N^{TH} ORDER CIRCULAR SYMMETRY PATTERN AND HEXOGANAL TESSELATION: TWO NEW LAYOUT TECHNIQUES CANCELLING NONLINEAR GRADIENT

Chengming He, Kuangming Yap, Degang Chen, Randall Geiger

Iowa State University, Ames, IA, 50011, USA

ABSTRACT

This paper discusses systematic mismatch due to gradient in IC fabrication and presents a generic layout strategy to enhance matching characteristics for passive and active devices in VLSI. Circular symmetry pattern, a new type of layout structure, was proposed and its effect on high order gradient cancellation was discussed. Analytical deviations suggested circular symmetry pattern has the potential to cancel high order gradient mismatching between two devices. This paper also proposed two novel layout structures, the second circular symmetry pattern and a hexagonal tessellation, which can cancel linear quadratic gradient in resistors, capacitors and threshold voltages of transistors. Simulation results of proposed second circular symmetry pattern and hexagonal tessellation have a better matching performance than existing layout structure.

1. INTRODUCTION

Matching characteristics of capacitors, resistors and current mirrors play a critical role in high performance analog and mixed-signal circuits. For example, match of two capacitors in a switched-capacitor amplifier determines its gain's accuracy; match of capacitors in a pipeline stage dramatically affects a pipeline ADC's performance such as INL, signal-to-distortion ratio. Previous studies have shown mismatch between devices could degrade a circuit's performance [1, 2]. Thus, it is important to maintain good match in layout to minimize mismatch after fabrication. Mismatch includes systematic and random mismatch. Random mismatch is typically modeled as Gaussian distribution. It can be compensated by increasing area [3]. Systematic mismatch, however, is determined by layout and process. Previous studies by Felt et al. [4] suggested that the systematic mismatch could be large enough to swamp out random mismatch. Systematic mismatch is less desirable than random mismatch because systematic mismatch can show up degrading at low specifications while random mismatch shows up typically at high-end specifications. In IC fabrication, systematic mismatch refers to a spatial gradient in component values. Common centroid pattern can be used to compensate linear gradient [5], however, no reported layout strategy claimed to compensate nonlinear gradient completely. Lan [6] proposed a current mirror layout technique to compensate drain current mismatch due to linear gradient in threshold voltage completely at certain degrees. Our study extends these discoveries and gives a more generic form. After investing a concept of circular symmetry, we proposed a generic layout strategy to handle mismatch due to high order nonlinear gradient.

2. GRADIENT MODELING

The effect of gradient on a parameter Ω is typically modeled in a distributed way through the area of interest. Two dimension functions are widely used in the modeling while more accurate modeling even uses three-dimensional functions. These models give a better approximation of real gradient but it is difficult to find the useful information from complex equations.

In our work, a two dimension polynomial function Ω(x,y) is used to represent the gradient. However, instead of using a distributed integral form, an ideal point is used to represent a device of interest. After obtaining the property of critical points, we extend the results to general cases. Equation (1) shows this gradient model.

$$\Omega(x, y) = \sum_{j=0}^{N} \sum_{i=0}^{j} a_{ij} x^i y^{j-i} \tag{1}$$

3. A N^{TH}-ORDER CIRCULAR SYMMETRY PATTERN AND A HEXAGONAL TESSELLATION

Before we go to study different layouts' effect on gradient cancellation, we first study some general graph patterns.

3.1. N^{th}-order Circular Symmetry Pattern

Figure 1 depicts a Nth-order circular symmetry pattern. As shown in Figure 1, 2^N points locates around a circle (centered at (x_0,y_0) with radius ρ). Their coordinates are expressed in equation (2)

$$\{X_i \mid (x_0 + \rho\cos\theta_i, y_0 + \rho\sin\theta_i), i = 1,...,2^N\}$$

$$\{\theta \mid \theta_i = \theta_0 + (i-1)\frac{2\pi}{2^N}, i = 1,...,2^N\} \tag{2}$$

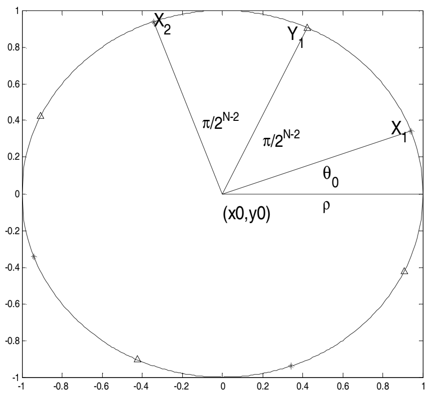

Figure 1 an Nth-order circular symmetry pattern

Using the gradient model (1), the effect of gradient on a point X_i can be expressed as (3) and the total effect can be expressed by equation (4)

$$\Omega(X_i) = \Omega(X = (x_0 + \rho\cos\theta_i, y_0 + \rho\sin\theta_i)) \quad (3)$$

$$Tot = \sum_{i=1}^{2^N} \Omega(X_i) \quad (4)$$

Using trigonometry theory, we can use equation (5) to represent (3) with appropriate coefficients. The coefficients are determined by the coefficients in equation (1).

$$\Omega(X_i) = g_0(x_0, y_0, \rho) + \sum_{j=1}^{N} g_j(x_0, y_0, \rho)\sin(j\theta_i + \phi_j) \quad (5)$$

Thus we can extend equation (4) to (6).

$$\begin{aligned} Tot &= 2^N g_0(x_0, y_0, \rho) + \sum_{i=1}^{2^N}\sum_{j=1}^{N} g_j(x_0, y_0, \rho)\sin(j\theta_i + \phi_j) \\ &= 2^N g_0(x_0, y_0, \rho) + \sum_{j=1}^{N} g_j(x_0, y_0, \rho)\sum_{i=1}^{2^N}\sin(j\theta_i + \phi_j) \end{aligned} \quad (6)$$

Define $VarP_j = g_j(x_0, y_0, \rho)\sum_{i=1}^{2^N}\sin(j\theta_i + \phi_j)$, then

$$Tot = 2^N g_0(x_0, y_0, \rho) + \sum_{j=1}^{N} VarP_j \quad (7)$$

We will show $VarP_j = 0$ using the well known equality $\sin(\theta + 2k\pi + \pi) + \sin\theta = 0$.

Because $\{\theta \mid \theta_i = \theta_0 + (i-1)\frac{2\pi}{2^N}, i = 1,...,2^N\}$, for $j \le N$,

$$\sum_{i=1}^{2^N}\sin(j\theta_i + \phi_j) = \sum_{i=1}^{2^N}\sin[j\theta_0 + j(i-1)\frac{2\pi}{2^N} + \phi_j] \quad (8)$$

It is not apparent that equation (8) is identical to zero. However, if j is odd, it is easy to show $\theta_{i+2^{N-1}} - \theta_i = j\pi, i = 1,2,\cdots,2^{N-1}$. At such a case, $VarP_j = 0$.

If j is even, we can express $j = 2^m A, A\ is\ odd$. It is easy to show that $\theta_{2^{N-m-1}+i} - \theta_i = A\pi$.

$\{\theta \mid \theta_i = \theta_0 + (i-1)\frac{2\pi}{2^N}, i = 1,...,2^N\}$ can be grouped to 2^m exclusive and complete sub sets. At both cases, $VarP_j = 0$.

Thus,

$$Tot(X) = 2^N g_0(x_0, y_0, \rho) \quad (9)$$

We call the pattern of $X_i, i = 1,2,\cdots,2^N$ an Nth-order circular symmetry pattern. One of the most important properties of this new pattern is **rotation-invariance**. This has been shown in our derivations since θ_0 can be any value. Figure 1 also displays another 2^N points {Y} $\{Y_i \mid (x_0 + \rho\cos(\theta_i + \frac{\pi}{2^N}), y_0 + \rho\sin(\theta_i + \frac{\pi}{2^N})), i = 1,...,2^N\}$ in the Nth-order circular symmetry layout.

Based on our discovery (9), $Tot(Y) = Tot(X)$. Thus, we demonstrated and proved a new layout pattern, which will cancel mismatch due to linear gradient and up to the Nth order nonlinear gradient.

Our discovery is consistent with previous studies [5]. Common centroid layout, which falls into 1st-order circular symmetry pattern, effectively cancels threshold voltage mismatch due to linear gradient.

The layout pattern shown in figure 1 is sufficient to cancel linear and up to the Nth order nonlinear gradient. However, this layout pattern is not necessary to be optimal at any level.

3.2. Hexagonal Tessellation

Hexagon, the basic cell of bee nest, has wide applications in communication, architecture, chemical engineering and so on because of its high mechanical strength, high spatial efficiency. We will show hexagon also is the most concise layout pattern that can cancel linear and quadratic gradient completely. Furthermore, we can extend hexagon to construct Hexagonal Tessellation easily without space-waste.

Figure 2 shows a hexagon in a circle. The coordinates of $\{X_i, i = 1,2,3\}$ can be annotated as

$$\{X_i, Y_i \mid (x_0 + \rho\cos\theta_i, y_0 + \rho\sin\theta_i), i = 1,2,3\}$$
$$\{\theta \mid \theta_i = \theta_0 + (i-1)\frac{2\pi}{3} + \frac{\pi}{3}a, i = 1,2,3\}\ a = 1 : Y; 0 : X \quad (10)$$

For a quadratic gradient (N=2 in equation 1), we can prove that the total gradient is not related to θ_0.

$$\sum_{i=1}^{3} \Omega(X_i) = f(x_0, y_0, \rho) \quad (11)$$

$$\sum_{i=1}^{3} \Omega(X_i) = \sum_{i=1}^{3} \Omega(Y_i) \quad (12)$$

This suggests hexagon structure can cancel linear and quadratic gradient.

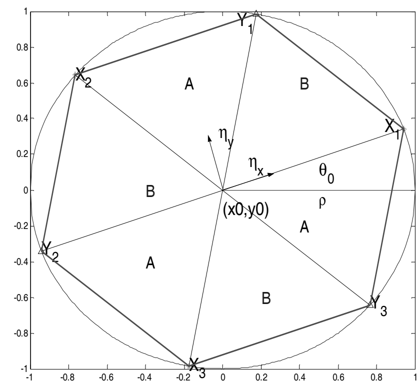

Figure 2 hexagonal tessellation

3.3. Layout of Nth-order Circular Symmetry and Hexagonal Tessellation

By *rotating a unit cell around a center* by $\pi/2^N$ for 2^N times, and connect alternations together, one can construct an Nth-order circular symmetry layout with good match. By *rotating a unit cell around a center* by $\pi/3$ for 6 times and connect alternations together(as shown in Figure 2 with label 'A' and 'B's), one can construct a hexagonal layout. Comparing to the second order circular symmetry layout with eight segments, hexagonal layout has only six segments. More importantly, hexagonal tessellation can be extended with arbitrary times wasting no area while no other structures can achieve the same area efficiency.

4. EVALUATION OF DIFFERENT LAYOUT PATTERNS AND SIMULATION RESULTS

In order to evaluate our proposed layouts' performance, a comparison between existing layout patterns and the new proposed structures is presented.

Figure 3 shows some existing layout patterns, including a mirror symmetric, a common centriod and a 'four-segment' layout proposed by Lan [6]. Mirror symmetric pattern (3a) suffers seriously from gradient; Common centriod layout pattern (3b), categorized as the first order circular symmetric pattern, effectively cancels linear gradient effect. Our prediction is consistent with circuit designers' experience.

Figure 3c is a layout proposed by Lan [6] with a name of 'four segment'. Figure 4 is our proposed second order circular symmetric pattern. They are quite similar. However, they are different. Proposed second order circular symmetry pattern will cancel parameter mismatch due to linear and quadratic gradient completely because of its rotation invariance, while Lan's 'four segment' can cancel most quadratic mismatch.

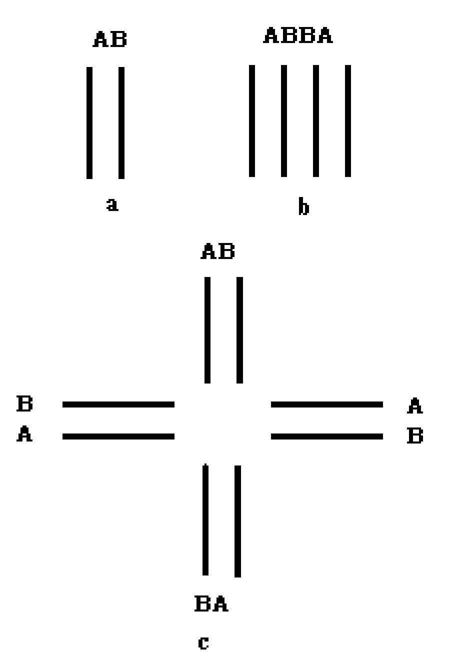

Figure 3 Existing Layout Patterns

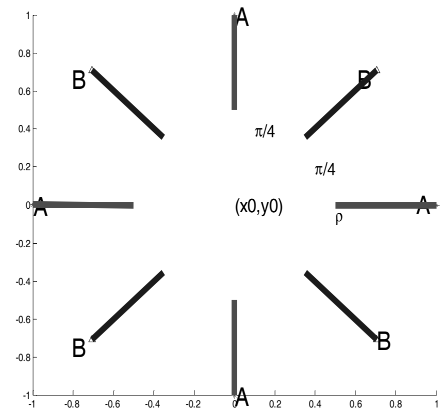

Figure 4 Proposed 2nd order circular symmetry pattern

Lan [6] proposed several current mirror layouts, which enhanced current match dramatically. He claimed to cancel current mismatch due to linear gradient in threshold voltage but simulation results suggested non-exact cancellation. This simulation result seems to contract to our prediction. However, after carefully studying his derivation and his proposed layout structures, we find the reason why the difference exists.

Lan's derivation is based on a special case of 2nd order circular symmetry with $\theta_0 = 0$. However, though Lan's layout patterns are quite close to our proposed 2nd order circular symmetric layout, they are not exactly the same. Figure 4 replicates one of these four segment structure. For a single device, both M1 and M2 are drawn in the form of circular symmetry. However, layouts of A and B have no property of rotation invariance, as shown in Figure 3c. Thus, it is not a second order circular symmetric layout. This discovery also explained why his proposed pattern could not completely cancel current mismatch determined by linear threshold voltage gradient in his simulations.

The second order circular symmetry pattern requires diagonal layout patterns, which is available in non deep-sub-micro CMOS processes but may not be available in DSM processes. It is a drawback.

Hexagonal tessellation is a better pattern to cancel linear and quadratic gradient for its high area efficiency and its ability to extension. Figure 2 is a possible capacitor layout using hexognal tessellation pattern. Figure 5 is a more realistic pattern with an abstract represent using stick graph. This layout pattern is most useful for match-critical resistor, capacitor and current mirror in high-end linear and mixed-signal circuits. This layout has a substantial application in critical match circuits such as switched-capacitor amplifiers, pipeline ADC stages and so on.

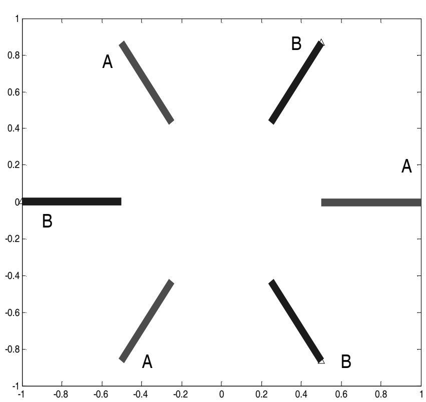

Figure 5 Hexagonal Tessellation

Based on equation (9), we can develop cubic or higher order circular symmetry patterns to compensate high order nonlinear gradient. However, high order nonlinear gradient is typically not serious in a die area. Furthermore, they need more flexible layout rules and tools.
As an example, a current mirror was used to illustrate the performance of different layout structures. In this example, only threshold voltage has mismatch due to linear gradient.

Table 1 summaries simulation results for different layout patterns. Gradient setup is the same as that in [6] and part of the results are cited from Lan's work [6]. Simulation results confirmed the better performance of our proposed new layout patterns.

Table 1 comparison between different layout patterns

Layout structure	Worst mismatch(%)	Effective resolution
Simple	4.8807	3-bit
Common centriod	1.6966e-2	12-bit
Four segment [6]	1.4090e-4	18-bit
2^{nd} order circular symmetric	0	∞
hexagonal tessellation	0	∞

5. CONLUSIONS

We studied systematic mismatch of two components due to linear and nonlinear gradient in the layout of integrated circuits, proposed a generic way called Nth-order circular symmetric pattern to cancel the linear to the nth order nonlinear gradient and proved our new method mathematically. We also proposed an area efficient hexagonal tessellation cell which can cancel mismatch due to linear and quadratic gradient. The new hexagonal tessellation can be extended easily without wasting die area. Numerical simulation results are consistent with our prediction well.
Our studies extend existing discoveries and experiences to a generic way, which is consistent with previous studies but more powerful, more knowledge-based. This work will help to enhance students, researchers and designers in VLSI fields to understand the effects of gradient (one of process variations) more precisely and helps them use systematic methods to compensate them, both in circuit level and in mathematical level.

6. REFERENCES

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