Robust Inversion-based Learning Control for Nonminimum Phase Systems

Xuezhen Wang and Degang Chen

Iowa State University
Department of Electrical and Computer Engineering
Ames, IA, 50011, U. S. A.
E-mail:xzwang@iastate.edu, djchen@iastate.edu

Abstract— **This paper introduces a new robust inversion-based learning algorithm for the repetitive tracking control of a class of unstable nonminimum phase systems. After each repetitive trial, the Least-Squares method is used to estimate the system parameters. The output tracking error and the identified system model are used through stable inversion to find the feed forward input, together with the desired state trajectories, for the next trial. A robust controller is used in each trial to ensure the stability of the systems and the output tracking error convergence. Sufficient conditions for learning control convergence are provided. Simulation studies on the systems with gain uncertainty and time constant uncertainty are also presented. In addition, simulation results demonstrate that the proposed learning control scheme is very effective in reproducing the desired trajectories.**

Keywords— **Learning Control, Inversion, Nonminimum Phase, Robust Control.**

I. Introduction

Iterative learning control (ILC) is a feed forward control approach aimed at achieving high performance output tracking control by "learning" from past experience so as to eliminate the repetitive errors from future execution [3]. The concept of iterative learning for generating the optimal input to a system was first introduced by Uchiyama [10]. Arimoto *et al.*[4] then developed the idea and first proposed a learning control method for linear time-varying, continuous-time systems. Moore [3] modified the Arimoto method and extended it to systems with relative degree larger than one. Furuta and Yamakita [12] presented a modification of Moore's method. Their algorithm provided convergence in the sense of the L_2 norm but required the complete knowledge of the adjoint system, which is equivalent to needing the complete knowledge of the system dynamics.

Hauser presented a nonlinear version of Arimoto's method for a class of nonlinear systems [15] and provided sufficient conditions for its uniform convergence. Hauser's method is more general than Arimoto's method. For a more specific structure, Sugie and Ono [9] provided the learning controller given by a linear time-varying system and showed its convergence under some conditions. Kuc *et al.* [7] presented an ILC scheme for a class of nonlinear dynamic systems. Saab [8] presented sufficient conditions for the convergence of P-type learning algorithm for a class of time-varying, nonlinear systems. Jang *et al.* [6] proposed an ILC method to achieve precise tracking control of a class of nonlinear systems. Comprehensive analysis, design, and applications of ILC could be found from [3] and [1].

Although existing learning algorithms have been theoretically proven to provide output error convergence with successful applications, many such algorithms have practical difficulties with nonminimum phase systems. Amann and Owens [11] showed that a zero of the plant in the RHP caused very slow convergence of the input sequence and resulted in a nonzero error for some iterative control algorithms . To remove the minimum phase requirement, Gao and Chen [13] developed a new adaptive learning algorithm for stable linear systems based on "stable inversion". Based on Gao and Chen's algorithm, Ghosh and Paden [14] developed an ILC algorithm for nonlinear nonminimum phase plants with input disturbances and output sensor noise. The algorithms developed by Ghosh and Paden assume that the plants are stable and assume the system parameter are known. Wang and Chen [16] presented an adaptive learning control algorithm for unstable nonminimum phase systems. In this paper, a robust learning algorithm that can guarantee the learning control convergence is developed to work for unstable nonminimum phase systems . Simulation studies are presented to show the effectiveness of the proposed robust learning algorithm.

The remainder of this paper is organized as follows: In the next section, a class of desired trajectories under consideration is defined and the problem of ILC is stated. The learning control convergence issue is also addressed. Section III presents the new robust learning control law and a sufficient condition for the convergence property of the proposed ILC. Section IV applies the proposed robust ILC to the linear systems with gain uncertainty and time constant uncertainty. Section V shows the simulation results for these two types of linear systems. Finally, some conclusions are given in Section VI.

II. Framework and Problem Statement

Consider a nonlinear time varying plant model in the k^{th} trial:

$$y_k(t) = \Phi(x_k(t), \theta, u_k(t)) \tag{1}$$

where, for all $t \in [0, T]$, $x_k(t) \in \Re^n, u_k(t) \in \Re^m, y_k(t) \in \Re^p$. And θ is a parameter vector.

In addition, we make the following assumptions:

TA1M1

(A1) The system has a well-defined relative degree $r = (r_1, \cdots, r_m)^T$ that is known. The linearization of the system about an equilibrium point, which is assumed to be the origin WLOG, is completely controllable.

(A2) The order of the system, n, is known.

(A3) The system parameter vector θ is unknown or known incompletely.

(A4) A desired output trajectory is given and is a sufficiently smooth function of t satisfying $y_d(t) = 0$ for any $t \in (-\infty, 0] \cup [T, \infty)$and finite for any $t \in (0, T)$, where $T > 0$.

(A5) The system can be represented in terms of control input $u_k(\cdot)$ and output $y_k(\cdot)$ in the k^{th} trial by means of a nonlinear time-varying operator Φ as follows:

$$y_k(\cdot) = \Phi\{u_k(\cdot)\} \tag{2}$$

And the operator $\Phi\{\cdot\}$ is uniformly globally Lipschitz in $u_k$on the interval $[0, T]$. That is, $\|\Phi u_k - \Phi u_{k+1}\| \leq L\|u_k(t) - u_{k+1}(t)\|$, $\forall t \in [0, T]$ with a Lipschitz constant $0 \leq L < \infty$.

Iterative Learning Control Problems:
Given a desired output trajectory $y_d(t)$ and a tolerance error bound ϵ for a class of system (1) and (2), starting from an arbitrary continuous initial control input $u_0^d(\cdot)$ and initial state $x_0^d(\cdot)$, iterative learning control will try to find a sequence of desired state trajectories $x_k^d(\cdot)$ and desired control inputs $u_k^d(\cdot)$, which when applied to the system, produces an output sequence $y_k(\cdot)$ such that

(1) $\|y_d(\cdot) - y_k(\cdot)\|_\infty \leq \epsilon$, as $k \to \infty$, where k is the trial number and $\|f\|_\infty = \sup_{t \in [0,T]} \|f(t)\|$.

(2) $\|u_k^d(t)\| \leq \epsilon, \|x_k^d(t)\| \leq \epsilon, \forall t \in (-\infty, 0] \cup [T, +\infty)$.

(3) $u_k^d(t)$, $x_k^d(t)$, $u_k(t)$, and $x_k(t)$ are uniformly bounded.

In this dynamic process, the functions have two arguments: continuous time $t$and the trial number k. In the sequel, it is assumed that the variation of the operator over two consecutive trials are slow and can be neglected. Then the operator obtained by the identification performed in the k^{th} trial can be used to determine the input for the $(k+1)^{th}$ trial. This general description of the problem allows a simultaneous description of linear or nonlinear dynamics, continuous or discrete plant, and time-invariant or time-varying systems.

When applying a linear ILC, however, the plant must fulfill the following conditions: (1) The desired trajectory $y_d(t)$ is identical for every trial and satisfies Assumption (A4). (2) Each trial has the fixed period T. (3) The system parameters are fixed or very slowly time-varying.

At any trial k, define a tracking error to be $e_k = y_d - y_k$. Learning control convergence means that $\|e_k\| \to 0$ as $k \to \infty$. The λ-norm defined in Arimoto *et al.* [4] has been adopted in many papers [3] as the topological measure in the proof of the convergence property for a newly proposed ILC. The formal definition [4] of the λ-norm for a function $f : [0, T] \to \Re^n$ is given by

$$\|f(\cdot)\|_\lambda \triangleq \sup_{t \in [0,T]} e^{-\lambda t} \|f(t)\| \tag{3}$$

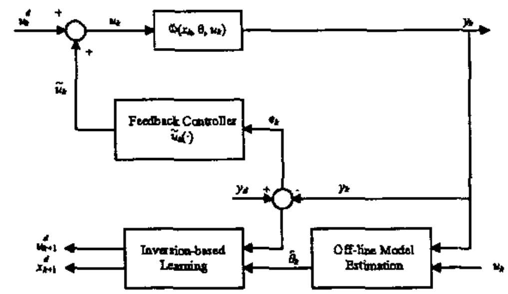

Fig. 1. Block diagram of inversion-based learning control system

It is easily observed that $\|f\|_\lambda \leq \|f\|_\infty \leq e^{\lambda T} \|f\|_\lambda$ for $\lambda > 0$, where $\|f\|_\infty \triangleq \sup \|f(t)\|_\infty$, implying the $\lambda-$norm is equivalent to the sup norm.

III. Inversion-Based Learning Controller Design

In Section II, we have given the general setup for learning control. In this section, a robust learning controller will be presented. The block diagram of the robust learning system is shown in Figure 1.

The proposed robust learning control strategy has three components: a parameter estimator, a stable inverse system, and a robust feedback controller. The parameter estimator is in charge of "learning" the parameterized model of the system. During each trial, the input and output trajectories are recorded. Then offline Least-Squares method [2] is applied to obtain the optimal estimate of parameters. Also obtained during each trial is the output tracking error signal. This error signal and the estimated model are used by the stable inverse system to learn the optimal input signal for the next trial. Although the estimated model may be nonminimum phase which normally leads to unbounded inverse solutions, stable inversion guarantees a unique and bounded inverse solution. This "learning" action is done "off-line" between two consecutive trials. Afterwards, the new feed forward input is used by a feedback controller to stabilize the system and to ensure regulation of the tracking error. The same feedback control algorithm is used during every trial.

In the following, sufficient conditions of learning convergence for linear systems are to beaddress ed. The stable inversion solution to nonminimum phase systems is provided as well.

A. *Sufficient condition of learning convergence for linear systems*

One of the advantages for linear systems is that one can obtain an explicit relation between $\|e_{k+1}\|_\infty$ and $\|e_k\|_\infty$.

For LTI systems, the learning control update law is chosen as

$$u_{k+1}^d = u_k^d + \hat{H}_k e_k$$

where $\hat{H}_k$ is a linear operator.

A fixed controller could be chosen. Thus, the output tracking error is described as follows:

$$\begin{aligned} e_{k+1} &= y_d - y_{k+1} \\ &= e_k + Gu_k - Gu_{k+1} \\ &= (I - G\hat{H}_k)e_k + G(\tilde{u}_k - \tilde{u}_{k+1}) \\ &= (I - G\hat{H}_k)e_k + G(Ke_k - Ke_{k+1}) \\ &= (I + GK - G\hat{H}_k)e_k - GKe_{k+1} \end{aligned}$$

it yields

$$\begin{aligned} e_{k+1} &= (I + GK)^{-1}(I + GK - G\hat{H}_k)e_k \\ &= I - (I + GK)^{-1}G\hat{H}_k)e_k \end{aligned}$$

Furthermore, taking the norms yields

$$\|e_{k+1}\|_\infty = \|I - (I + GK)^{-1}G\hat{H}_k)\|_\infty \|e_k\|_\infty$$

Then the sufficient condition for learning convergence is

$$\|I - (I + GK)^{-1}G\hat{H}_k)\|_\infty \leq \rho < 1 \tag{4}$$

with $\rho \in (0, 1)$.

There are several options for choosing $\hat{H}_k$ and controller K. Among these options, $\hat{H}_k$can be selected as the stable inverse of $(I + \hat{G}K)^{-1}\hat{G}$. And K can be chosen as a robust controller if the systems have some uncertainties.

In the following section, a solution to stable inversion of linear nonminimum phase systems is presented.

B. *Solution to stable inversion of nonminimum phase systems*

Consider a LTI system in the form:

$$\sum\nolimits_1 : \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

If $\hat{G}$ is nonminimum phase, this will lead to unbounded solutions. The stable inversion theory [5] provides an avenue to overcome this difficulty. The procedure to obtain a unique stable inverse solution $u^d = \hat{H}y_d$ is illustrated below. There are four steps:

(1) Find the time-domain state-space model of $\hat{G}$:

Since $\hat{G}u^d = \hat{G}\hat{H}y_d = y_d$, a state-space representation of $\hat{G}$ yields

$$\begin{aligned} \dot{x}^d(t) &= \hat{A}x^d(t) + \hat{B}u^d(t) && (5) \\ y_d(t) &= \hat{C}x^d(t) + \hat{D}u^d(t) && (6) \end{aligned}$$

where x^d is the state and $\hat{A}, \hat{B}, \hat{C}$, and $\hat{D}$ are matrices with suitable sizes.

(2) Find its inverse in state space:

Differentiate $y_d(t)$ until u^d appears explicitly in the right hand side. Solvefor u^d and substitute into (5) and (6) to obtain

$$\begin{aligned} \dot{x}^d(t) &= \bar{A}x^d(t) + \bar{B}y_d^{(r)}(t) && (7) \\ u^d(t) &= \bar{C}x^d(t) + \bar{D}y_d^{(r)}(t) && (8) \end{aligned}$$

where $\bar{A}, \bar{B}, \bar{C}$, and $\bar{D}$ are defined according to the substitution.

(3) Decompose the inverse system into center, stable, and unstable subsystems:

Perform a change of variables so that

$$x^d = Pz = P[z^c, z^s, z^u]^T \tag{9}$$

which leads to

$$\begin{aligned} \dot{z}^c &= A^c z^c + B^c y_d^{(r)} && (10) \\ \dot{z}^s &= A^s z^s + B^s y_d^{(r)} && (11) \\ \dot{z}^u &= A^u z^u + B^u y_d^{(r)} && (12) \\ u^d &= [C^c \ \ C^s \ \ C^u][z^c \ \ z^s \ \ z^u]^T + \bar{D}y_d^{(r)} && (13) \end{aligned}$$

where A^c, A^s, and A^u are real Jordan matrices of suitable dimensions; A^c has reigen values at zero; A^s has all eigenvalues in the open left-half plane; A^u has all eigenvalues in the open right-half plane.

(4) Obtain the stable inverse system:

Pick the transformation matrix P so that the center subsystem is a simple chain of r integrators. Solve that and impose two boundary conditions on the stable and unstable subsystems to yield

$$z^c = [y_d, \dot{y}_d, \cdots, y_d^{(r-1)}]^T \tag{14}$$

$$\dot{z}^s = A^s z^s + B^s y_d^{(r)}, t \geq 0; z^s(t) = 0, \ \ \forall\, t \leq 0 \tag{15}$$

$$\dot{z}^u = A^u z^u + B^u y_d^{(r)}, t \leq T; z^u(t) = 0, \ \ \forall\, t \geq T \tag{16}$$

These together with (9) and (13) define the desired stable inverse system.

The stable inverse system always yields bounded solutions for bounded and smooth y_d. This can be clearly seen from (14, 15, 16) since the center solution z^c is clearly bounded, the stable subsystem is in the forward time, and the unstable subsystem is in the reverse time, all leading to bounded solutions.

Based on the stable solutions outline presented above, to facilitate iterative learning, the inversion process is slightly modified as follows.

Since G is unknown and $\hat{G}_k$ is the best estimate model after the k^{th} trial, one would select

$$\begin{aligned} u_{k+1}^e &= \hat{H}_k e_k && (17) \\ x_{k+1}^e &= P_k z = P_k[z^c, z^s, z^u]^T && (18) \end{aligned}$$

so that $e_k = \hat{G}_k u_{k+1}^e$. Then the learning algorithm becomes

$$u_{k+1}^d = u_k^d + u_{k+1}^e \tag{19}$$

There are various methods to design the feedback controller. For systems with uncertainties, a robust controller will be chosen to stabilize the systems.

In the following section, the implementation of the robust learning algorithm for two types of LTI systems is presented.

IV. Learning Control of Linear Systems with Uncerntainty

Sufficient conditions for learning control convergence for linear systems with gain uncertainty and time constants uncertainty are provided in this section.

A. Learning Control of Systems with Gain Uncertainty

In this section, the following type of linear nonminimum phase system with gain uncertainty is considered. Let the set of possible plants be

$$G_p(s) = k_p G_0(s), \qquad k_{min} < k_p < k_{max}$$

where $G_0(s) = \frac{s-z}{s^2+as+b}$ with $z > 0, a < 0, b > 0$, and $b \geq -az$.

For the above system, the uncertainty can be expressed as the following multiplicative uncertainty: $G_p(s) = \bar{k}(1+r\Delta)G_0(s) \quad |\Delta| \leq 1$ where $\bar{k} = \frac{k_{max}+k_{min}}{2}$ and $r = \frac{k_{max}-k_{min}}{k_{max}+k_{min}}$.

The closed-loop transfer function is given as

$$\frac{k_p(s-z)}{s^2+(a+k_pK)s+b-k_pKz}$$

To guarantee the stability of the system, the following inequalities should be satisfied:

$$\begin{cases} k_pKz > -az \\ k_pKz < b \end{cases}$$

The robust controller can be chosen as

$$K = \frac{\sqrt{-abz}}{\bar{k}_p z} \tag{20}$$

which means

$$\begin{cases} \sqrt{-abz}(1-r) > -az \\ \sqrt{-abz}(1+r) < b \end{cases}$$

Thus, to guarantee the stability, r should satisfy

$$r < \min(1 + \frac{az}{\sqrt{-abz}}, \frac{b}{\sqrt{-abz}} - 1) \tag{21}$$

For a specific example, setting $a = -1, z = 3, b = 12, k_{min} = 0$, and $k_{max} = 4$, the following system is considered

$$G_p(s) = k_p \frac{s-3}{s^2-s+12}, \qquad 0 < k_p < 4 \tag{22}$$

where $k_p = 2.15$.

This system has a zero at 3, and has poles at $0.5 \pm 3.4278i$. The causal reference output trajectory is given by:

$$y_d = \begin{cases} 5 - 5\cos(0.4\pi t), & t \in [0, 5] \\ 0, & \text{otherwise} \end{cases}$$

as shown by the solid curve in Figure 2. Then by (21), to guarantee the systems stability,r should satisfy $r < 0.5$.

If $\hat{H}_k$ is chosen as the stable inverse of $(I+\hat{G}_pK)^{-1}\hat{G}_p$, then by (4) and (20), to guarantee the learning convergence, the following condition should be satisfied,

$$\begin{aligned}
&\|1 - (1+G_pK)^{-1}G_p\hat{H}_k\|_\infty \\
&= \|1 - \tfrac{k_p(s-3)}{s^2-s+12+Kk_p(s-3)} \tfrac{s^2-s+12+K\hat{k}_p(s-3)}{\hat{k}_p(s-3)}\|_\infty \\
&= \|1 - \tfrac{k_p}{\hat{k}_p} - \tfrac{k_pK(\hat{k}_p-k_p)}{\hat{k}_p} \tfrac{s-3}{s^2-s+12+K\hat{k}_p(s-3)}\|_\infty \\
&\leq 2r + 4r(1+r)\|\tfrac{s-3}{s^2-s+12+K\hat{k}_p(s-3)}\|_\infty \\
&\leq \rho < 1
\end{aligned}$$

Hence the range for r to guarantee the learning convergence is $r \in [0, 0.092]$. Combined with $r < 0.5$, to guarantee both the learning convergence and the stability of the system, r should satisfy $r \in [0, 0.092]$. Thus $\hat{k}_p \in [\bar{k}(1-r), \bar{k}(1+r)]$, i.e., $\hat{k}_p \in [1.816, 2.184]$, which means the estimated parameter $\hat{k}_p$ should be restricted in the above range in order to guarantee the learning convergence.

B. Learning Control of Systems with Time Constant Uncertainty

In this section, the following single-input single-output linear nonminimum phase system with time constant uncertainty is considered. Let a set of plants are given by

$$G_p(s) = \frac{G_0(s)}{\tau_p s+1}, \qquad \tau_{min} < \tau_p < \tau_{max}$$

where $G_0(s) = \frac{s-z}{s-p}$ with $z - p \geq \frac{\tau_{max}+\tau_{min}}{2}pz > 0$ and $\tau_{min} \geq 0$. For the abovesys tem, the uncertainty can be expressed as the following inverse multiplicative uncertainty:

$G_p(s) = \frac{G_0(s)}{\bar{\tau}(1+r\Delta)} \quad |\Delta| \leq 1$ where $\bar{\tau} = \frac{\tau_{max}+\tau_{min}}{2}$ and $r = \frac{\tau_{max}-\tau_{min}}{\tau_{max}+\tau_{min}}$. The closed-loop transfer function is given as

$$\frac{s-z}{\tau_p s^2+(1-\tau_p p+K)s-p-Kz}$$

To guarantee the stability of the system, the following inequalities should be satisfied:

$$\begin{cases} k_pKz > -az \\ k_pKz < b \end{cases}$$

The robust controller can be chosen as

$$K = -\frac{\sqrt{pz[1-\bar{\tau}(1+r)p]}}{z} \tag{23}$$

Then it yields

$$\begin{cases} r < \frac{z-p}{\bar{\tau}pz} - 1 \\ 1 - \bar{\tau}(1+r)p > 0 \end{cases}$$

Thus, to guarantee the stability, r should satisfy

$$r < \min(\frac{z-p}{\bar{\tau}pz} - 1, \frac{1}{\bar{\tau}p} - 1) \tag{24}$$

For a specific example, setting $z = 4, p = 1, k_{min} = 0$, and $k_{max} = 1$, the following system is considered

$$y = \frac{s-4}{(\tau_p s+1)(s-1)}u, \qquad 0 < \tau_p < 1 \tag{25}$$

where $\tau_p = 0.473$. This system has a zero at 4, and has poles at -2.1142 and 1. The causal reference output trajectory is given by:

$$y_d = \begin{cases} 10 - 10\cos(0.2\pi t), & t \in [0, 10] \\ 0, & \text{otherwise} \end{cases}$$

as shown by the solid curve in Figure 5.

Then by (24), to guarantee the systems stability, r should satisfy

$$r < 0.5 \tag{26}$$

If $\hat{H}_k$ is chosen as the stable inverse of $(I+\hat{G}_p K)^{-1}\hat{G}_p$, then by (4) and (23), to guarantee the learning convergence, the following condition should be satisfied,

$$\begin{aligned}
&\|1-(1+G_pK)^{-1}G_p\hat{H}_k\|_\infty \\
&= \|1 - \tfrac{s-4}{(\tau_p s+1)(s-p)+K(s-4)} \tfrac{(\hat{\tau}_p s+1)(s-p)+K(s-4)}{s-4}\|_\infty \\
&= \|1 - \tfrac{\hat{\tau}_p}{\tau_p} - (1-\tfrac{\hat{\tau}_p}{\tau_p})\tfrac{(1+K)s-1-4K}{(\tau_p s+1)(s-1)+K(s-4)}\|_\infty \\
&\le 2r + 2r\|\tfrac{(1+K)s-1-4K}{(\tau_p s+1)(s-1)+K(s-4)}\|_\infty \\
&\le \rho < 1
\end{aligned}$$

Thus the range for r to guarantee the learning convergence is $r \in [0, 0.273]$. Combined with (26), to guarantee both the learning convergence and the stability of the system, r should satisfy $r \in [0, 0.273]$. Hence $\hat{\tau}_p \in [\bar{\tau}(1-r), \bar{\tau}(1+r)]$, i.e., $\hat{\tau}_p \in [0.3635, 0.6365]$, which means the estimated parameter $\hat{\tau}_p$ should be restricted in the above range in order to guarantee the learning convergence.

V. Simulation results

In this section, two specific examples (22) and (25) are simulated. Here, suppose only output and input signals can be measured. And there is random noise on output measurement, which has mean 0 and deviation 0.01.

For both two examples, three cases are simulated. For all the three cases, the least-squares method is used to estimate the unknown parameter. For Case 1, take the initial condition within the range. The estimated parameter is always enforced within the range by projection. For Case 2, take the initial condition beyond the range. The true estimated parameter is used without projection. For Case 3, take the initial condition beyond the range. The estimated parameter is always enforced outside the range.

The initial conditions of the unknown parameters for these two examples are shown in the following table:

TABLE I
Initial Condition of the Unknown Parameters for Two Examples.

	Example 1($\hat{k}_p$)	*Example* 2($\hat{\tau}_p$)
Case 1	1.98	0.6
Case 2	6	1.5
Case 3	6	1.5

Given an initial input $u_0^d(t) = 0$, simulation results for the trial $k = 1$ and $k = 2$ are shown in Figure(2-4) for Example 1 and shown in Figure(5-7) for Example 2.

For both two examples, at the 2^{nd} trial, the output $y_2(t)$ converges to the desired $y_d(t)$ exactly shown by the dotted curve for Case 1 and Case 2. But the output $y_2(t)$ diverges tremendously for Case 3. Table II and Table III shows the infinity norm of the output tracking error at each trial for three cases for Example 1 and Example 2

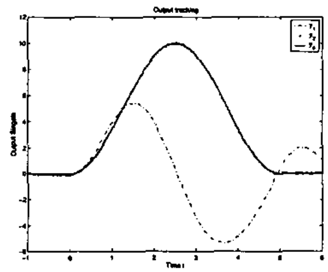

Fig. 2. Tracking of nonminimum phase systems with gain uncertainty for Case 1

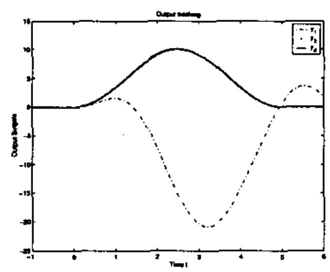

Fig. 3. Tracking of nonminimum phase systems with gain uncertainty for Case 2

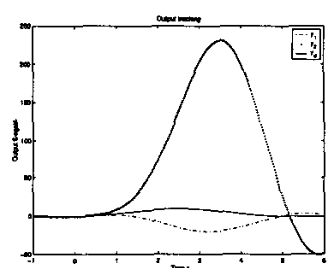

Fig. 4. Tracking of nonminimum phase systems with gain uncertainty for Case 3

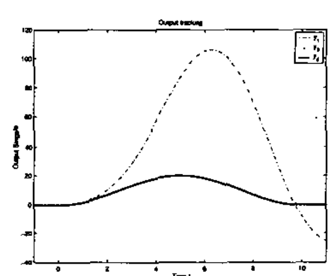

Fig. 5. Tracking of nonminimum phase systems with time constant uncertainty for Case 1

respectively. The infinity norm of the output tracking error decreases for Case 1 and Case 2, while it increases

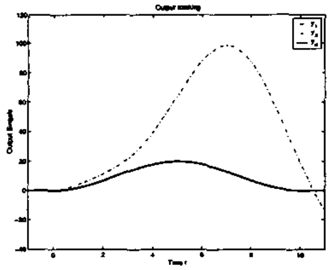

Fig. 6. Tracking of nonminimum phase systems with time constant uncertainty for Case 2

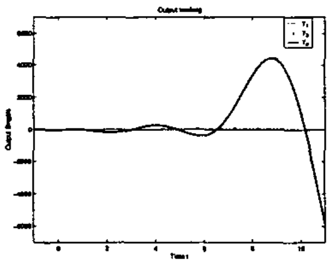

Fig. 7. Tracking of nonminimum phase systems with time constant uncertainty for Case 3

largely for Case 3.

The above results demonstrate that the proposed learning control is very effective in reproducing the desired trajectories. Simulation results also show that the provided condition is only a sufficient condition not a necessary condition.

VI. Conclusions

A new inversion-based robust learning algorithm has been developed for unstable nonminimum phase systems. The sufficient condition for the convergence of the proposed ILC is also provided. The robust feedback control law is employed to guarantee the system stability and the convergence of tracking error, and a stable inverse system is used to update the feed forward input for the next trial. Simulation studies on two types of linear systems with gain uncertainty and time constant uncertainty are presented. Given a desired trajectory, the learning controller is able to learn and eventually drive the closed-loop dynamics to track the desired trajectory. Simulation results demonstrate the effectiveness of the proposed method.

TABLE II
Output Tracking Error of Nonminimum Phase Systems with Gain Uncertainty.

k	*Case* 1	*Case* 2	*Case* 3
1	12.6169	29.4802	29.4802
2	0.1238	0.1908	225
3	0.1233	0.1446	1662
4	0.1106	0.1267	12334

TABLE III
Output Tracking Error of Nonminimum Phase Systems with Time Constant Uncertainty.

k	*Case* 1	*Case* 2	*Case* 3
1	89.5941	86.4392	86
2	0.6494	0.6407	710
3	0.0391	0.0260	5813
4	0.0159	0.0173	56337

References

[1] Z. Bien and J. X. Xu, *Iterative learning control*, Kluwer Academic Publishers, 1998.

[2] R. Johansson, *System Modelling Identification*, Prentice Hall, Englewood Cliffs, New Jersey, 1993.

[3] K. L. Moore, *Iterative learning control for deterministic systems*, Springer-Verlag, New York, 1993.

[4] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, Vol. 1, pp. 123-140, 1984.

[5] D. Chen and B. Paden, "Stable inversion of nonminimum phase nonlinear systems," *International Journal of Control*, Vol. 64, pp. 81-96, 1996.

[6] T. J. Jang and C. H. Ahn, "Iterative learning control in feedback systems," *Automatica*, Vol. 31, pp. 243-248, 1995.

[7] J. E. Kuc and M. B. Zaremba, "An iterative learning control theory for acl ass of nonlinear dynamics systems," *Automatica*, Vol. 28, pp. 1215-1221, 1992.

[8] S. Saab, "On the P-type learning control," *IEEE Transactions on Automatic Control*, Vol. 39, pp. 2298-2302, 1994.

[9] T. Sugie and T. Ono, "An iterative learning control law for dynamical systems," *Automatica*, Vol. 27, pp. 729-732, 1991.

[10] M. Uchiyama, "Formation of high speed motion pattern of mechanical arm by trial," *Transactions of the Society of Instrumentation and Control Engineers*, Vol. 19, pp. 706-712, 1978.

[11] N. Amann, D. H. Owens, and E. Rogers, "Nonminimum phase plant in iterative learning control," *Proceedings of the 2th Conference on Intelligent System Engineering*, pp. 107-112, 1994.

[12] K. Furuta and M. Yamakita, "The design of a learning control system for multivariable systems," *Proceedings of IEEE International Symposium on Intelligent Control*, pp. 371-376, 1987.

[13] J. Gao and D. J. Chen, "Iterative learning control for nonminimum phase systems," *American Control Conference*, 1998.

[14] J. Ghosh and B. Paden, "Iterative learning control for nonlinear nonminimum phase plants with input disturbances," *American Control Conference*, Vol. 4, pp. 2584-2589, 1999.

[15] J. Hauser, "Learning control for a class nonlinear systems," *Proceedings of the 26th Conference on Decision and Control*, pp. 859-860, 1987.

[16] X. Z. Wang and D. J. Chen, "Adaptive learning control for nonminimum phase systems," *2000 IEEE International Conference on Systems, Man, and Cybernetics*, pp. 26-31, Vol. 1, 2000.