Explicit Characterization of Bandgap References

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***Abstract*—Conspicuously absent in the literature are explicit relationships between the output voltage and temperature of bandgap references. In this paper an explicit relationship for the output voltage of a popular bandgap reference structure is developed. Within the context of this explicit relationship, temperature stability properties of references are explored.**

I. INTRODUCTION

A stable and precision voltage reference is an integral part in many analog and mixed-signal integrated circuits and systems such as PLLs, memories, and data converters. Since their introduction by Widlar [1] in the 1970's, bandgap references [2-5] have become the reference of choice for precision on-chip voltage sources in both bipolar and CMOS processes. Considerable effort has been devoted to the improvement of performance of the basic bandgap circuits to meet increasingly stringent temperature stability requirements. Different techniques such as laser trimming [2,3], op amp offset cancellation [7], curvature compensation [6-8], etc, have been employed individually or jointly to minimize the temperature drift of the reference voltage in continuous- or discrete-time systems. New architectures [5,9] have been developed which enabled implementations in low voltage or current mode applications.

In spite of innovations and the ensuing performance improvements, little attention has been focused on the theoretical analysis and characterization of bandgap references in the past three decades. The formulation that almost every author adopts is almost the same as that developed by Widlar [1] for the basic bandgap reference concept more than 30 years ago and this trend continues to be followed by authors of some of the most recent textbooks in the field [10-12] as well. In Widlar's work as well as essentially all that followed, the expression for the reference voltage includes a base-emitter voltage of a BJT at some temperature, T_0. This base-emitter voltage is itself a function of the current flowing in the device, making it impossible to determine the actual reference voltage in terms of model parameters, temperature, and supply voltages from the expression that is generally given. Essentially all authors have avoided addressing the question of how the base-emitter voltage is obtained from either the circuit or the device model. Although the formulation of Widlar that has been widely adopted by others is mathematically correct, additional insight into the performance of bandgap references can be obtained if an explicit expression for the reference voltage is obtained and such insight is becoming increasingly important as temperature stability requirements become more stringent.

This paper addresses the issue of analytical characterization of bandgap references. An explicit model of the reference voltage of bandgap circuits involving only process and model parameters will be introduced. The temperature sensitivity of bandgap reference is then analyzed in the context of the explicit model.

The rest of the paper is organized as follows. Part II gives a brief review of bandgap references. Analytical modeling and explicit characterization of a bandgap reference, as well as the discussion of temperature stability, is given in Part III. Part IV concludes the work.

II. REVIEW OF BANDGAP REFENCES

The principle of the operation of a bandgap circuit is to make the reference output be a weighted addition of two signals that have temperature coefficients with opposite signs but equal magnitude. The concept is illustrated in Fig. 1.

With this structure, the two outputs X_P and X_N of the positive and negative temperature coefficient blocks satisfy

$$\frac{\partial X_P(T)}{\partial T} = \psi_P > 0 \tag{1}$$

$$\frac{\partial X_N(T)}{\partial T} = \psi_N < 0 \tag{2}$$

The output is given by

$$X_{OUT} = X_P + KX_N \,. \tag{3}$$

The output will have a zero temperature coefficient at a temperature T_0 if K in (3) is adjusted so that

$$\left.\frac{\partial X_{OUT}}{\partial T}\right|_{T=T_0} = \left.\frac{\partial X_P}{\partial T}\right|_{T=T_0} + K\left.\frac{\partial X_N}{\partial T}\right|_{T=T_0} = 0 \,. \tag{4}$$

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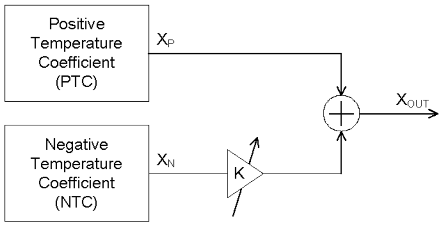

Figure 1. Basic structure of temperature stable reference circuits

It is well known that the collector current and the base-emitter voltage of a BJT have the exponential relationship

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right), \tag{5}$$

where I_S is the saturation current, $V_T = kT/q$, k is Boltzman's constant, T is the temperature in K, and q is the charge of an electron. The parameter I_S is a process and design parameter and can be expressed as

$$I_S = J_S A\,, \tag{6}$$

where J_S is a geometry-independent process parameter and A is the area factor determined by the sizing of the device.

If the temperature dependence of I_S is included, it can be shown that the relationship between collector current density and the base-emitter voltage at a temperature T is related to that at a temperature T_0 by the expression [1]

$$V_{BE}(T) = V_{G0}\left(1-\frac{T}{T_0}\right) + V_{BE}(T_0)\frac{T}{T_0} + \frac{mkT}{q}\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\ln\left(\frac{J_C(T)}{J_C(T_0)}\right), \tag{7}$$

where V_{G0} is the bandgap voltage, J_C is the collector current density and m is a temperature independent constant.

The temperature dependence of V_{BE} at temperature T_0 can be calculated as (assuming J_C has a temperature dependence of T^{α})

$$\left.\frac{\partial V_{BE}}{\partial T}\right|_{T=T_0} = \frac{V_{BE}(T_0) - V_{G0}}{T_0} + (\alpha - m)\left(\frac{k}{q}\right), \tag{8}$$

which turns out to be approximately -2.2mV/°C at room temperature.

If two transistors are operating at the same temperature, it follows from (7) that the difference of the base-emitter voltage of the two devices is given by the expression

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q}\ln\left(\frac{J_{C2}}{J_{C1}}\right)\right]T\,. \tag{9}$$

If $J_{C2} > J_{C1}$, ΔV_{BE} has a positive temperature coefficient. This is generally termed a PTAT (Proportional to Absolute Temperature) source. A bandgap reference can be built if V_{BE} is used for the negative temperature coefficient block of Fig. 1 and ΔV_{BE} is used for the positive temperature coefficient block. A zero temperature coefficient at $T=T_0$ can be achieved with an appropriate gain K.

Equation (7) is useful for relating the operating point at a temperature T to that at a temperature T_0. The terms $J_C(T_0)$ and $V_{BE}(T_0)$ in (7) are convenient to use but are related to each other and are both functions of the current flowing in the device. As such, neither are process parameters nor design parameters. Almost all authors simply gloss over this fact and do not address what information is really carried in these terms or where they come from. Regardless, the functional form of (7) is widely and almost exclusively used when bandgap references are discussed [1-3,10-12]. It is important to emphasize that (7) should not be viewed as an equation that models the BJT (or diode) because the parameters $J_C(T_0)$ and $V_{BE}(T_0)$ are not model or port variable parameters.

III. Characterization of Bandgap References

Equation (7) does not look like the standard diode equation but the relationship to the standard diode equation becomes apparent by rewriting (7) as

$$I(T) = J_C(T_0)Ae^{-\frac{V_{BE}(T_0)}{V_{T_0}}}\left(\frac{T}{T_0}\right)^m e^{\frac{V_{G0}}{V_T}\left(\frac{T}{T_0}-1\right)} e^{\frac{V_{BE}(T)}{V_T}}, \tag{10}$$

where $I(T)$ is the collector (or diode) current and $V_{T0} = kT_0/q$. The standard parameter I_S of (5) that appears in the diode equation is thus

$$I_S(T) = J_C(T_0)Ae^{-\frac{V_{BE}(T_0)}{V_{T_0}}}\left(\frac{T}{T_0}\right)^m e^{\frac{V_{G0}}{V_T}\left(\frac{T}{T_0}-1\right)}. \tag{11}$$

The temperature dependence on I_S is explicitly shown in this equation. Actually, SPICE uses a slightly different partitioning of (10) as shown in (12)

$$I(T) = \left\{\left[J_C(T_0)Ae^{-\frac{V_{BE}(T_0)}{V_{T_0}}}\right]\left(\frac{T}{T_0}\right)^m e^{\frac{V_{G0}}{V_T}\left(\frac{T}{T_0}-1\right)}\right\} e^{\frac{V_{BE}(T)}{V_T}}. \tag{12}$$

If the term in brackets in (12) is defined to be the parameter I_{SX}, that is

$$I_{SX} = J_C(T_0)Ae^{-\frac{V_{BE}(T_0)}{V_{T_0}}}, \tag{13}$$

then it follows from (12) and (13) that the current can be expressed as

$$I(T) = \left\{I_{SX}(T_0)\left(\frac{T}{T_0}\right)^m e^{\frac{V_{G0}}{V_T}\left(\frac{T}{T_0}-1\right)}\right\} e^{\frac{V_{BE}(T)}{V_T}}. \tag{14}$$

The parameter $I_{SX}(T_0)$ is a model parameter that is only dependent upon the process and the geometry of the device and (14) is a model equation that characterizes the device. The choice of the temperature at which the parameter I_{SX} is defined is arbitrary. Note that neither of the parameters $J_C(T_0)$ and $V_{BE}(T_0)$ appears in (14) however a relationship between these parameters can be seen by referring to (13). A second equation will still be needed to obtain these parameters and that will be determined from the circuit in which the device is embedded.

Consider now the bandgap reference circuit shown in Fig. 2. A routine analysis, under the assumption that the op amp is ideal, gives the following set of equations

$$I_{E1}R_2 + V_{BE1} = V_{BE2} \quad (15)$$

$$V_{REF} = V_{BE2} + \left(I_{E1} + I_{E2}\right)R_1 \quad (16)$$

$$I_{C1} = \frac{V_{DD} - V_X}{R_3} \quad (17)$$

$$I_{C2} = \frac{V_{DD} - V_X}{R_4} \quad (18)$$

Since the collector current density is given by

$$J_C = \frac{I_C}{A_E}, \quad (19)$$

solving equations (15)~(19) gives

$$V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right]T. \quad (20)$$

Thus the voltage ΔV_{BE} is a PTAT voltage for this circuit. Considering that the collector and emitter currents of a BJT are related by

$$I_C = \left(\frac{\beta}{1+\beta}\right)I_E = \alpha I_E, \quad (21)$$

it follows from (15)~(21) that

$$V_{REF} = V_{BE2} + \frac{R_1}{R_2}\left(1 + \left(\frac{\alpha_1}{\alpha_2}\right)\frac{R_3}{R_4}\right)\cdot\Delta V_{BE}. \quad (22)$$

Equation (22) shows that the reference voltage is the sum of a V_{BE} voltage and a ΔV_{BE} voltage in which the weight of the ΔV_{BE} voltage can be adjusted by resistor ratios. As long as the current in Q_2 is reasonably constant, the weights can be chosen to have a zero temperature coefficient at a desired temperature.

Unfortunately (22) is not a closed-form expression for V_{REF} in terms of circuit and model parameters. Substituting for V_{BE2} and ΔV_{BE} from (7) and (20) into (22) gives

$$V_{REF} = \left[V_{G0}\left(1 - \frac{T}{T_0}\right) + V_{BE2}\left(T_0\right)\frac{T}{T_0} + \frac{mkT}{q}\ln\left(\frac{T_0}{T}\right) + \frac{kT}{q}\ln\left(\frac{J_{C2}(T)}{J_{C2}(T_0)}\right)\right] + \left\{\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\cdot\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right\}T. \quad (23)$$

It follows from (15), (17), (18) and (21) that

$$I_{C2} = \alpha_1\frac{R_3}{R_4}\frac{\Delta V_{BE}}{R_2}. \quad (24)$$

Observe from (20) and (24) that I_{C2} is a PTAT current. Since the current ratios for Q_2 at two temperatures are equal to the current density ratios, it follows that

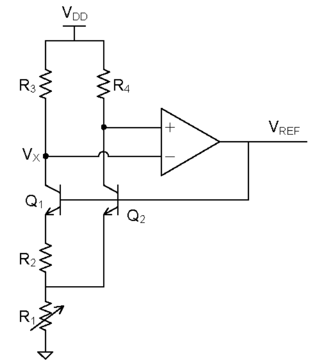

Figure 2. A typical bandgap reference circuit

$$\frac{J_{C2}(T)}{J_{C2}(T_0)} = \frac{T}{T_0}. \quad (25)$$

This can now be substituted into (23) to obtain

$$V_{REF}(T) = V_{G0}\left(1 - \frac{T}{T_0}\right) + V_{BE2}\left(T_0\right)\frac{T}{T_0} + \frac{mkT}{q}\ln\left(\frac{T_0}{T}\right) + \left\{\frac{R_1}{R_2}\left(1 + \frac{\alpha_1}{\alpha_2}\frac{R_3}{R_4}\right)\cdot\frac{k}{q}\ln\left(\frac{A_{E1}}{A_{E2}}\frac{R_3}{R_4}\right)\right\}T. \quad (26)$$

Equation (26) is very similar to that introduced by Widlar [1] in 1971 and is where most authors stop in the analysis of the bandgap reference circuit but, unfortunately, it still contains the term $V_{BE2}(T_0)$ which is neither a model parameter nor a circuit parameter. To remove the $V_{BE2}(T_0)$ term, first take the natural logarithm of (14) and the device model equation can be expressed as

$$V_{BE}(T) = V_T\ln\left(I_C(T)\right) + \theta, \quad (27)$$

where the parameter θ is defined as

$$\theta = -V_T\left[\ln\left(I_{SX}\left(T_0\right)\right) + m\ln\left(\frac{T}{T_0}\right)\right] + V_{G0}\left(1 - \frac{T}{T_0}\right). \quad (28)$$

This parameter is dependent only upon the device model and temperature, not dependent upon the circuit in which the device is used.

Equation (27) is similar to (7) but now contains only process and model parameters and thus does define the I-V characteristics of the device.

It follows from (17), (18), (20) and (24) that

$$I_{C1} = \frac{\alpha_1}{R_2}\frac{kT}{q}\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right) \quad (29)$$

Substituting (29) into (27) gives

$$V_{BE1} = \frac{kT}{q}\ln\left(\frac{\alpha_1}{R_2}\frac{kT}{q}\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right)\right) + \theta_1 \quad (30)$$

Combining (20), (26), (28) and (30) finally gives the close-form explicit expression of V_{REF} as

$$V_{REF} = a + bT + cT\ln\left(\frac{T_0}{T}\right). \tag{31}$$

where

$$a = V_{G0}, \tag{32}$$

$$b = -\frac{V_{G0}}{T_0} + \frac{k}{q}\left(\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right)\left(1+\frac{R_1}{R_2}\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\right)\right)$$
$$+\frac{k}{q}\ln\left(\frac{k}{q}\frac{\alpha_1 T_0}{I_{SX1}(T_0)R_2}\ln\left(\frac{R_3}{R_4}\frac{A_{E1}}{A_{E2}}\right)\right), \tag{33}$$

$$c = \frac{(m-1)k}{q}. \tag{34}$$

This expression is modestly more involved than (26) but is now a closed-form solution that contains only circuit and model parameters.

Inflection point analysis can be done based on (31). Since the parameters a, b and c are not dependent upon temperature, differentiating V_{REF} with respect to T and setting the derivative equal to 0 gives

$$\frac{dV_{REF}}{dT} = b + c\left(-1+\ln\left(\frac{T}{T_0}\right)\right) = 0. \tag{35}$$

This expression can be solved for T to determine the inflection point, T_{INF} to be

$$T_{INF} = T_0 e^{\left(\frac{b}{c}-1\right)}. \tag{36}$$

If the temperature coefficient at T_0 is set to be 0, it follows that $b=c$. When this is achieved by resistor trimming, it follows from (31)~(34) that

$$V_{REF} = V_{G0} + \frac{(m-1)k}{q}T\left(1+\ln\frac{T_0}{T}\right). \tag{37}$$

The sensitivity of the inflection point to the trimmable parameters is of interest as it gives an indication of how much care must be exercised in the trimming operation.

Equation (35) can be solved for T_{INF} and differentiated with respect to the desired component. A straightforward calculation yields

$$S_{R_1}^{T_{INF}}\Big|_{T=T_{INF}} = \frac{1}{m-1}\ln\left(\frac{R_3 A_{E1}}{R_4 A_{E2}}\right)\cdot\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right) \tag{38}$$

$$S_{R_2}^{T_{INF}}\Big|_{T=T_{INF}} = -\frac{1}{m-1}\left[\ln\left(\frac{R_3 A_{E1}}{R_4 A_{E2}}\right)\cdot\left(1+\frac{R_3\alpha_1}{R_4\alpha_2}\right)\frac{R_1}{R_2}+1\right] \tag{39}$$

The corresponding sensitivities to R_3 and R_4 can be calculated similarly. Since bandgap references are sensitive to component ratios and degradation in the performance will be experienced with errors in circuit parameters, to obtain a reasonable yield after trim, it must be recognized that the trim range and resolution must be established to compensate for a worst-case deviation in the circuit parameters. This can be done based on the sensitivity analysis which gives an approximation about how much shift of the inflection point will be caused by errors in trimmable parameters.

If the inflection point is placed at the desired operating temperature, T_0, then different references can be compared by considering how rapidly they open up away from the inflection point. A good measure of this is the second derivative of the reference voltage evaluated at the inflection point. It follows from (35) that

$$\frac{\partial^2 V_{REF}}{\partial T^2}\Big|_{T=T_0} = -\frac{c}{T}\Big|_{T=T_0} = -\frac{c}{T_0} = -\frac{(m-1)k}{qT_0} \tag{40}$$

Equation (40) shows that the second derivative of the output voltage is independent of circuit parameters assuming perfect matching and an ideal op amp.

IV. Conclusion

It has been shown that commonly used formulations of the output voltage of a bandgap reference involve a $V_{BE}(T_0)$ term which itself is dependent upon the circuit in which the BJT is embedded. An explicit closed-form expression for the reference voltage of bandgap references which is only dependent on process and model parameters has been introduced. This model provides more insight into how a bandgap circuit operates and is useful for accurately characterizing properties of bandgap circuits.

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