Minimization of Total Area in Integrated Active RC Filters

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***Abstract*— A useful theorem for the design of integrated active RC filters with minimum passive area is introduced and proven. A necessary condition for a minimum-area design is presented. Design examples based on the established condition are discussed.**

I. Introduction

Analog filters are indispensable for interfacing digital systems and the physical world. Many well-known continuous-time analog filter structures inherently require resistors or conductors for adding and amplifying signals and capacitors as frequency-dependent elements which realize the Laplace variable s that appears in the system transfer function. Active elements are also necessary for achieving quality factors larger than $1/\sqrt{2}$ if only resistors and capacitors are used as passive elements. One of the most popular classes of continuous-time filters is comprised of circuits that are termed RC active filters. An RC active filter employs operational amplifiers, resistors, and capacitors and can provide high linearity. These filters can also operate with low supply voltages. RC active filters are also used as a prototype where resistors are replaced by equivalent circuits comprised of MOSFETs [1] in attempts to reduce the total area required for the filter in integrated applications. Nevertheless, for low-frequency applications, e.g. audio and speech equipment, relatively large resistances and capacitances must be integrated to realize low pole frequencies. Since the area required for large resistances and large capacitances in monolithic implementations is inherently large, an important issue is how to determine resistances and capacitances to make the physical area of the filter as small as possible.

To minimize a total area of an RC active filter, a conjecture that the minimum area is achieved when the total area of resistors is equal to that of capacitors was presented in Ref. [2]. This conjecture was motivated by considering a small number of design examples in which it was shown that the passive area in these examples was minimized when the total resistor area was equal to the total capacitor area. Although the conjecture seemed plausible since several substantially different circuit structures were considered, the authors in [2] did not provide a proof for the conjecture nor attempt to identify what constraints, if any, on the filter topology may be necessary if the conjecture is valid. It can be observed that each of the design examples presented in [2] was a system with only one degree of freedom.

In this paper it is shown that the conjecture as stated in [2] is not always true but is true if some additional constraints on the characteristics of the circuit are added. In the next section the following theorem is logically proven.

Theorem

If an integrated active RC filter structure that satisfies a dimensionless transfer function specification is given, then the total area for the resistors is equal to that of capacitors when the minimum total-area for the passive components in this structure is achieved.

Some design examples using this theorem as well as several counterexamples showing the conjecture does not apply in the most general sense are given in Section III.

II. Theorem for Area Minimization

In this optimization problem it is assumed that there exists at least one design satisfying specifications and the number of design parameters (resistors and capacitors) is larger than that of specifications.

Let the numbers of resistors and capacitors be N_R and N_C, respectively. If all resistors in an integrated active RC filter are realized with the same preferred thin film layer that has an associated sheet resistance $R_\square$ and it is assumed that all resistors in an integrated active RC filter are implemented by a series connection of unit resistors of width W_R, then the resistance density can be defined as $R_d = R_\square W_R^{-2}$ and the area required for the k-th resistor of value R_k $(k = 1, 2, \ldots, N_R)$ is given by R_k/R_d. Correspondingly, the area of the l-th capacitor $(l = 1, 2, \ldots, N_C)$ is by C_l/C_d where C_l is its capacitance and C_d is a capacitance density, respectively. The total area A_T is the sum of A_R and A_C where A_R is the total areas of resistors $\sum_{k=1}^{N_R}(R_k/R_d)$ and A_C is that of capacitors $\sum_{l=1}^{N_C}(C_l/C_d)$. R_k's and C_l's cannot be independent and should satisfy equations corresponding to specifications:

$$F_i(R_1, R_2, \ldots, R_{N_R}, C_1, C_2, \ldots, C_{N_C}) = F_i^{(spec)} \qquad (1)$$
$$(i = 1, 2, \ldots, N_{(spec)}, \quad N_{(spec)} < N_R + N_C)$$

where a function F_i is the i-th circuit function with a bilinear form of each parameter and $F_i^{(spec)}$ is the i-th specified value. From the assumption described at the beginning of this section, these simultaneous equations have a solution which can be written as $R_k = \hat{R}_k$ and $C_l = \hat{C}_l$.

The considered problem is minimization of the total area A_T in a solution space with one or more degree of freedom. Fortunately one degree of freedom can be known from the theorem of impedance scaling [3]. The theorem guarantees that $R_k = \alpha_1 \hat{R}_k$ and $C_l = \hat{C}_l/\alpha_1$ satisfy the simultaneous equations for an arbitrary α_1 which is a parameter for one degree of freedom. This scaling does not change any non-dimensional circuit function F_i. Although input and output impedances vary with α, they are not taken by F_i's and the effect of variation in impedances can be avoided by use of buffers. Since the degree of freedom $N_{(frdm)}$ is not greater than $N_R + N_C - N_{(spec)}$, the other parameters α_m $(m = 2, \ldots, N_{(frdm)})$ for $N_{(frdm)} - 1$ degree of freedom can be introduced. Then resistances and capacitances are functions of these freedom parameters as

$$R_k = \alpha_1 \hat{R}_k(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}}) \tag{2}$$

and

$$C_l = \frac{1}{\alpha_1}\hat{C}_k(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}}) \tag{3}$$

and areas can be written by

$$A_R = \sum_{k=1}^{N_R} \alpha_1 \frac{\hat{R}_k(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})}{R_d} = \alpha_1 \hat{A}_R, \tag{4}$$

$$A_C = \sum_{l=1}^{N_C} \frac{1}{\alpha_1} \frac{\hat{C}_l(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})}{C_d} = \frac{1}{\alpha_1}\hat{A}_C \tag{5}$$

and

$$A_T = \alpha_1 \hat{A}_R + \frac{1}{\alpha_1}\hat{A}_C \tag{6}$$

where

$$\hat{A}_R = \sum_{k=1}^{N_R} \frac{\hat{R}_k(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})}{R_d} \tag{7}$$

and

$$\hat{A}_C = \sum_{l=1}^{N_C} \frac{\hat{C}_l(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})}{C_d}. \tag{8}$$

An inequality relating to the total area is obtained using a mathematical formula $x + y \geq 2\sqrt{xy}$ for arbitrary positive real numbers x and y. Here the both sides are equal if and only if $x = y$. From substitution of the first and the second terms of the right-hand side of Eq. (6) for x and y, respectively, an inequality

$$A_T \geq 2\sqrt{\hat{A}_R \hat{A}_C} \tag{9}$$

is obtained and it is known that the total area for fixed $\alpha_2, \alpha_3, \ldots,$ and $\alpha_{N_{(frdm)}}$ becomes the minimum

$$\begin{aligned} & A_T^*(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}}) \\ &= 2\sqrt{\hat{A}_R(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})\hat{A}_C(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})} \end{aligned} \tag{10}$$

when

$$\alpha_1 = \sqrt{\frac{\hat{A}_C(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})}{\hat{A}_R(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})}}, \tag{11}$$

that is

$$A_R(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}}) = A_C(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}}) \tag{12}$$

Letting the minimum or the lower bound of $A_T^*(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}})$ be $A_{T,min}$, since

$$A_T \geq A_T^*(\alpha_2, \alpha_3, \ldots, \alpha_{N_{(frdm)}}) \geq A_{T,min} \tag{13}$$

for all R_k's and C_l's, $A_{T,min}$ is the globally minimum total area in the solution space. $A_{T,min}$ is realized by using appropriate values α_m^* $(m = 2, 3, \ldots, N_{(frdm)})$ and α_1 given in Eq. (11) under $\alpha_m = \alpha_m^*$. It is known that Eq. (12) is satisfied for the case of the minimum area but that the total area does not become the minimum even with α_1 of Eq. (12) when non-optimum solutions α_m's are used. Conclusively it has been proven that Eq. (12) is a necessary condition for the minimization. ∎

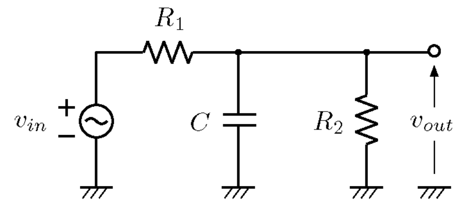


Fig. 1. Example of RC circuits.

From the practical point of view the theorem can be rephrased that, when a design problem is minimizing the total area on an arbitrarily fixed topology subject to specified poles and zeros, gains, phases, and group delays, which are issues of filter designs except for input and output impedances, the total area of resistors should always equal that of capacitors. If degree of freedom is 1, the global minimum is easily brought by adding a restriction $A_R = A_C$.

III. Examples

A. Doubly-terminated 1st-order passive filter

The proven theorem requires that no specification restricting a circuit impedance is included. For seeing that $A_R = A_C$ is not a sufficient condition, a simple counterexample for the inverse of the theorem as well as an example where Eq. (12) gives the minimum is shown in this section. Moreover an example where A_R is equal to A_C but A_T is not the minimum is also presented.

Figure 1 shows a simple example of RC circuits. Design parameters are R_1, R_2, and C and the transfer function of this filter is

$$\frac{v_{out}}{v_{in}} = \frac{R_2}{R_1 + R_2} \frac{1}{1 + sC\dfrac{R_1 R_2}{R_1 + R_2}}. \tag{14}$$

Three different designs for a circuit topology shown in Fig. 1 are following.

1) Minimum known from the impedance scaling: Realizing a specified transfer function is often requested in a filter design. As the first case design with a DC gain G and a pole $-p$ given is considered. This design does not restrict any impedances. Solving

$$R_2/(R_1+R_2)=G \quad \text{and} \quad CR_1R_2/(R_1+R_2)=1/p \quad (15)$$

for R_1 and R_2,

$$R_1=1/(GpC) \quad \text{and} \quad R_2=1/[(1-G)pC] \qquad (16)$$

are obtained and there is one degree of freedom as written by C. It is also verified that the impedance scaling is true in this example. The total area is

$$\begin{aligned} A_T &= \frac{R_1}{R_d}+\frac{R_2}{R_d}+\frac{C}{C_d}=\frac{1}{R_d}\left(\frac{1}{Gp}+\frac{1}{(1-G)p}\right)\frac{1}{C}+\frac{C}{C_d} \\ &\geq 2\sqrt{\frac{1}{R_dC_d}\left(\frac{1}{Gp}+\frac{1}{(1-G)p}\right)} \qquad (17) \end{aligned}$$

and the minimum is achieved only when

$$C=\sqrt{\frac{C_d}{R_d}\left(\frac{1}{Gp}+\frac{1}{(1-G)p}\right)}. \qquad (18)$$

This capacitance requires the total area of the resistors equal to an area of the capacitor.

2) Impedance fixed: In some cases matching of impedances are important for connecting circuits. Let us consider a case that an arbitrary DC gain is available but a DC input impedance Z_{in} and a pole are fixed. Simultaneous equations for the specifications becomes

$$R_1+R_2=Z_{in} \quad \text{and} \quad CR_1R_2/(R_1+R_2)=1/p \qquad (19)$$

and R_2 and C are given as

$$R_2=Z_{in}-R_1 \quad \text{and} \quad C=Z_{in}/[pR_1(Z_{in}-R_1)]. \qquad (20)$$

In this case resistors and capacitors cannot be expressed with a parameter α_1 for scaling. The areas A_R and A_C are equal to Z_{in}/R_d and $Z_{in}/[C_dpR_1(Z_{in}-R_1)]$, respectively, and the total area is to

$$A_T=\frac{Z_{in}}{R_d}+\frac{Z_{in}}{C_dpR_1(Z_{in}-R_1)}. \qquad (21)$$

This area becomes the minimum for $R_1=Z_{in}/2$ and $A_R=Z_{in}/R_d$ and $A_C=4/(C_dpZ_{in})$ may be different depending on the given Z_{in} and p.

3) Counterexample for sufficiency: The theorem states that, even if $A_R=A_C$, A_T may not become the minimum. When only a pole is specified in a design, the equal areas of resistors and capacitors do not always result in the minimum A_T. In this design two degrees of freedom exist for satisfying Eq. (15). From the additional condition $A_R=A_C$ two equations

$$R_1+R_2=(R_d/C_d)C \quad \text{and} \quad R_1R_2=(R_d/C_d)(1/p) \quad (22)$$

are deduced. If C is set as an arbitrary number in a region $C\geq\sqrt{(4C_d)/(pR_d)}$, the positive resistances are solved for. Then the total area is $A_T=2C/C_d$. Therefore, when C is in

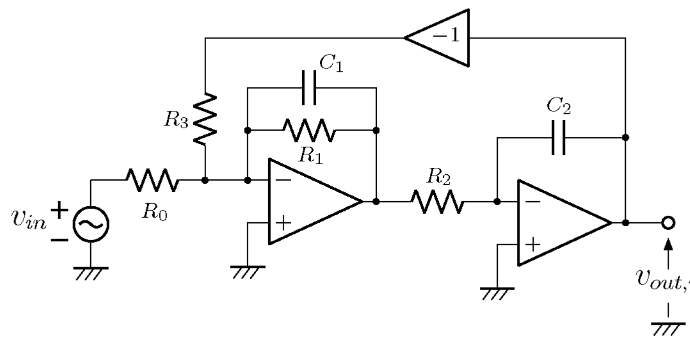


Fig. 2. 2nd-order active RC filter.

the region except for its boundary, $A_R=A_C$ is satisfied but A_T is not the minimum. Although $A_R=A_C$ is not sufficient for the minimum-area design, this necessary condition should be used for the design.

B. 2nd-order active RC filter

1) Design using $A_R=A_C$: Another filter is shown in Fig. 2. A transfer function is equal to

$$\frac{v_{out}}{v_{in}}=\frac{R_3/R_0}{s^2C_1C_2R_2R_3+sC_2R_2R_3/R_1+1}. \qquad (23)$$

The element values must satisfy relations

$$C_1C_2R_2R_3=1/\omega_c^2 \quad \text{and} \quad R_1^2C_1/(C_2R_2R_3)=Q^2 \quad (24)$$

where ω_c is a specified cutoff angular frequency and Q is a quality factor. Let a DC gain R_3/R_0 be set as 0 dB $=1$ and a design with $R_1=R_2$ and $C_1=C_2$ be assumed for small element value spreads. Then design solutions have 1 degree of freedom and can be expressed by

$$R_1{=}R_2{=}Q/(C_1\omega_c),\ R_0{=}R_3{=}1/(C_1Q\omega_c),\ \text{and}\ C_2{=}C_1. \qquad (25)$$

An inverting buffer with a gain -1 can be composed of an operational amplifier and two resistors with equal values. Since its area can be made small with small resistors, the total area of the other components is minimized here. The minimization requires equal areas of resistors and capacitors. This necessary condition is written by an equation

$$\frac{2}{R_dC_1\omega_c}\left(Q+\frac{1}{Q}\right)=\frac{2C_1}{C_d} \qquad (26)$$

by use of the above expressions of resistors and capacitors. Eventually the optimum values are solved as

$$C_1=C_2 = \sqrt{\frac{C_d}{R_d}\frac{Q+1/Q}{\omega_c}}, \qquad (27)$$

$$R_1=R_2 = Q\sqrt{\frac{R_d}{C_d}\frac{1}{\omega_c(Q+1/Q)}}, \qquad (28)$$

and

$$R_0=R_3=\frac{1}{Q}\sqrt{\frac{R_d}{C_d}\frac{1}{\omega_c(Q+1/Q)}}. \qquad (29)$$

2) Numerical examples: Filters with $\omega_c = 50$ krad/s and $Q = 1$ are designed.

a) Passive resistors realized in an active region: Let a sheet resistance be 100 $\Omega/\square$ and a capacitance density C_d be 1 fF/μm^2 as an example. If the minimum width of an active region is W [μm], a resistance density R_d is $100/W^2$ [Ω/μm^2]. Capacitances and resistances should be chosen as

$$C_1{=}C_2{=}20\ W\ \text{[pF]} \quad \text{and} \quad R_0{=}R_1{=}R_2{=}R_3{=}1/\text{W [M}\Omega] \tag{30}$$

and then the total area of resistors and capacitors is $A_T = 8 \times 10^4 \cdot W$ [μm^2]

b) Resistors realized by non-saturated MOSFETs: An MOSFET is sometimes used as a resistor with a value controllable with a gate-source voltage. Its equivalent resistance is proportional to an aspect ratio of its channel. Since this feature is the same with that of a passive resistor, the results obtained in the previous section is still useful in the design with MOSFET resistors. Here let 10 k$\Omega/\square$ be assumed as an equivalent sheet resistance. Using the same capacitance density with the last example,

$$C_1{=}C_2{=}2\ W\ \text{[pF]} \quad \text{and} \quad R_0{=}R_1{=}R_2{=}R_3{=}10/\text{W [M}\Omega] \tag{31}$$

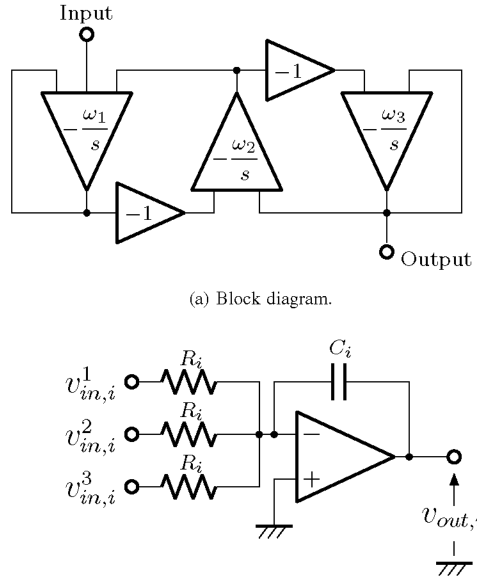
are obtained and A_T is equal to $8 \times 10^3 \cdot W$ [μm^2]. If results of the last example ware used in this case also, the total area would become $40.4 \times 10^3 \cdot W$ [μm^2] which is larger by a factor of about 5 than the minimum A_T. It should be noticed that a non-optimum solution can be easily recognized from unequal areas of resistors and capacitors even if the minimum total area is not known.

C. 3rd-order leapfrog filter

Total-area minimization of the 3rd low-pass filter based on the leapfrog structure shown in Fig. 3 (a) is considered. As a simple example, a Butterworth characteristic with a 50-krad/s cutoff angular frequency. $\omega_1 = 2\omega_2 = \omega_3 = 50$ krad/s is required. Figure 3 (b) is a schema of each integrator. When a resistor and a capacitor of the i-th integrator are denoted as R_i and C_i, respectively, ω_i $(i = 1, 2, 3)$ is realized by $1/(R_i C_i)$. Let us consider the case using passive resistors in active layer where $W = 1$ and set 20 pF to initial values of capacitors for minimization. Resistors of $R_1 = R_2/2 = R_3 = 1$ MΩ are necessary and each of those elements occupies area tabulated in Table I (a). An impedance scaling with $\alpha = \sqrt{6/9}$ gives a smaller total area as summarized in Table I (b). In the case of (b) the total area is not the minimum while the sum of resistor areas is equal to that of capacitor ones. At a design where characteristics of building blocks are fixed, minimization should be performed in each building block with a different appropriate scaling factor. Results of such minimization for each integrator are shown in Table I (c). This design achieves the globally minimum area.

IV. Conclusions

It was proven that equal total areas of resistors and capacitors is a necessary condition for the minimum total area of RC active filters and is not always a sufficient one. If a design has only one degree of freedom, the sufficiency is true. It was mentioned that the theory is applicable to circuits where resistors are replaced by MOSFETs in the non-saturation region. As a future work how to divide a circuit into building blocks should be considered for global minimization.



(b) 3-input integrator.

Fig. 3. 3rd-order leapfrog filter.

TABLE I

Areas of the 3rd leapfrog low-pass filters ($\times 10^4 \mu$m^2).

(a) Initial design.

Integrator ♯	1	2	3	total
Resistors	1×3	2×2	1×2	9
Capacitors	2	2	2	6
Total	5	6	4	15

(b) Scaled for the whole circuit.

Integrator ♯	1	2	3	total
Resistors	$\sqrt{2/3} \times 3$	$2\sqrt{2/3} \times 2$	$\sqrt{2/3} \times 2$	$3\sqrt{6}$
Capacitors	$\sqrt{6}$	$\sqrt{6}$	$\sqrt{6}$	$3\sqrt{6}$
Total	$2\sqrt{6}$	$7\sqrt{6}/3$	$5\sqrt{6}/3$	$6\sqrt{6}$

(c) Scaled for each integrator.

Integrator ♯	1	2	3	total
Resistors	$\sqrt{2/3} \times 3$	$\sqrt{2} \times 2$	1×2	$2 + 2\sqrt{2} + \sqrt{6}$
Capacitors	$\sqrt{6}$	$2\sqrt{2}$	2	$2 + 2\sqrt{2} + \sqrt{6}$
Total	$2\sqrt{6}$	$4\sqrt{2}$	4	$2(2 + 2\sqrt{2} + \sqrt{6})$

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