Reliable Computing **9**, pp. 1-12, 2003.

Dependable Handling of Uncertainty

Daniel Berleant, Mei-Peng Cheong, Chris Chu, Yong Guan, Ahmed Kamal
Department of Electrical and Computer Engineering, Iowa State University, Ames, Iowa 50011, USA, e-mail: {berleant, mpcheong, cnchu, yguan, kamal, gsheble}@iastate.edu

Scott Ferson
Applied Biomathematics, 100 North Country Road, Setauket, NY 11733 USA, email: scott@ramas.com

James F. Peters
Department of Electrical snd Computer Engineering, University of Manitoba, Winnipeg, Manitoba, R3T 5V6 Canada, e-mail: jfpeters@ee.umanitoba.ca

Abstract

Uncertainty quantification is an important approach to modeling in the presence of limited information about uncertain quantities. As a result recent years have witnessed a burgeoning body of work in this field. The present paper gives some background, highlights some recent work, and presents some problems and challenges.

1 Introduction

Traditional methods emphasize uncertainty due to intrinsic variability in values of parameters of interest. Yet there is growing interest in also handling uncertainty due to lack of knowledge about values that often could be better known if more information was somehow obtained about them. Because such information can be expensive or difficult to obtain, we wish to be able to reason in the presence of incomplete information. This kind of uncertainty due to incomplete knowledge has several names, including epistemic uncertainty, 2nd order uncertainty, reducible uncertainty, and subjective uncertainty. These kinds of uncertainty are distinguished from uncertainty sometimes considered to be intrinsic to the phenomena themselves rather than to our knowledge about it, which has been called by such terms as randomness, natural variability, aleatory uncertainty, irreducible uncertainty, objective uncertainty, and stochastic uncertainty. Philosophically the distinctness of these two categories is not completely clear (more knowledge can reduce what might be thought to be natural variability, and to take this to the limit a quantity can be measured, making its value a historical record and removing most of the uncertainty associated with it even if its source is natural variability). However from a computational perspective there is a clear distinction between:

- uncertainty described with a distribution function (of which a specific probability value is a special case), and
- uncertainty described with both a distribution function and error bounds of some sort (giving, for example, dependency bounds/envelopes/p-boxes, confidence intervals, or interval-valued probabilities).

All too often, incomplete knowledge requires that one work with the latter case. Thus non-classical computational methods are needed that can handle quantities described with something less specific than a distribution function.

Diverse problems in important fields such as risk, reliability, measurement interpretation, signal processing, control, and decision theory, and in applications as varied as insurance, finance, mathematical ecology, and many others can benefit from inference under conditions in which the uncertainty that is present includes vague, conflicting, or otherwise incomplete knowledge of the problem itself.

Traditional approaches, such as making assumptions as needed to allow problem solution by traditional means, are often thought necessary or used without sufficient attention to the dependability of the results. However there is growing interest as well as a growing body of results on innovative ways to dependably handle uncertainties that are incompletely characterized, vague, imprecise, etc.

Relevant work falls under various overlapping subject categories. A non-exhaustive list of such relevant areas would include, for example, approximate reasoning [31], interval-valued probabilities [16], fixed marginals of unknown dependency [24], rough sets [28], robust statistics [11], imprecise probabilities [29], copulas [22], fuzzy set computations [30], and possibility theory [9]. Common themes and problems in the dependable handling of uncertainty imply potential opportunities for convergences, interrelationships, and cross-fertilization, as well as recognition and exposure among the various areas.

In the next section some current research paths are synopsized. Following that, Section 3 describes some problems for future investigation.

2 Some Current Investigations

It would take literally volumes to give a full account of the field. The following paragraphs thus do not give a comprehensive view, but rather focus on some notable current research paths related to papers in this special issue.

Benford's Law, first described not by Benford (1938 [1]) but by Newcomb (1881 [23]), describes how in many data sets the probability that a datum begins with the numeral "1" is greater than the probability that it begins with "2," and so on with "9" having the lowest probability of being the most significant digit. It is likely that this tendency explains in part the fact that on Google.com, a leading Web search engine, the query term "18" has more hits than "28," which has more hits than "38," a trend that continues on with occasional exceptions and generally seems to apply to other sequences of numbers $1x_1x_2x_3...x_k$, $2x_1x_2x_3...x_k$, $3x_1x_2x_3...x_k$, ... One application of this is detecting faked data. Hill (1999 [13]) suggests that using Benford's law for this purpose is only the beginning, and that the future holds great potential for advances in the detection of data fraud.

Credal sets (Levi 1980 [18]) provide a way to express partial knowledge of probability using probability intervals and to reason with such knowledge. Credal networks (Cozman 2000 [6]; Fagiuoli and Zaffalon 1998 [10]) are a recent advance which generalizes Bayesian networks to handle evidence in the form of credal sets. As the field advances, we may expect to see advances in such areas as elicitation of expert knowledge in the form of credal sets, and extensions to handle partial knowledge of a probability that is non-convex in that it may consist of a disjoint set of intervals. Solutions to such problems would be valuable advances toward the goal of widespread application.

Interval Probability Theory, or IPT (Cui and Blockley 1990 [7]) has recently been applied to combining evidence for decision-making (Davis and Hall 2003 [8]). A compelling approach to combining evidence needs to provide a way of describing the dependency relationship between pieces of evidence. IPT addresses this problem with a parameter, ρ, that states the degree of overlap between two propositions with interval-valued probabilities. Mutually exclusive propositions have ρ=0. If one proposition is a subset of another, then ρ=1. Intermediate degrees of overlap have intermediate values of ρ. If future work shows that the quality of decisions produced with this technique is superior to those produced using conventional techniques, that would be a significant result with great practical value.

The **Imputation toward Directional Extremes** (IDE) algorithm (Vansteelandt and Goetghebeur 2001 [26]) addresses the problem of understanding the sensitivity of results to the range of possible values of missing data (Little and Rubin 1987 [19]). This is distinct from ranges (e.g. confidence intervals) implied by noise due to sampling that is addressed by standard statistical methods, although both degrade the quality of conclusions derived from the data. The IDE algorithm is tuned to handle generalized linear data models efficiently. As further advances occur toward the goals of algorithms that are both fast and general, better understanding of the implications of missing data will be increasingly available, thus supporting better results in design, decision-making, and other important tasks.

Nonparametric predictive inference (NPI) is a technique for predicting the value of a future observation based on previous observations. Previous observations are listed in order from smallest to largest, and a future observation is assumed to have equal probabilities of being smaller than the smallest previous observation, larger than the largest, or between any two adjacent ones (Hill 1968 [12]). A result of the minimal structural assumptions underlying NPI is that uncertainty quantifications are mostly via envelopes around families of distributions. Application of NPI to reliability problems is reported by Coolen et al. (2002 [5]). The method they describe for handling right-censored data enables further progress in applying NPI to replacement and maintenance decisions. (Right-censored failure times derive from observations of survival at discrete time points, which bound the failure time from below).

In **mathematical finance**, such concepts as risk and risk tolerance, decision theory, game theory, utility theory, freedom of action (as in real options theory), and information are all translated into monetary terms. As one example, research into the value of information has inspired investigation of the financial implications of fixed marginal distributions when information about the joint distribution is lacking or is limited to restrictions on the values of Spearman [32] or Pearson [2] correlation coefficients. Spearman correlation is one of those indicated when the investigation is based on the mathematical theory of copulas (Nelsen 1999 [22]), while Pearson correlation is one of those indicated when marginals are used without the normalization implied by copulas. Results can apply to such financial problems as bounding value at risk (VaR). Another example is the problem of asset pricing in incomplete markets. Recent work by Staum (to appear [25]) and others has led not only to valuable theorems but has also revealed parallels with such concepts as the previsions of the imprecise probabilities community. In that work as in other work, the assumption of convexity is important. If it is established that non-convex problems have sufficient practical importance, further work may lead to results in which the convexity assumption is relaxed.

3 Some unsolved problems

The problems in this section appear to require non-traditional handling of uncertainty. Further work is expected to shed more light on the challenges they pose. Section 3.1 gives simply stated (but not necessarily easily solved) problems related to the mathematics and/or software implementation of computations on incompletely specified distributions (p-boxes, envelopes, imprecise probabilities, etc.). Section 3.2 explains some challenging application problems in more detail.

3.1 Challenges to computations on incompletely specified distributions

The following challenges involve envelopes (Figure 1) as problem inputs and/or outputs.

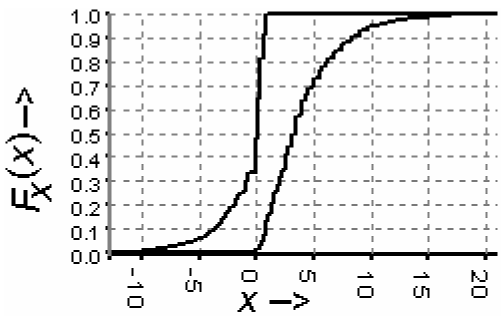

Figure 1. Example of envelopes (i.e. a p-box) around the cumulative distribution of *x*, bounding a family of cumulative distributions for *x* consistent with our state of knowledge about *x*.

1) **Backcalculation (deconvolution)**. Suppose $z=g(x,y)$ where x and y are samples of distributions $F_x(.)$ and $F_y(.)$ and z is a sample of derived distribution $F_z(.)$. If we have envelopes for $F_x(.)$ and $F_z(.)$, then determine envelopes for $F_y(.)$. A similar problem is the constraint updating problem: given $z=g(x,y)$, how can envelopes for the distribution of any two variables be used to derive (or tighten) envelopes for the third?

2) **Limit theorems.** If random variables described using envelopes are added or otherwise combined sufficiently many times, what form will the result converge to?

3) **Non-rigorous envelopes.** One source of envelopes is confidence limits, which can be determined from sets of samples [15]. Given two distributions, each described using confidence limits, how can these be arithmetically combined, as with function g characterized above?

4) **Algorithm equivalence.** Various algorithms have been proposed for calculating function g. To what degree are these algorithms equivalent?

5) **Discretization schemes.** The finer the discretization used to represent envelopes, the more computation is required to compute with them. For a given degree of discretization, and allowing variable step sizes, what is the best discretization to use from the standpoint of quality of the envelopes computed when other envelopes used as inputs are combined with function g?

6) **Partial information about dependency relationships.** The envelopes for $z=g(x,y)$ are affected not only by the envelopes for x and y but also by the dependency relationship between x and y. No information about the dependency tends to lead to envelopes for z that are wider apart than a specific dependency relationship. There are many possible ways to describe dependency besides the commonly used correlation coefficient (Hutchinson and Lai 1990 [14]). A way to seek to narrow the envelopes for z when justification for fully specifying a dependency relationship between x and y is lacking is to *partially* specify a dependency relationship. Research on applying this concept to the case where x and y are described using envelopes is still in its

infancy despite its considerable promise.

7) **Partial information about marginals.** Moments and unimodality are examples of how marginals might be characterized, and these characteristics need to be propagated through g to help determine envelopes for $z=g(x,y)$. Some recent work appears in Manski (2003 [20]).

8) **Properties of the space of CDFs.** A focus on envelopes can take attention away from other properties of a family of distributions that might usefully be propagated through g (in the case of marginals) or used for decisions (in the case of z). Convexity – or lack of it – is an example of such a property (Kyberg and Pittarelli 1996 [17]). Is there a "natural" sense in which the density of functions in the family is greater in some areas within the bounds defined by the envelopes than in other areas? If so, what are the implications of this?

3.2 Applications challenge problems

In this section four problems in separate application domains are described. Such problems illustrate the potential for use of flexible and dependable use of uncertainty on important and challenging problems.

3.2.1 The Networking domain

Providing quality of service (QoS) guarantees to flows whose traffic distributions are unknown is an example of a type of system where there is a need to flexibly deal with uncertainty. Guarantees are of interest for a number of performance measures, such as minimum bandwidth, maximum delay, maximum delay jitter, and maximum data loss rate. QoS guarantees take two forms, deterministic and stochastic. Deterministic guarantees provide absolute bounds on the performance offered to the traffic. This typically requires some knowledge of upper bounds of the traffic characteristics, which is provided in practice through traffic regulation at the ingress to the network. This effectively acts to keep the demands on the network within a certain envelope [27]. For example, the leaky bucket is a prominent and simple mechanism for traffic regulation which is used in ATM networks [27]. Service guarantees are then provided to the regulated traffic by routers that provide minimal service guarantees for each of the input flows [4]. The combination of the upper bound on input traffic, and the lower bound on service provides the guarantees. Figure 2 shows an example of an arrival stream and the input traffic curve that it should not exceed. The regulated input traffic curve is also shown, where packets are delayed at the ingress to the network.

Stochastic guarantees, in contrast to the deterministic guarantees just mentioned, provide probabilistic bounds rather than deterministic ones. For example, the delay can be guaranteed to be less than a certain value with a probability that is less than, but close to 1. The ingress traffic in this case is a stochastic process (whose tail function needs to be within a certain envelope).

Multiplexing different flows at routers introduces a degree of correlation between the traffic served from such flows, even if the flows are independent. The use of non-FIFO service strategies, e.g. prioritized service, may further increase the degree of correlation. In networks of routers where the route for each of the flows is determined before the start of the session (often called *route pinning*), a router operates on each of the flows potentially with knowledge of its traffic envelope. This traffic envelope corresponds to worst case conditions and, as correlation between flows is rarely taken into account, the guarantees provided on an end-to-end basis tend to be more conservative than necessary and, usually, unrealistically weak. For example, using (σ,ρ)-

calculus (chapter 1 of [27]) it can be shown that if two flows are multiplexed in a queue, the sum of the envelopes of the output processes of the two flows is much higher than the envelope of the total output process. This is due to the assumption of worst case conditions on the service strategy. Using knowledge about the correlation between the different flows would lead to better envelopes, hence more informed service strategies and ultimately better guarantees.

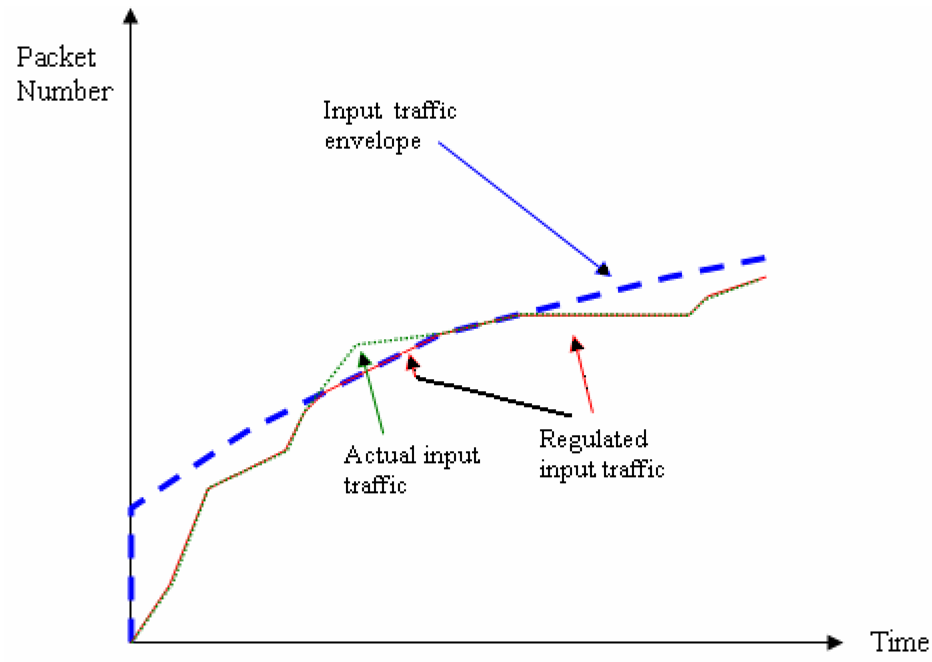

Figure 2. An example of an input traffic stream which is shaped in order to conform to a certain traffic envelope.

3.2.2 Modeling sensor nodes and clusters in large-scale wireless sensor networks

Large-scale distributed sensor networks are comprised of many small sensing devices equipped with memory, processors, and short-range wireless communication. They can be used in a wide range of challenging environments in which sensor nodes are linked together to monitor and report distributed event occurrences. Optimizing use of such sensor networks will require simple, scalable, self-organizing, and energy-efficient algorithms for data dissemination, discovery of conclusions by humans interacting with the network, routing, and aggregation. However the efficiency and effectiveness of an algorithm relies heavily on how the behaviors of sensor networks in real world settings are modeled. For example, many previous theoretical studies assume a circular connectivity model, in which the transmission range of a sensor forms a circular region, called a connectivity cell, with the sensor at its center. Such a model simplifies the analysis and allows geometric approaches to be used.

However in real sensor networks such simplifications may lead to problematic conclusions. For example the connection radius cannot be accurately modeled as a circular cell, since sensor transmission signals are stronger in some directions than in others. Other sources of uncertainty and variability include the fact that ability to transmit successfully can vary over time as environmental conditions change, as can the ability of other sensors to receive the transmissions, and that batteries deteriorate over time, reducing transmission power and/or limiting the lifetime of the sensor. In addition, coupling between different layers (such as the data link layer, MAC layer, and application layer) results in complex unanticipated behavior in large-scale wireless sensor networks. For example, the interactions among asymmetric links (at the link layer) and contention and collision (at the MAC layer) often complicate the modeling of the collision behavior of the sensors. These make modeling the behavior of sensor networks challenging and difficult, since for example there might be dependencies among these characteristics. Manipulation of distributions whose details and dependency relationships are imperfectly known may find application in modeling complex behaviors of sensor networks and may provide a basis for subsequent algorithmic studies of sensors and sensor networks.

For example, consider the following basic problem. Suppose a sensor network consists of two large sections each containing many sensors, and a small section containing a small number of sensors that must act as relays in order for messages originating in either one of the large sections to be transmitted to the other. For purposes of transmission between the large sections, the lifetime of the relay section determines the lifetime of the entire network. We might seek to address the lifetime of the relay section as a reliability problem, modeling it as an n-component system which fails when k of its components fail, resulting in network partition where sensors between two partitions cannot communicate with each other. However it may not be known whether the lifetimes of its components are positively correlated, negatively correlated, or perhaps independent. This is because some factors suggest positive correlation (such as environmental conditions), some suggest negative correlation (such as if there is a tendency for traffic to go through some relay sensors in preference to others), and so on. Thus calculating the cumulative probability of network failure over time becomes more challenging. Handling of this challenge by the algorithm that the sensors use could enable it to make better decisions as it seeks to optimize network lifetime. Designing such algorithms poses a further interesting challenge.

3.2.3 From 2nd-order uncertainty to bidding decisions in electric energy trading

We consider a problem in the deregulated electric energy market, that of bidding in an electric energy auction to sell electricity. Let two electricity generation companies (GENCOs), GENCO 1 and GENCO 2, be competing to sell the 1000MWh of power needed by the buyer to supply retail customers over a particular 1-hour time period. Both GENCOs submit bids, each consisting of a price and an amount of power, to the Independent System Operator (ISO). Bids are accepted starting from the one with the lowest price per MWh, and proceeding to successively higher priced bids until the total need of 1000 MWh has been reached. A profit-maximizing strategy for GENCO 1 is to try to bid as high as possible while still undercutting the bid of GENCO 2. To do this, GENCO 1 must model its competitor, GENCO 2.

Assume that GENCO 2 owns two generators, 2A and 2B. Generator 2A has a capacity of 300 MW and 2B has a capacity of 700 MW. Generator 2A has a lower cost than generator 2B. We do not know the precise generation cost of each, but model them with probability density functions:

$f_{2A}(.)$: uniform distribution from \$95-105/MWh, and

$f_{2B}(.)$: normal distribution from \$145-155/MWh.

GENCO 1 has only one generator and knows its own generation cost, $f_1(.)$. At this point we have a problem that can be solved without reference to non-classical methods of manipulating uncertainty. However what if precise distribution functions for $f_{2A}(.)$ and $f_{2B}(.)$ are not available? This could be if they were to model not only the generation costs of generators 2A and 2B, but also a profit margin beyond that cost whose value we do not know very well and hence wish to model using an interval (that is, a minimum and a maximum for the range of profit margins we believe GENCO 2 might incorporate into their bids). Then the cumulative distributions corresponding to $f_{2A}(.)$ and $f_{2B}(.)$, call them $F_{2A}(.)$ and $F_{2B}(.)$, will be envelopes (Figure 1). Such 2^{nd}-order uncertainties lead to three questions regarding optimal bidding. These are as follows.

1) What is the optimal bid when intelligence about the competitor, GENCO 2, contains 2^{nd}-order uncertainty, and how might this bid be ascertained?
2) What is the expected monetary value (EMV) of obtaining additional information that would eliminate or reduce the 2^{nd}-order uncertainty, and can that information be obtained at or below that cost?
3) What is the expected cost of ignoring the presence of 2^{nd}-order uncertainty during the bid determination process, instead making assumptions that are not justified by intelligence but are made for the purpose of making the problem tractable, *as is often done*?

Ultimately, understanding the degree to which these questions can be answered will provide the motivation and the means for GENCO 1 to determine its optimal (profit-maximizing) bid. Clearly this problem is an archetype for a large class of competitive bidding problems. The need for solution strategies that properly account for 2^{nd}-order uncertainty is real, and success will have real monetary value.

3.2.4 Integrated circuit fabrication

Process variation has always been a key concern in Integrated Circuit (IC) fabrication as it can lead to yield loss. This is the failure of a manufactured chip to meet functional and performance specifications. With continued shrinking of transistor dimensions, device characteristics will become increasingly sensitive to variations in the fabrication process. Process variations are commonly characterized with probability distributions and analyzed with statistical techniques. Thus advances in reasoning under uncertainty may be expected to enable advances in process control and circuit design.

As an example, Metal-Oxide-Semiconductor Field Effect Transistor (MOSFET) devices are well known to be particularly sensitive to effective channel length (chapter 6 of [3]). Channel length variation has direct impact on device output current characteristics. Channel length variation can be decomposed into inter-die and intra-die components. Inter-die variation is the variation across nominally identical dice. It can be caused by several physical and independent sources like variations in etch rate, implant dose, and implant energy. Intra-die variation is the spatial variation within any one die. It can be caused by variations in mask, lens or photo system, photolithography proximity effects, etc.

It is usually sufficient to lump the contributions of different sources of inter-die variation together into a single effective die-to-die variation component with a single mean and variance. On the other hand, intra-die variation may be systematic [21]. By systematic, we mean the variation follows a non-random pattern. Bounds can often be derived for systematic variations. For example, the variation of channel length can be viewed as a probability distribution with uncertain but bounded mean. The effect of changes in the process technology to the channel length and hence important device characteristics, and the resulting downstream effects on yield, implications for yield optimization, and cost issues, become difficult to analyze dependably. We hypothesize that advanced techniques for reasoning about uncertainty can lead to improved understanding of, and solutions for, such problems.

4 Conclusion

Some problems and challenges relating to reasoning in the presence of uncertainty have been presented. Many opportunities for advancement exist. Several advances are given in the papers of this special issue. We expect that continued advances will have increasingly felt practical effects as practice follows growing awareness of the value of modeling 2nd-order uncertainty.

5 Acknowledgements

The authors wish to thank Vyacheslav Nesterov for his critical role in initiating and supporting this special issue. The authors' relevant work is supported in part by the Power Systems Engineering Research Center (PSERC), and by Natural Sciences and Engineering Research Council of Canada (NSERC) research grant 185986.

6 References

[1] Benford, F., The Law of Anomalous Numbers, *Proceedings of the American Philosophical Society* **78** (1938), pp. 551-572.

[2] Berleant, D. and J. Zhang, Using correlation to improve envelopes around derived distributions, *Reliable Computing*, to appear. http://class.ee.iastate.edu/berleant/home/.

[3] Chandrakasan, A., W.J. Bowhill, and F. Fox, eds., *Design of High-Performance Microprocessor Circuits*, IEEE Press, 2001.

[4] Chang, C.-S., *Performance Guarantees in Communication Networks*, Springer-Verlag, 2000.

[5] Coolen, F.P.A., P. Coolen-Schrijner, and K.J. Yan, Nonparametric predictive inference in reliability, *Reliability Engineering and System Safety* **78** (2002), pp. 185-193.

[6] Cozman, F., Credal Networks, *Artificial Intelligence* **120** (2000), pp. 199-233.

[7] Cui, W.C. and D.I. Blockley, Interval probability theory for evidential support, *International Journal of Intelligent Systems* **5** (1990), pp. 183-192.

[8] Davis, J.P. and J.W. Hall, A software-supported process for assembling evidence and handling uncertainty in decision-making, *Decision Support Systems* **35** (2003), pp. 415-433.

[9] Dubois, D. and H. Prade, *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, Plenum Press, 1988.

[10] Fagiuoli, E. and M. Zaffalon, An exact interval propagation algorithm for polytrees with binary variables, *Artificial Intelligence* **106** (1) (1998), pp. 77-107.

[11] Hampel, F., Robust statistics: A brief introduction and overview, Research Report No. 94, 2001, Seminar für Statistik, Edgenössische Technische Hochschule (ETH), Switzerland, stat.ethz.ch/Research-Reports/94.pdf. See also Huber, P.J., *Robust Statistics*, Wiley, 1981. See also *Int. Conf. on Robust Statistics 2003*, http://win-www.uia.ac.be/u/icors03/.

[12] Hill, B.M., Posterior distribution of percentiles: Bayes' theorem for sampling from a population, *Journal of the American Statistical Association* **63** (1968), pp. 677-691.

[13] Hill, T.P., The difficulty of faking data, *Chance* **12** (3) (1999), pp. 27-31.

[14] Hutchinson, T.P. and C.D. Lai, *Continuous Bivariate Distributions Emphasizing Applications*, Rumsby Scientific Publishing, Adelaide, Australia, 1990.

[15] Kolmogoroff, A., Confidence limits for an unknown distribution function, *Annals of Mathematical Statistics* **12** (4) (1941), pp. 461-463.

[16] Kyburg, H.E., Interval-valued probabilities, http://ippserv.rug.ac.be/documentation/interval_prob/interval_prob.html (as of 6/03).

[17] Kyberg, H.E. and M. Pittarelli, Set-based Bayesianism, *IEEE Trans. On Systems, Man, and*

Cybernetics, **26** (3) (1996), pp. 324-339.
[18] Levi, I., *The Enterprise of Knowledge, an Essay on Knowledge, Credal Probabiliy, and Chance*, MIT Press, 1980.
[19] Little, R.J. and D.B. Rubin, *Statistical Analysis with Missing Data*, Wiley, 1987.
[20] Manski, C.F., *Partial Identification of Probability Distributions*, Springer-Verlag, 2003.
[21] Mehrotra, V., Modeling the Effects of Systematic Process Variation on Circuit Performance, dissertation, MIT, 2001.
[22] Nelsen, R.B., *An Introduction to Copulas*, Lecture Notes in Statistics Vol. 139, Springer-Verlag, 1999.
[23] Newcomb, S., Note on the frequency of use of the different digits in natural numbers, *American Journal of Mathematics* **4** (1881), pp. 39-40.
[24] Sherwood, H. quoted at http://gro.creditlyonnais.fr/content/rd/home_copulas.htm as of 6/03, notes the great, yet often under-recognized overlap among the areas of joint probability distributions with fixed marginals, copulas, doubly stochastic measures, Markov operators, and dependency relations.
[25] Staum, J., Fundamental theorems of asset pricing for good deal bounds, *Mathematical Finance*, forthcoming. See also Technical Report 1351, Dept. of ORIE, Cornell University, 2002.
[26] Vansteelandt, S. and E. Goetghebeur, Analyzing the sensitivity of generalized linear models to incomplete outcomes via the IDE algorithm, *Journal of Computational and Graphical Statistics* **10** (4) (2001), pp. 656-672.
[27] Wang, Z., *Internet QoS: Architectures and Mechanisms for Quality of Service*, Morgan-Kaufmann, 2001.
[28] *Electronic Bulletin of the Rough Set Community*. http://www2.cs.uregina.ca/~roughset/.
[29] The Imprecise Probabilities Project (home page), http://ippserv.rug.ac.be/home/ipp.html (updated Nov. 2000).
[30] *Fuzzy Sets and Systems*, journal, Elsevier. http://www.elsevier.nl/locate/fss.
[31] *International Journal of Approximate Reasoning*, Elsevier.
[32] www.gloriamundi.org, gro.creditlyonnais.fr, and www.risklab.ch are sources for reports on mathematical finance, copulas, and related items, including a few mentioning Spearman correlation (as of 6/03).