Bounds on the Number of Slicing, Mosaic, and General Floorplans

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***Abstract*—A floorplan can be defined as a rectangular dissection of the floorplan region. Simple and tight asymptotic bounds on the number of floorplans for different dissection structures help us to evaluate the size of the solution space of different floorplan representation. They are also interesting theoretically. However, only loose bounds exist in the literature. In this paper, we derive tighter asymptotic bounds on the number of slicing, mosaic and general floorplans. Consider the floorplanning of n blocks. For slicing floorplan, we prove that the exact number is $n!((-1)^{n+1}/2)\sum_{k=0}^{n}(3+\sqrt{8})^{n-2k}\binom{1/2}{k}\binom{1/2}{n-k}$ and the tight bound is $\Theta(n!2^{2.543n}/n^{1.5})$ [9]. For mosaic floorplan, we prove that the tight bound is $\Theta(n!2^{3n}/n^4)$. For general floorplan, we prove a tighter lower bound of $\Omega(n!2^{3n}/n^4)$ and a tighter upper bound of $O(n!2^{5n}/n^{4.5})$.**

***Index Terms*—Asymptotic bounds, general floorplan, mosaic floorplan, slicing floorplan.**

I. Introduction

FLOORPLANNING is a major step in the physical design cycle of VLSI circuits. It is the step to plan the positions and the shapes of the top-level blocks of a hierarchical design. With circuit sizes keep on increasing, floorplanning becomes more and more critical in determining the quality of a layout.

Floorplanning can be viewed as the problem of placing flexible blocks, that is, blocks with fixed area but unknown dimensions. There are many variations in the problem formulation [1]–[3]. Unfortunately, all practical floorplanning formulations are NP-complete [1], [2]. As a result, many floorplanners adopt simulated annealing [4] or other stochastic techniques. A code, called a floorplan representation, is usually used to represent the geometrical relationship among the blocks. The code is perturbed repeatedly by the stochastic techniques to search for a good floorplan. The run time and the quality of the solutions depend strongly on the size of the solution space, i.e., the number of possible codes.

The geometrical relationship among the blocks is commonly specified by a rectangular dissection of the floorplan region. The floorplan region is first dissected into rectangular rooms and each block is then mapped to a different room. In order to restrict the size of the solution space, three different ways of dissection are proposed. The corresponding floorplanning structures are called slicing [5], mosaic [6] and general floorplan [7]. Slicing floorplan is a special case of mosaic floorplan, and mosaic floorplan is a special case of general floorplan. The relationship among the solution spaces of slicing, mosaic and general floorplans is illustrated in Fig. 1. However, only very loose lower and upper bounds on the size of these three sets are available. The details are discussed below.

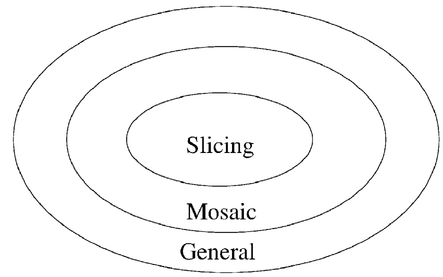

Fig. 1. Relationship among the solution spaces of slicing, mosaic and general floorplans.

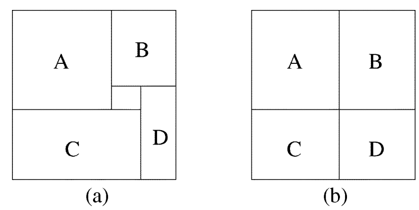

Fig. 2. Structures that cannot be represented in mosaic floorplan. Floorplan (a) with empty room and (b) with crossing cut.

Slicing floorplan is a rectangular dissection that can be obtained by recursively cutting a rectangle horizontally or vertically into two smaller rectangles. In [8], Otten first proposed to represent slicing floorplan using a binary tree representation called slicing tree. Each leaf of the slicing tree corresponds to a block and each internal node represents a vertical or horizontal merge operation on the two descendents. Note that one slicing floorplan may correspond to more than one slicing tree. Later, the redundancy was identified by Wong and Liu in [5], where Normalized Polish Expression (NPE) was proposed to represent any slicing structure without redundancy. An upper bound on the number of NPEs, which is also an upper bound on the number of slicing floorplans, is $O(n!2^{3n}/n^{1.5})$[1]. The best lower bound on the number of slicing floorplans is given by the number of binary trees without labels on internal nodes, which is $\Omega(n!2^{2n}/n^{1.5})$[9].

Mosaic floorplan was proposed by Hong *et al.* in [6]. In mosaic floorplan, nonslicing structures (e.g., a wheel structure) are allowed. However, the floorplan region is dissected into exactly n rooms so that each room is occupied by one and only one block. In addition, there is no crossing cut in the mosaic floorplan. See Fig. 2 for some structures that cannot be represented in mosaic floorplan. Corner block list (CBL) was proposed in

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[1]In this paper, we let n be the number of blocks in the floorplanning problem.

TABLE I
A SUMMARY OF RESULTS

	Previous Bounds		Our Bounds	
	Lower	Upper	Lower	Upper
Slicing	$\Omega(n!2^{2n}/n^{1.5})$ [9]	$O(n!2^{3n}/n^{1.5})$ [5]	$\Theta(n!2^{2.543n}/n^{1.5})$	
Mosaic	$\Omega(n!2^{2n}/n^{1.5})$ [9]	$O(n!2^{3n})$ [10]	$\Theta(n!2^{3n}/n^{4})$	
General	$\Omega(n!2^{2n}/n^{1.5})$ [9]	$O((n!)^2)$ [12]	$\Omega(n!2^{3n}/n^{4})$	$O(n!2^{5n}/n^{4.5})$

[6] to represent mosaic floorplan. The size of the solution space for CBL is $\Theta(n!2^{3n})$. Notice that some CBLs do not correspond to any floorplan. At about the same time, Sakanushi *et al.* [10] introduced the Quarter-State Sequence (Q-Sequence) representation for mosaic floorplan. Q-Sequence is a concatenation of room names and two kinds of positional symbols, with the total length equals 3n. It is a nonredundant representation of mosaic floorplan. An upper bound on the size of the solution space for Q-sequence is $O(n!2^{3n})$. There is no previously available result in literature on the lower bound on the number of mosaic floorplans. So the best lower bound is the same as the one for slicing floorplan.

General floorplan is similar to mosaic floorplan in that nonslicing structures are allowed. However, the floorplan region can be dissected into more than n rooms such that some rooms are not occupied by any block. Many representations have been proposed during the 1990s [11]. In, Onodera used Branch-and-Bound algorithm to solve the general floorplan problem. An upper bound on the size of the solution space for this approach is $O(2^{n(n+2)})$, which is extremely huge. In [12], Murata *et al.* introduced the sequence pair (SP) representation for general floorplan. SP is one of the most elegant representations for general floorplan and has been widely used. Unfortunately, redundancy still exists in this representation. The number of different SP is $\Theta((n!)^2)$. In [13], Nakatake, *et al.* proposed the bounded-slice-line grid (BSG) representation. In BSG, n blocks are randomly placed in a special n-by-n grid. The corresponding size of the solution space is $O(n!C(n^2,n))$, which is even larger than that of SP. The huge solution spaces of SP and BSG restrict the applicability of these representations in large floorplan problems. Later, O-tree [14] and B^*-tree [15] were proposed to represent a compacted version of general floorplan. Compared to SP and BSG, these two representations have a much smaller solution space of $\Theta(n!2^{2n}/n^{1.5})$. However, they represent only partial topological information, and the dimensions of all blocks are required in order to describe an exact floorplan. In addition, not all possible rectangular dissections can be represented by O-tree and B^*-tree. For the lower bound on the number of general floorplans, there is no previously available result in literature. So the best lower bound is again the same as the one for slicing floorplan.

Recently, several representations have been proposed to construct general floorplans by inserting empty rooms into mosaic floorplans. They make use of mosaic floorplan as an intermediate step to represent nonslicing structures. In such approach, the number of empty rooms is crucial because it changes the size of the solution space significantly. In [16], Zhou *et al.* proved that n^2-n empty rooms are enough to produce all general floorplans. As a result, the size of the solution space is as huge as $O(n!C(n^2,n)2^{3n^2})$. In [17], Zhuang *et al.* proved that $n-\lfloor\sqrt{4n-1}\rfloor$ empty rooms are enough to generate all general floorplans. But the size of the solution space of $O(2^{6n}(2n)!/n!)$ is still quite large. Recently, Young *et al.* introduced Twin Binary Sequences (TBS) [18]. TBS is a nonredundant mosaic floorplan representation in which the exact positions for irreducible empty room insertion can be found in linear time. So, by upper-bounding the number of ways to insert empty rooms into each TBS, we can derive an upper bound on the number of general floorplans. We use this idea to derive the bound in Section V.

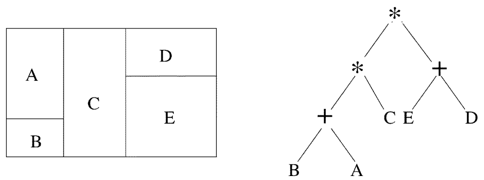
In [19], Yao *et al.* showed that the exact number of slicing floorplans is given by the Super Catalan number and the exact number of mosaic floorplans is given by the Baxter number. However, Super Catalan number is given as a recurrence relation and Baxter number is given as a very complicated summation. The growth rate of those numbers are hard to comprehend. The asymptotic bounds derived in this paper give us a better understanding on those numbers as well as on the number of slicing and mosaic floorplans.

II. CONTRIBUTIONS

Although many representations of these three types of floorplan have been studied intensively and several upper bounds on the number of combinations of those representations have been reported, it is still theoretically interesting and practically useful to find the tight bounds on the number of slicing, mosaic and general floorplans. In this paper, we show that the exact number of the slicing floorplans is $n!((-1)^{n+1}/2)\sum_{k=0}^{n}(3+\sqrt{8})^{n-2k}\binom{1/2}{k}\binom{1/2}{n-k}$. Also we prove that the tight bound on this number is $\Theta(n!(3+\sqrt{8})^n/n^{1.5}) = \Theta(n!2^{2.543n}/n^{1.5})$. For the number of mosaic floorplans, based on the Baxter number, we show that the tight bound is $\Theta(n!2^{3n}/n^4)$. For the number of general floorplans, based on the idea of inserting the empty rooms into TBS, we derive a tighter upper bound of $O(n!2^{5n}/n^{4.5})$. Based on the bound for mosaic floorplan, we also get a tighter lower bound of $\Omega(n!2^{3n}/n^4)$ on the number of general floorplans. The results are summarized in Table I.

These bounds give us a better understanding on the relative sizes of these three types of floorplan. In addition, these bounds could be utilized as a criterion to evaluate the size of the solution space of different floorplan representation.

The organization of this paper is as follows. In Section III, we will show the detailed proof of the exact number and the tight asymptotic bound on the number of slicing floorplans. In Section IV, we will present the tight bound on the number of mosaic floorplans. In Section V, a tighter upper bound on the number of general floorplans will be derived. In Section VI, we will conclude the paper.



(a) (b)

Fig. 3. (a) Slicing floorplan and (b) its corresponding SST.

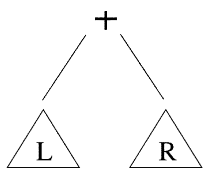


Fig. 4. A "+"-rooted Skewed Slicing Tree Γ.

III. Tight Bound for Slicing Floorplan

In [8], Otten *et al.* introduced a kind of binary tree called Slicing Tree (ST) to represent slicing structure. A ST is a hierarchical description of the direction of the cuts (vertical or horizontal) in a slicing floorplan. However, for a given slicing floorplan, there may be more than one slicing tree representation. In order to nonredundantly represent all slicing floorplans, Wong and Liu [5] proposed a special kind of slicing tree named Skewed Slicing Tree (SST). A SST is a slicing tree in which no node and its right child have the same label in $\{*, +\}$ (Fig. 3), where they interpreted the symbols $*$ and $+$ as two binary operators between slicing floorplans. They used the postorder traversal of SST called the Normalized Polish Expression as the floorplan representation. Wong and Liu noted that there is a one-to-one correspondence between the set of NPEs of length $2n-1$ and the set of SSTs with n leaves. Thus, A one-to-one correspondence also exists between all SSTs with n leaves and all slicing structures with n rectangular rooms. Therefore, we could obtain the number of slicing floorplan configurations with n blocks by counting the number of SSTs with n leaves. Before we explore the tight bound on the number of SSTs, we will first show how the exact number of SSTs can be obtained in the Section III-A.

A. Exact Number of SSTs

Suppose there are overall t_n different SSTs with n leaves. When $n=1$, obviously

$$t_1 = 1. \tag{1}$$

When $n \geq 2$, we classify them into two types of SSTs as follows: a_n "+"-rooted SSTs with n leaves and b_n "$*$"-rooted SSTs with n leaves. Then

$$t_n = a_n + b_n. \tag{2}$$

Given a "+"-rooted SST Γ (see Fig. 4), according to the definition of SST, the left subtree L of Γ could be either a SST ("$*$"-rooted or "+"-rooted) with fewer than n leaves or a single leaf. The right subtree R of Γ could be either a "$*$"-rooted SST with fewer than n leaves or a single leaf. Then the number of "+"-rooted SSTs with n leaves becomes:

$$a_n = t_1 b_{n-1} + t_2 b_{n-2} + \cdots + t_{n-2} b_2 + t_{n-1} \cdot 1. \tag{3}$$

Similarly, the number of "$*$"-rooted SSTs with n leaves is

$$b_n = t_1 a_{n-1} + t_2 a_{n-2} + \cdots + t_{n-2} a_2 + t_{n-1} \cdot 1. \tag{4}$$

According to (2)–(4), the number of SSTs with n leaves becomes

$$\begin{aligned} t_n =& t_1(a_{n-1}+b_{n-1}) + t_2(a_{n-2}+b_{n-2}) + \\ & \cdots + t_{n-2}(a_2+b_2) + 2t_{n-1} \\ =& t_1 t_{n-1} + t_2 t_{n-2} + \cdots + t_{n-2} t_2 + t_{n-1} t_1 + t_{n-1}. \end{aligned} \tag{5}$$

In order to solve the recurrence (5) with the initial condition (1), we define the generating function as

$$T(z) = t_1 + t_2 z + t_3 z^2 + \cdots. \tag{6}$$

Then, we have

$$T^2(z) = t_1^2 + (t_1 t_2 + t_2 t_1)z + (t_1 t_3 + t_2 t_2 + t_3 t_1)z^2 + \cdots.$$

and so

$$\begin{aligned} T^2(z) + T(z) =& (t_1^2 + t_1) + (t_1 t_2 + t_2 t_1 + t_2)z \\ & + (t_1 t_3 + t_2 t_2 + t_3 t_1 + t_3)z^2 + \cdots. \end{aligned} \tag{7}$$

Combining (6) and (7) yields

$$t_1 + [T^2(z) + T(z)]z = T(z).$$

Since $t_1 = 1$, then

$$zT(z)^2 + (z-1)T(z) + 1 = 0. \tag{8}$$

Solving (8) with initial condition $T(0) = t_1 = 1$ yields

$$T(z) = \frac{1 - z - \sqrt{z^2 - 6z + 1}}{2z}.$$

Let $\alpha = 3 + \sqrt{8}$ and $\beta = 3 - \sqrt{8}$. Notice that $\alpha\beta = 1$. Thus

$$\begin{aligned} T(z) =& \frac{1 - z - \sqrt{\alpha - z}\sqrt{\beta - z}}{2z} \\ =& \frac{1}{2z}\left[1 - z - \sqrt{\alpha\beta}\sqrt{1 - \frac{z}{\alpha}}\sqrt{1 - \frac{z}{\beta}}\right] \\ =& \frac{1}{2z}\left[1 - z - \sum_{i=0}^{\infty}\binom{\frac{1}{2}}{i}\left(\frac{-1}{\alpha}\right)^i z^i \right. \\ & \left. \times \sum_{j=0}^{\infty}\binom{\frac{1}{2}}{j}\left(\frac{-1}{\beta}\right)^j z^j\right]. \end{aligned}$$

By the definition of generating function (6), for $n \geq 2$, we get the coefficient of z^{n-1} as

$$\begin{aligned} t_n =& -\frac{1}{2}\sum_{k=0}^{n}\binom{\frac{1}{2}}{k}\left(\frac{-1}{\alpha}\right)^k \binom{\frac{1}{2}}{n-k}\left(\frac{-1}{\beta}\right)^{n-k} \\ =& \frac{(-1)^{n+1}}{2}\sum_{k=0}^{n}\alpha^{n-2k}\binom{\frac{1}{2}}{k}\binom{\frac{1}{2}}{n-k}. \end{aligned} \tag{9}$$

The exact number of SSTs thus has the general form

$$t_n = \begin{cases} 1, & \text{if } n = 1 \\ \frac{(-1)^{n+1}}{2} \sum_{k=0}^{n} \alpha^{n-2k} \binom{\frac{1}{2}}{k} \binom{\frac{1}{2}}{n-k}, & \text{if } n \geq 2 \end{cases}.$$

In Section III-B, we try to get the tight bound on t_n.

B. The Tight Bound on the Number of Slicing Floorplans

In order to obtain the tight bound on t_n $(n \geq 2)$, we rewrite (9) as

$$t_n = \sum_{k=0}^{n} F(k)$$

where

$$F(k) = \frac{(-1)^{n+1}}{2} \alpha^{n-2k} \binom{\frac{1}{2}}{k} \binom{\frac{1}{2}}{n-k}.$$

First, we bound $F(0)$ as follows:

$$\begin{aligned} F(0) =& \frac{(-1)^{n+1}}{2} \alpha^n \binom{\frac{1}{2}}{n} \\ =& \frac{(-1)^{n+1}}{2} \alpha^n \frac{\frac{1}{2}(\frac{1}{2}-1)\cdots(\frac{1}{2}-n+1)}{n!} (-1)^n \\ =& \frac{\alpha^n (1 \times 3 \times \cdots \times (2n-3))}{2^{n+1} n!} \\ =& \frac{\alpha^n (2n)!}{2^{2n+1} (n!)^2 (2n-1)} \end{aligned}$$

then, by Stirling's approximation[2]

$$\begin{aligned} =& \Theta\left(\frac{\alpha^n \sqrt{2\pi 2n} (\frac{2n}{e})^{2n}}{2^{2n+1} (\sqrt{2\pi n})^2 (\frac{n}{e})^{2n} (2n-1)} \right) \\ =& \Theta\left(\frac{\alpha^n}{n^{1.5}} \right) \\ =& \Theta\left(\frac{2^{2.543n}}{n^{1.5}} \right). \end{aligned}$$

Second, we will bound $\sum_{k=1}^{n-1} F(k)$. For $1 \leq k \leq n-1$, let

$$r_k = \frac{F(k)}{F(k-1)} = \frac{(n-k+1)\left(k - \frac{3}{2}\right)}{\alpha^2 \left(n-k-\frac{1}{2}\right) k}.$$

When $n \rightarrow \infty$, it is not difficult to observe that

$$\begin{aligned} r_1 \approx& -0.0147 \\ 0 <& r_2 > r_3 < \cdots < r_{n-1} < 1 \\ r_{n-1} \approx& 0.1177. \end{aligned}$$

Therefore, $F(1) = r_1 F(0) < 0$ and

$$\begin{aligned} \sum_{k=1}^{n-1} F(k) =& F(1) + F(1) r_2 + F(1) r_2 r_3 \\ &+ \cdots + F(1) r_2 \cdots r_{n-1} \\ >& F(1) + F(1) r_{n-1} + F(1) r_{n-1}^2 \\ &+ \cdots + F(1) r_{n-1}^{n-2} \\ >& \frac{F(1)}{1 - r_{n-1}} \\ \approx& -0.0166 F(0). \end{aligned}$$

Thus, we bound $\sum_{k=1}^{n-1} F(k)$ as

$$-0.0166 F(0) < \sum_{k=1}^{n-1} F(k) < 0.$$

Third, we bound $F(n)$ as

$$F(n) = \alpha^{-2n} F(0) = o(F(0)).$$

We thus get the tight bound on t_n as

$$\begin{aligned} t_n =& F(0) - 0.0166 F(0) + o(F(0)) \\ =& \Theta(F(0)) \\ =& \Theta\left(\frac{2^{2.543n}}{n^{1.5}} \right). \end{aligned}$$

If we consider the labels of the leaves of SSTs, there are $n!$ combinations for labeling of the leaves of SSTs. Thus the total number of combinations of slicing floorplan is $\Theta(n! 2^{2.543n} / n^{1.5})$.

IV. Tight Bound for Mosaic Floorplan

In paper [19], Yao *et al.* first proved that the number of combinations of mosaic floorplan with n blocks is equal to the number of Baxter permutations on $\{1, \ldots, n\}$. Then, $M(n) = B(n)$, where $M(n)$ is the exact number of combinations of mosaic floorplans with n blocks, and $B(n)$ is a Baxter number, which can be represented as follows:

$$B(n) = \binom{n+1}{1}^{-1} \binom{n+1}{2}^{-1} \sum_{k=1}^{n} \binom{n+1}{k-1} \times \binom{n+1}{k} \binom{n+1}{k+1}. \quad (10)$$

In order to get the tight bound on $B(n)$, we first simplify (10) as follows:

$$\begin{aligned} B(n) =& \frac{2}{n(n+1)^2} \\ &\times \sum_{k=1}^{n} \frac{((n+1)!)^3}{(k-1)!(n-k+2)!k!(n-k+1)!(k+1)!(n-k)!} \\ =& \sum_{k=1}^{n} \frac{2(n!)^3(n+1)}{n(k-1)!k!(k+1)!(n-k)!(n-k+1)!(n-k+2)!} \\ =& \sum_{k=1}^{n} G(k) \end{aligned}$$

where

$$G(k) = \frac{2(n!)^3(n+1)}{n(k-1)!k!(k+1)!(n-k)!(n-k+1)!(n-k+2)!}.$$

Without lost of generality, we assume n is even. Note that for $1 \leq k \leq n$

$$G(k) = G(n-k+1)$$

and for $2 \leq k \leq n/2$

$$\frac{G(k)}{G(k-1)} = \frac{(n-k+1)(n-k+2)(n-k+3)}{(k-1)k(k+1)} > 1.$$

Therefore, for $1 \leq k \leq n/2$, $G(k)$ will increase with the increasing of k; for $n/2+1 \leq k \leq n$, $G(k)$ will decrease with the

[2]Stirling's approximation for $n! = \Theta(\sqrt{2\pi n}(n/e)^n)$ [9].

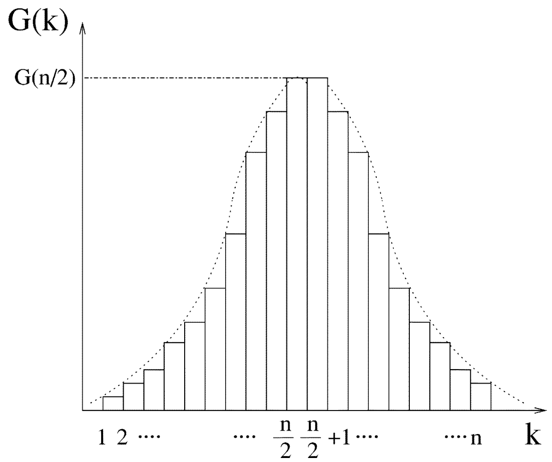


Fig. 5. Distribution of $G(k)$.

increasing of k. Thus, the distribution of $G(k)$ will be roughly like Fig. 5.

First, we bound $G(n/2)$ as

$$\begin{aligned} G\left(\frac{n}{2}\right) =& \frac{2(n!)^3(n+1)}{n(\frac{n}{2}-1)!(\frac{n}{2})!(\frac{n}{2}+1)!(\frac{n}{2})!(\frac{n}{2}+1)!(\frac{n}{2}+2)!} \\ =& \frac{(n!)^3(n+1)}{\left((\frac{n}{2})!\right)^6\left(\frac{n}{2}+1\right)^3\left(\frac{n}{2}+2\right)} \\ =&\Theta\left(\frac{(2\pi n)^{3/2}(\frac{n}{e})^{3n}}{(\pi n)^3(\frac{n}{2e})^{3n}n^3}\right) \\ & \text{by Stirling's approximation} \\ =&\Theta\left(\frac{2^{3n}}{n^{4.5}}\right). \end{aligned}$$

Second, we try to bound $\sum_{k=1}^{n/2} G(k)$.

Lemma 1: For $n/2 - \lceil\sqrt{n\ln n}\rceil \leq k \leq n/2$, when $n \to \infty$, $G(k) = G(n/2)/M$, where $M = \Theta((2k/n)^{3k}(2-2k/n)^{3(n-k)})$.

Proof: For $n/2 - \lceil\sqrt{n\ln n}\rceil \leq k \leq n/2$, when $n \to \infty$, notice that $k \to \infty$, $n-k \to \infty$. By Stirling's approximation, we work out $G(k)$ as

$$\begin{aligned} G(k) =&\Theta\left(\frac{(n!)^3}{n^3(k!)^3((n-k)!)^3}\right) \\ =&\Theta\left(\frac{(2\pi n)^{3/2}(\frac{n}{e})^{3n}}{(2\pi k)^{3/2}(\frac{k}{e})^{3k}\left(2\pi(n-k)\right)^{3/2}\left(\frac{n-k}{e}\right)^{3(n-k)}n^3}\right) \\ =&\Theta\left(\frac{n^{3n}}{n^{4.5}k^{3k}(n-k)^{3(n-k)}}\right) \\ =&\Theta\left(\frac{2^{3n}}{n^{4.5}(\frac{2k}{n})^{3k}(2-\frac{2k}{n})^{3(n-k)}}\right) \\ =&\frac{G(\frac{n}{2})}{M} \end{aligned} \tag{11}$$

where $M = \Theta((2k/n)^{3k}(2-2k/n)^{3(n-k)})$. ■

Lemma 2: When $k = n/2 - \lceil 1/\sqrt{12}\sqrt{n\ln n}\rceil$, $G(k) = \Theta(n^{-1})G(n/2)$.

Proof: Assume

$$k = \frac{n}{2} - \lceil c\sqrt{n\ln n}\rceil \tag{12}$$

where $0 \leq c \leq 1$. By Lemma 1, we have

$$M = \Theta\left(\left(1-2c\sqrt{\frac{\ln n}{n}}\right)^{3(n/2-c\sqrt{n\ln n})} \times\left(1+2c\sqrt{\frac{\ln n}{n}}\right)^{3(n/2+c\sqrt{n\ln n})}\right).$$

Using the limit of function

$$\lim_{x\to 0}(1\pm x)^{k/x} = e^{\pm k}$$

we simplify M as

$$\begin{aligned} M =&\Theta\left(e^{-\left(3c\sqrt{n\ln n}-6c^2\ln n\right)}\cdot e^{\left(3c\sqrt{n\ln n}+6c^2\ln n\right)}\right) \\ =&\Theta\left(n^{12c^2}\right). \end{aligned} \tag{13}$$

Let $c = 1/\sqrt{12}$, by (11) and (13), we get

$$G\left(\frac{n}{2} - \lceil\frac{1}{\sqrt{12}}\sqrt{n\ln n}\rceil\right) = \Theta\left(n^{-1}\right)G\left(\frac{n}{2}\right).$$

■

Noticing that

$$G(1) < G(2) < \cdots < G\left(\frac{n}{2} - \lceil\frac{1}{\sqrt{12}}\sqrt{n\ln n}\rceil\right)$$

we thus bound $\sum_{k=1}^{n/2-\lceil 1/\sqrt{12}\sqrt{n\ln n}\rceil} G(k)$ as follows:

$$\begin{aligned} 0 < \sum_{k=1}^{n/2-\lceil 1/\sqrt{12}\sqrt{n\ln n}\rceil} G(k) <& \left(\frac{n}{2} - \lceil\frac{1}{\sqrt{12}}\sqrt{n\ln n}\rceil\right) \\ & \times\Theta(n^{-1})G(\frac{n}{2}) \\ =& O\left(G\left(\frac{n}{2}\right)\right). \end{aligned}$$

Lemma 3: For $n/2 - \lceil 1/\sqrt{12}\sqrt{n\ln n}\rceil \leq k \leq n/2$, $\sum_{k=n/2-\lceil 1/\sqrt{12}\sqrt{n\ln n}\rceil}^{n/2} G(k)$ is bounded by $\Theta(\sqrt{n}G(n/2))$.

Proof: For $n/2 - \lceil 1/\sqrt{12}\sqrt{n\ln n}\rceil \leq k \leq n/2$,we take $G(k)$ as an continuous function. Thus we bound the summation of $G(k)$ by bounding the integration of continuous function $G(k)$ where $n/2 - 1/\sqrt{12}\sqrt{n\ln n} \leq k \leq n/2$. By (11)–(13)

$$\begin{aligned} \sum_{k=n/2-\lceil(1/\sqrt{12})\sqrt{n\ln n}\ \rceil}^{(n/2)} G(k) &= \Theta\left(\int_{n/2-(1/\sqrt{12})\sqrt{n\ln n}}^{n/2} G(k)dk\right) \\ &= \sqrt{n\ln n}G\left(\frac{n}{2}\right)\Theta\left(\int_{1/\sqrt{12}}^{0} -n^{-12c^2}dc\right) \\ &= \sqrt{n\ln n}G\left(\frac{n}{2}\right)\Theta\left(\int_{0}^{1/\sqrt{12}} n^{-12c^2}dc\right). \end{aligned}$$

In order to solve the integration $\int_0^{1/\sqrt{12}} n^{-12c^2}dc$, we use the property of Normal Distribution with the probability density function as

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2}$$

for $0 \leq x \leq 3\sigma$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_0^{3\sigma} e^{-x^2/2\sigma^2} dx \approx 0.5$$

i.e.,

$$\int_0^{3\sigma} e^{-x^2/2\sigma^2} dx \approx \sqrt{\frac{\pi}{2}}\sigma.$$

Then

$$\int_0^{1/\sqrt{12}} n^{-12c^2} dc = \int_0^{1/\sqrt{12}} e^{-12c^2 \ln n} dc.$$

Notice that when $n \to \infty$

$$\frac{1}{\sqrt{12}} \gg 3 \cdot \frac{1}{\sqrt{24 \ln n}}$$

thus

$$\int_0^{1/\sqrt{12}} n^{-12c^2} dc \approx \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{24 \ln n}}.$$

Then, we have

$$\sum_{k=n/2-\lceil 1/\sqrt{12}\sqrt{n \ln n}\,\rceil}^{n/2} G(k) = \Theta\left(\sqrt{n}G\left(\frac{n}{2}\right)\right).$$

■

We thus obtain the tight bound on $B(n)$ as

$$\begin{aligned} &B(n) \\ &= \sum_{k=1}^{n} G(k) \\ &= 2\left[\sum_{k=1}^{n/2-\lceil 1/\sqrt{12}\sqrt{n \ln n}\,\rceil} G(k) + \sum_{k=n/2-\lceil 1/\sqrt{12}\sqrt{n \ln n}\rceil}^{n/2} G(k)\right] \\ &- 2G\left(\frac{n}{2} - \left\lceil \frac{1}{\sqrt{12}}\sqrt{n \ln n}\right\rceil\right) \\ &= 2\left[\Theta\left(G\left(\frac{n}{2}\right)\right) + \Theta\left(\sqrt{n}G\left(\frac{n}{2}\right)\right) - \Theta(n^{-1})G\left(\frac{n}{2}\right)\right] \\ &= \Theta\left(\sqrt{n}G\left(\frac{n}{2}\right)\right) \\ &= \Theta\left(\frac{2^{3n}}{n^4}\right). \end{aligned}$$

If we consider the labeling of block names, the final tight bound on the number of mosaic floorplans with n blocks is $\Theta(n!2^{3n}/n^4)$.

V. Upper Bound for General Floorplan

In paper [18], a general floorplan F' can be constructed from a mosaic floorplan F by inserting some irreducible empty rooms into a mosaic floorplan at right places in F. There are two kinds of empty rooms. One is called reducible empty room which is resulted from assigning a small block into a big room [see an example in Fig. 6(a)]. Another kind of empty room is called irreducible empty rooms which can not be removed by merging with another room in the packing [see an example in Fig. 6(b)]. In addition, a wheel structure always exists in every irreducible empty room (see Fig. 7). We discuss this idea in more detail in the Section V-A.

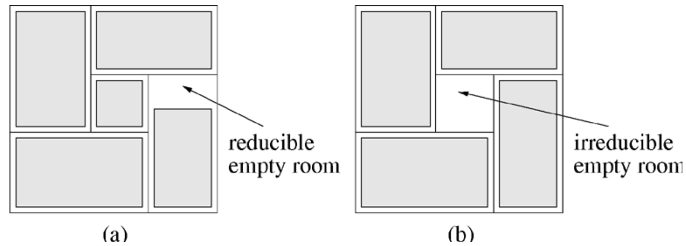


Fig. 6. Example of reducible and irreducible empty rooms.

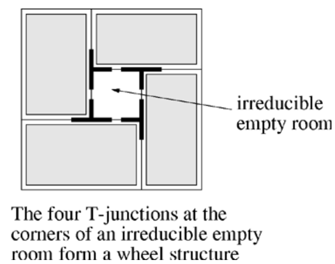


Fig. 7. A wheel structure.

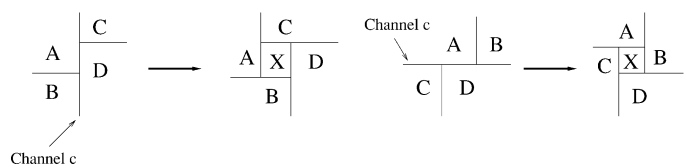


Fig. 8. Pair of T junctions produce a wheel structure.

A. Empty Room Insertion

Observation 1: A wheel structure can be produced from and only from the following mosaic structure: a pair of T junctions share the same channel on each side, respectively. It is shown in Fig. 8.

Based on the Observation 1, it is not difficult to prove the following Lemma.

Lemma 4: For a channel with p and q blocks on each side, respectively, the maximum number of irreducible empty rooms which could be inserted along the channel is $\min(p, q) - 1$.

Proof: Without lost of generality, we assume $p \leq q$. Then, from p blocks on one side of a channel, we could find out $p - 1$ T junctions. Similarly, for q blocks along the other side of the channel, there is $q - 1$ T junctions. Therefore, at most we could pick p pairs of T junctions from each side at one time. According to the Observation 1, an empty room could be produced by any pair of T junctions with one from each side of the channel, respectively. We label these T junctions as $T_1, T_2, \ldots, T_{p-1}$ on each side from top to bottom or from left to right, and then match them one by one according to the order from T_1 to T_p. $p - 1$ empty rooms could thus be inserted along the channel. An example with 4 blocks and 5 blocks along a channel c on each side, respectively, is shown in Fig. 9. Maximumly, 3 irreducible empty rooms (represented by X) are inserted in this example. ■

Lemma 5: For a channel with p and q blocks on each side, respectively, the number of ways to insert empty rooms along the channel is $\begin{pmatrix} p+q-2 \\ q-1 \end{pmatrix}$.

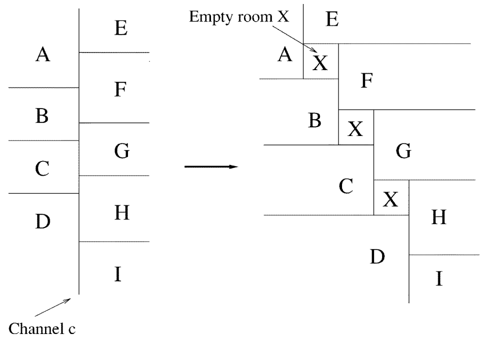


Fig. 9. Example of inserting maximum number of irreducible empty rooms.

Proof: Without lost of generality, we assume $p \le q$. According to Lemma 4, we can insert at most $p-1$ empty rooms along the channel, which means j $0 \le j \le p-1$ empty rooms could possibly inserted along the channel. Similar to the PROOF of Lemma 4, we pick j pairs of T junctions from each side of the channel, and label them from top to bottom or left to right and then match them one by one according to the order. We can thus insert j empty rooms into those T junctions.

Let $C(p,q)$ denote the number of ways to insert empty rooms along the channel with p and q blocks on each side, respectively. We have

$$\begin{aligned} C(p,q) &= \sum_{j=0}^{p-1} \binom{p-1}{j}\binom{q-1}{j} \\ &= \sum_{j=0}^{p-1} \binom{p-1}{p-1-j}\binom{q-1}{j} \\ &= \binom{p+q-2}{p-1} \text{ by the definition of Combination} \\ &= \binom{p+q-2}{q-1}. \end{aligned}$$

■

In the Section V-B, we will formulate the total number of ways to insert empty rooms into a mosaic floorplan with n blocks.

B. An Upper Bound on the Number of General Floorplans

Given a mosaic floorplan with n blocks, by counting the total number of ways to insert empty rooms into the mosaic floorplan, we can obtain the total number of general floorplan generated from the mosaic floorplan.

For a mosaic floorplan with n blocks, it has overall $n+3$ channels. Without lost of generality, we assume it has k $(2 \le k \le n+1)$ horizontal channels with the uppermost boundary as the 1st horizontal channel and downmost boundary as kth horizontal channel. In addition, it has $n+3-k$ vertical channels with the leftmost boundary as 1st vertical channel and rightmost boundary as $(n+3-k)^{th}$ vertical channel.

Let h_i $(i = 1, 2, \ldots, k)$ be the number of blocks which touch the ith horizontal channel on the top, h'_i be the number of blocks which touch the ith horizontal channel on the bottom. Let v_j $(1 \le j \le n+3-k)$ be the number of blocks which touch the jth vertical channel on the left, v'_j be the number of blocks which touch the jth vertical channel on the right.

Assuming $h_1 = h'_k = v_1 = v'_{n+3-k} = 1$, we have

$$\sum_{i=1}^{k} h_i = \sum_{i=1}^{k} h'_i = \sum_{j=1}^{n+3-k} v_j = \sum_{j=1}^{n+3-k} v'_j = n+1.$$

We denote $L(n)$ as the total number of ways to insert empty rooms into a mosaic floorplan with n blocks, according to Lemma 5

$$\begin{aligned} L(n) &= \prod_{i=1}^{k} C(h_i, h'_i) \cdot \prod_{j=1}^{n+3-k} C(v_j, v'_j) \\ &= \prod_{i=1}^{k} \binom{h_i + h'_i - 2}{h_i - 1} \prod_{j=1}^{n+3-k} \binom{v_j + v'_j - 2}{v_j - 1}. \end{aligned} \tag{14}$$

In order to get an upper bound on $L(n)$, we notice that $\forall p, q, s, t \in \{0, 1, 2, \ldots, n\}$

$$\binom{p}{q} \cdot \binom{s}{t} \le \binom{p+s}{q+t}.$$

We thus bound (14) as

$$\begin{aligned} L(n) &\le \binom{\sum_{i=1}^{k}(h_i + h'_i - 2)}{\sum_{i=1}^{k}(h_i - 1)} \cdot \binom{\sum_{j=1}^{n+3-k}(v_j + v'_j - 2)}{\sum_{j=1}^{n+3-k}(v_j - 1)} \\ &= \binom{2n+2-2k}{n+1-k} \cdot \binom{2n+2-2(n+3-k)}{n+1-(n+3-k)} \\ &\le \binom{2n-2}{n-1} \\ &= \frac{(2n-2)!}{((n-1)!)^2} \text{ then, by Stirling's approximation} \\ &= O\left(\frac{\sqrt{2\pi(2n-2)}\left(\frac{(2n-2)}{e}\right)^{2n-2}}{(\sqrt{2\pi(n-1)})^2\left(\frac{(n-1)}{e}\right)^{2n-2}}\right) \\ &= O\left(\frac{2^{2n}}{n^{0.5}}\right). \end{aligned}$$

Since the tight bound on the number of mosaic floorplan with n blocks is $\Theta(n!2^{3n}/n^4)$, we obtain an upper bound on the number of general floorplan with n blocks as $O(n!2^{5n}/n^{4.5})$.

VI. Conclusion

We have successfully obtained tight bounds of $\Theta(n!2^{2.543n}/n^{1.5})$ on the number of slicing floorplans and $\Theta(n!2^{3n}/n^4)$ on the number of mosaic floorplans. However, for the number of general floorplans, the lower bound $\Omega(n!2^{3n}/n^4)$ is still significantly smaller than the upper bound $O(n!2^{5n}/n^{4.5})$. We will work on the tight bound on the number of general floorplans in the future.

Regarding floorplan representations, NPE is a nonredundant representation for slicing floorplan. Q-sequence and TBS are two nonredundant representations for mosaic floorplan. However, there is no nonredundant representation for general floorplan. Although all general floorplans can be produced by inserting empty rooms into TBSs, the information describing which empty room to be inserted is not uniform. Hence TBS cannot be easily extended to a succinct representation which describes a general floorplan completely. We will also work on the problem of designing an elegant and nonredundant general floorplan representation in the future.

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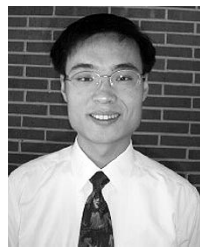

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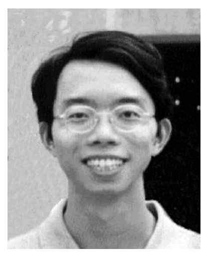

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