot{\theta}_{2d} \end{bmatrix} \tag{27}$$

The nominal input is then calculated by

$$\begin{aligned} \bar{u}_d = c_1(\cdot) + d_1\dot{\theta}_{1d} + \frac{b_{11}(\theta_{2d})-2b_{12}(\theta_{2d})}{2b_{22}-b_{12}(\theta_{2d})} \\ (c_2(\cdot) + k\theta_{2d} + d_2\dot{\theta}_{2d}) + \frac{2detB(\theta_{2d})}{2b22-b12(\theta_{2d})}\ddot{y}_d \end{aligned}$$

The controller is composed in the following structure

$$u = \bar{u}_d + K(\bar{x}_d - x) \tag{28}$$

where $\bar{x}_d$ denotes the state variables of the forward dynamics, $\bar{x}_d = (\theta_{1d}, \theta_{2d}, \dot{\theta}_{1d}, \dot{\theta}_{2d})$. The feedback gain, K, is chosen so as to stabilize the forward dynamics linearized at the origin. With the parameters listed in the next section, $K = [21.6660 \;\; 6.5543 \;\; 4.6608 \;\; 1.4568]$ is a simple choice from pole placement.

IV. Simulation Results

The parameters for the one-link flexible manipulator were chosen the same as in De Luca [14] $l = 1$ m, $m = 0.2$ kg, $k = 5$ Nm/rad, and $d_1 = d_2 = 0.01$ $Nm \cdot sec/rad$.

Let the desired output trajectory be defined as follows:

$$y_d = \begin{cases} \pi^2(\frac{1}{2\pi t} - \frac{1}{(2\pi)^2 \sin(2\pi t)}), & 0 \le t \le t_f \\ \frac{\pi}{2}, & t > t_f \end{cases}$$

as shown by the solid curve in Figure 2.

For the given trajectory, the following data were used: $y_0 = 0^o, y_f = 90^o$. The initial conditions are $\theta_1 = \theta_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0$.

Following the controller design strategy described in Section III-B, all the simulation results are shown in Figures (2-7).

Figure 2 shows the desired and actual trajectories for the output. The output tracking error is shown in Figure 3. The maximum error during transients is relatively small (around 0.7^o).

The difference of nominal control input generated by stable inversion and causal inversion is shown in Figure 4. Since the stable inversion needs preloading, Figure 4 shows that the nominal control input error is not zero at $t = 0$.

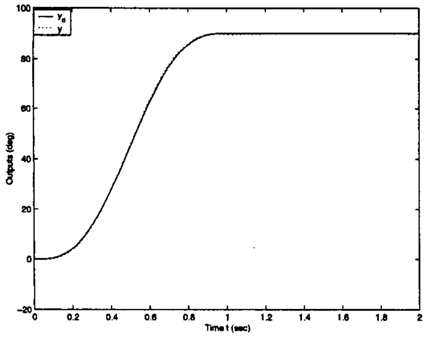

Fig. 2. Desired (solid) and actual output trajectories for Case 1

Notice that the nominal control input error is relatively small. The tracking performance for the angular position and the link deflection can be seen from Figure 5 to Figure 8 respectively. The deflection remains limited and reaches a peak value of around 2.5°.

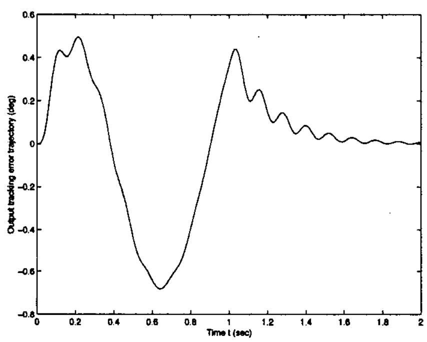

Fig. 3. Output tracking error trajectory

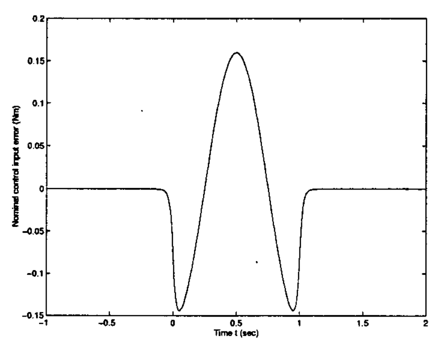

Fig. 4. Nominal control input error

Causal inversion applied to a one-link flexible manipulator has the advantages of not requiring the preloading for the links (as does stable inversion), as well as eliminating the need to solve the nontrivial solution of a set of partial differential algebraic equations (as required by the nonlinear regulation approach).

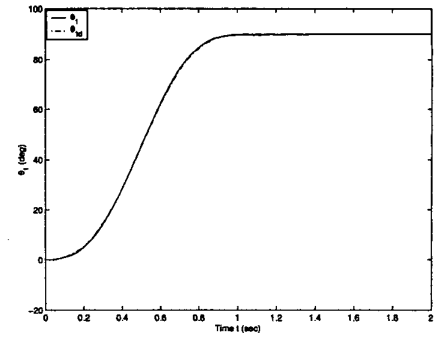

Fig. 5. Desired (solid) and actual angular position trajectory

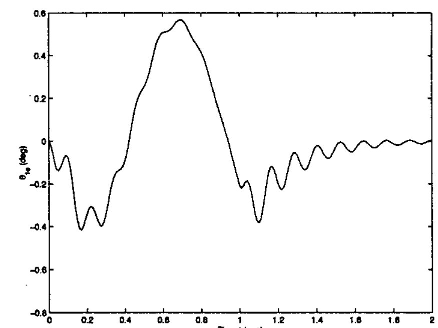

Fig. 6. Angular position tracking error trajectory

V. Conclusion

In this paper, a causal inversion-based controller is designed for tip trajectory tracking of a flexible link robot manipulator. This approach generates the causal nominal input torque that enables reproduction of the desired trajectory, while not requiring the usual preloading of the flexible link to the proper initial state. At the same time, this new approach avoids solving the nontrivial PDEs. Simulation results demonstrate that the causal inversion approach is very effective for obtaining output tracking for flexible manipulators. Future work will continue on new applications of causal inversion.

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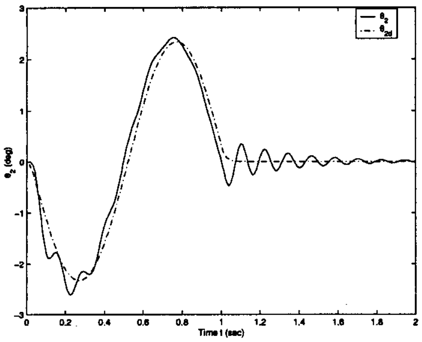

Fig. 7. Desired (solid) and actual link deflection trajectory

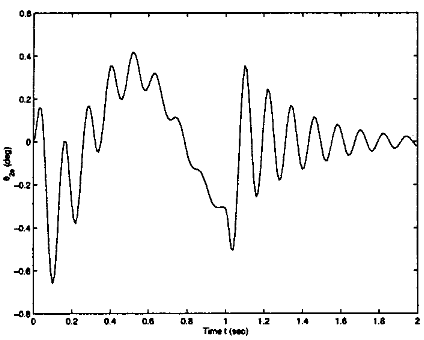

Fig. 8. Link deflection tracking error trajectory

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