A Detailed Analysis of Nonideal Effects on High Precision Bandgap Voltage References

Jun He
Department of Electrical and Computer Engineering
Iowa State University
Ames, IA 50011, USA
sirenehe@iastate.edu

Randall Geiger and Degang Chen
Department of Electrical and Computer Engineering
Iowa State University
Ames, IA 50011, USA
rlgeiger@iastate.edu and djchen@iastate.edu

***Abstract*—Until recently a closed-form expression for the output voltage of the most basic bandgap references was not available making it difficult to analytically and systematically determine the effects of the temperature dependence of non-ideal components on the magnitude of the output voltage, on the inflection point location, and on the curvature of these bandgap circuits. In this paper several non-ideal components that can adversely affect the performance of bandgap references are identified. A systematic approach is proposed to analytically determine the effects of the temperature dependence of non-ideal components. Analytical expressions for the effects of two of the most common non-ideal components, the temperature-dependent gain-determining resistors and the amplifier offset voltage, on the temperature characteristics of basic bandgap circuits are developed. The effectiveness of the analytical expression is validated by comparing analytical results with simulation results obtained from Spectre.**

I. INTRODUCTION

High precision bandgap voltage references are used in a wide range of emerging systems, such as high performance data converters, PLLs, and monolithic sensors. Despite the reported improvement in performance of bandgap voltage references [1][2], little attention has been focused on theoretical characterization of nonideal effects on the bandgap output voltage. Only recently an explicit model of the output voltage of a bandgap reference appeared that involves only process and model parameters. This explicit model provides insight into how a bandgap operates [3]. However, the explicit model does not include non-ideal error sources that degrade temperature stability in high precision reference application. A detailed knowledge of error sources' influences on the bandgap reference behavior, such as on the inflection point, the curvature, and the value of the reference output is fundamental in providing designers with a better understanding of the major performance limitations and in providing the insight needed to improve the performance of high precision voltage references.

In this paper, emphasis will be placed on the Kujik's bandgap voltage reference topology [4] because it is modestly less complicated than some other structures and because its performance is similar to that of several new structures. The non-ideal error sources are illustrated in Fig.1 and include the temperature-dependent offset voltage $V_{os}(T)$, the temperature dependence of resistors $R_0(T)$-$R_2(T)$, matching between $R_0(T)$-$R_2(T)$, the matching between diodes D_1 and D_2, the finite gain of the op amp A(s), and the parasitic resistors in the pn junctions $R_{d1}(T)$, $R_{d2}(T)$. In addition, the temperature dependence of V_{GO} [5] and package stress [6] will also introduce errors in the bandgap reference output. This paper will focus on the effects of temperature dependence of gain-determining resistors $R_0(T)$-$R_2(T)$ and the offset voltage of the op amp $V_{os}(T)$. Most authors ignore the temperature dependence of R_0-R_2, presumably because in the expressions reported for the output reference voltage, R_0-R_2 always appear in a form of resistor ratio suggesting that the effects of the temperature coefficients (TCs) of R_0-R_2 will be canceled out and thus will not affect the thermal stability of the reference voltage output [4][7][8]. However, the explicit closed form model in [3] shows that there is a part of the output voltage that is dependent upon a resistor value not just a resistor ratio. Correspondingly, the temperature dependence of the offset voltage $V_{os}(T)$ is often the biggest error source that is usually neglected when predicting the temperature coefficient of the reference output voltage [1].

In section II, a systematic explicit analysis approach that includes the temperature dependence of R_0-R_2 along with the temperature dependence of V_{os} will be introduced. The influence of these variables on the inflection point, the curvature of the bandgap curve, and the value of the bandgap output are discussed separately. In section III, Spectre simulation results are compared with theoretical analysis from the model and show a good consistency. Section IV concludes the work.

II. DETAILED CHARACTERIZATION OF BANDGAP REFERENCES WITH NON-IDEALITIES

In the circuit in Fig.2, assume that R_0-R_2 have a temperature dependence that can be modeled by (1)

978-1-4244-2167-1/08/$25.00 ©2008 IEEE

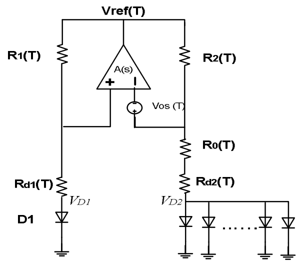

Figure 1. Bandgap circuit with error sources

$$R_i(T) = R_{iN} \cdot (1 + TCR \cdot (T - T_0))_{i=0,1,2} \quad (1)$$

where R_{iN} is the nominal value of R_0-R_2 at temperature T_0. Also assume that input referred offset voltage V_{os} has the first order temperature coefficient TCV_{os} and can be modeled as

$$V_{os}(T) = V_{os}(T_0) + TCV_{os}(T - T_0) \quad (2)$$

Five equations can be determined to completely characterize the circuit in Fig. 2.

$$V_{ref} = I_{D1}R_1(T) + V_{D1} \quad (3)$$

$$V_{D1} = V_{D2} + I_{D2}R_0(T) - V_{os}(T) \quad (4)$$

$$I_{D2} = [V_{ref} - V_{D1} - V_{os}(T)]/R_2(T) \quad (5)$$

$$V_{D1} = V_t \ln(I_{D1}) + V_{GO} - V_t[\ln(J_{sx} A_1) + m \ln T] \quad (6)$$

$$V_{D2} = V_t \ln(I_{D2}) + V_{GO} - V_t[\ln(J_{sx} A_2) + m \ln T] \quad (7)$$

where J_{sx} are process parameters and independent of temperature. m is also process parameter equal to 2.3. V_{GO} is bandgap voltage 1.205V. A_1 and A_2 are the area factors for diodes D_1 and D_2 respectively. $Vt=kT/q$, k is Boltzman's constant, T is temperature in K, and q is the charge of an electron.

The set of equations (3)-(7) in the unknowns $\{I_{D1}, I_{D2}, V_{D1}, V_{D2}, V_{ref}\}$ completely characterize the circuit in Fig. 2. Eliminate V_{D1}, V_{D2}, I_{D2} we reduce (3)-(7) to a set of 2 equations in the unknowns $\{I_{D1}, V_{ref}\}$.

$$V_{ref} = I_{D1}R_1(T) + V_t \ln(I_{D1}) + V_{GO} - V_t[\ln(J_{sx} A_1) + m \ln T] \quad (8)$$

$$I_{D1}R_1(T) = \frac{R_2(T)}{R_1(T)} V_t \ln\left(\frac{I_{D1}R_2(T)}{I_{D1}R_1(T) - V_{os}(T)}\right) + \frac{R_2(T)}{R_0(T)} V_t \ln\left(\frac{A_2}{A_1}\right)$$
$$+ V_{os}(T) \cdot \left(1 + \frac{R_2(T)}{R_0(T)}\right) \quad (9)$$

These two equations are highly non-linear and highly coupled. We will first linearize (9) from which I_{D1} can be obtained. The first logarithmic term in (9) can re-arranged as

$$\ln\left(\frac{I_{D1}R_2(T)}{I_{D1}R_1(T) - V_{os}(T)}\right) = \ln\left[\frac{R_2(T)}{R_1(T)} \cdot \left(\frac{1}{1 - \frac{V_{os}(T)}{I_{D1}R_1(T)}}\right)\right] \quad (10)$$

From (10), note that $I_{D1}R_1(T)$ is the voltage drop over R_1, and also the voltage difference between bandgap reference output voltage V_{ref} and diode voltage V_{D1}. Normally, V_{ref} is around 1.2V and V_{D1} is around 0.6V, so

$$I_{D1}R_1(T) = V_{ref} - V_{D1} >> V_{os}(T) \quad (11)$$

Apply the condition in (11), (10) can be expressed as

$$\ln\left[\frac{R_2(T)}{R_1(T)} \cdot \left(\frac{1}{1 - \frac{V_{os}(T)}{I_{D1}R_1(T)}}\right)\right] = \ln\frac{R_2(T)}{R_1(T)} + \ln\left(\frac{1}{1 - \frac{V_{os}(T)}{I_{D1}R_1(T)}}\right)$$

$$\approx \ln\frac{R_2(T)}{R_1(T)} + \frac{V_{os}}{I_{D1}R_1(T)} \approx \ln\frac{R_2(T)}{R_1(T)} \quad (12)$$

With the approximations, we have linearized (10) as

$$I_{D1}R_1(T) = \frac{R_2(T)}{R_1(T)} V_t \ln\left(\frac{R_2(T)}{R_1(T)}\right) + \frac{R_2(T)}{R_0(T)} V_t \ln\left(\frac{A_2}{A_1}\right)$$
$$+ V_{os}\left(1 + \frac{R_2(T)}{R_0(T)}\right) \quad (13)$$

Eliminate I_{D1}, finally gives a general close form explicit expression for V_{ref} as

$$V_{ref} = a + bT + cT\ln(T) + d\ V_{os}(T) \quad (14)$$

where $a = V_{G0}$ (15)

$$b = \frac{k}{q}\left(\frac{R_2(T)}{R_0(T)} \ln\left(\frac{R_2(T)}{R_1(T)} \frac{A_2}{A_1}\right) + \ln\left(\frac{R_2(T)}{R_1(T)} \frac{1}{R_0(T)} \frac{\ln\left(\frac{R_2(T)}{R_1(T)} \frac{A_2}{A_1}\right)}{A_1 J_{sx}}\right)\right) \quad (16)$$

$$c = \frac{k}{q}(1 - m) \quad (17)$$

$$d = 1 + \frac{R_2(T)}{R_0(T)} + \frac{\left(1 + \frac{R_2(T)}{R_0(T)}\right)}{R_2(T)/R_0(T) \cdot \ln(R_2(T)/R_1(T) \cdot A_2/A_1)} \quad (18)$$

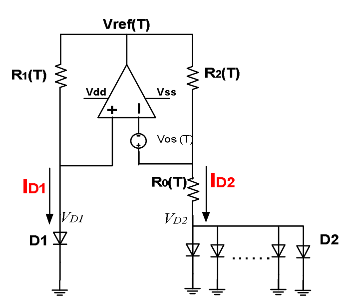

Figure 2. Bandgap circuit with temperature-dependent R_0-R_2 and Vos(T)

Non-ideal effects from Temperature dependent offset voltage V_{os} (T) and gain-determining resistor R_0(T)-R_2(T) are inherently involved in (14). Inflection point, the curvature of the bandgap curve and the value of the bandgap output are readily to be analyzed.

A. Temperature dependence of R_0-R_2

First, investigate non-ideal effect from temperature dependent gain-determining resistor R_0(T)-R_2(T) by assuming that V_{os}(T)=0. Equation (14) can be simplified as

$$V_{ref} = a + bT + cT\ln(T) \quad (19)$$

Note that resistors are not always shown as a form of resistor ratio. In (16), R_0(T) appear as a single resistor value dependent on temperature variation, which intuitively explain why temperature dependence of R_0(T)-R_2(T) is necessary to be considered.

According to (1), b in (16) can be re-written as

$$b = b_1 = \frac{k}{q}\left(\frac{R_{2N}}{R_{0N}}\ln\left(\frac{R_{2N}}{R_{1N}}\frac{A_2}{A_1}\right) + \ln\left(\frac{R_{2N}}{R_{1N}}\frac{1}{R_0(T)}\frac{\ln(\frac{R_{2N}}{R_{1N}}\frac{A_2}{A_1})}{A_1 J_{sx}}\right)\right) \quad (20)$$

Based on (19), (15), (17) and (20), inflection point analysis can be done by differentiating V_{ref} with respect to T and setting the derivative equal to 0 gives

$$\frac{\partial V_{ref}}{\partial T} = b_1 + T\frac{\partial b_1}{\partial T} + c\ln(T) + c = 0 \quad (21)$$

From (20), differentiate b_1 with respect to T gives

$$\frac{\partial b_1}{\partial T} = \frac{k}{q}R_{0N}\cdot TCR\cdot\frac{-1}{R_0(T)} \approx -\frac{k}{q}TCR \quad (22)$$

Substitute (22) into (21) to solve T, and T is the new inflection point T_{infN1} when TC of R_0-R_2 is considered. Equation (21) is re-arranged as

$$b_1 - T_{\inf N1}\cdot\frac{k}{q}\cdot TCR + c\ln(T_{\inf N1}) + c = 0 \quad (23)$$

It has been derived in [3] that if no error sources are considered, the desired inflection point T_{infI} satisfies the following equation

$$b_I + c_I + c_I\ln T_{\inf I} = 0 \quad (24)$$

where $b_I \approx b_1, c_I = c$.

Take the difference between (23) and (24) and rearrange the expression. It gives

$$T_{\inf I} = T_{\inf N1}\cdot\exp\left(T_{\inf N1}\cdot\frac{TCR}{m-1}\right) \quad (25)$$

From (25), it can be concluded that the new inflection point T_{infN1} is readily to be solved when TCR is known.

By doing the second derivative of the reference voltage evaluated at the inflection point, it can be measured that how rapidly the reference curve opens up away from the inflection point. It is also called the curvature of the reference curve. According to (21) and (22), it follows that

$$\frac{\partial^2 V_{ref}}{\partial T^2}\Big|_{T=T\inf N1} = \frac{\partial b_1}{\partial T} + \left(\frac{\partial b_1}{\partial T}\right)^2 + \frac{c}{T_{\inf N1}}$$
$$= \left(-\frac{k}{q}TCR\right) + \left(-\frac{k}{q}TCR\right)^2 + \frac{c}{T_{\inf N1}} \quad (26)$$

B. Offset voltage of op amp and its temperature dependence

When the effect from offset voltage V_{os}(T) is considered, assume all the other non-ideal effect are not existing. b and d in the general expression can be rewritten as

$$b = b_2 = \frac{k}{q}\left(\frac{R_{2N}}{R_{0N}}\ln\left(\frac{R_{2N}}{R_{1N}}\frac{A_2}{A_1}\right) + \ln\left(\frac{R_{2N}}{R_{1N}}\frac{1}{R_{0N}}\frac{\ln(\frac{R_{2N}}{R_{1N}}\frac{A_2}{A_1})}{A_1 J_{sx}}\right)\right) \quad (27)$$

$$d = d_2 = 1 + \frac{R_{2N}}{R_{0N}} + \frac{(1 + R_{2N}/R_{0N})}{R_{2N}/R_{0N}\cdot\ln(R_{2N}/R_{1N}\cdot A_2/A_1)} \quad (28)$$

It can be seen that b_2 and d_2 are independent of temperature. Then apply the same method to (14) as in part *A* to analyze the inflection point. It follows that

$$\frac{\partial V_{ref}}{\partial T} = b_2 + c\ln(T) + c + d_2\frac{\partial V_{os}(T)}{\partial T} = 0 \quad (29)$$

This expression can be solve for T to determine the inflection point T_{infN2} to be

$$T_{\inf N2} = \exp\left[-b_2/c - 1 - (d_2/c_2)\frac{\partial V_{os}(T)}{\partial T}\right]$$
$$= \exp(-b_2/c - 1)\cdot\exp\left[-(d_2/c_2)\frac{\partial V_{os}(T)}{\partial T}\right]$$
$$= T_{\inf I}\cdot\exp[-(d_2/c_2)\cdot TCV_{os}] \quad (30)$$

The curvature around inflection point is calculated as

$$\frac{\partial^2 V_{ref}}{\partial T^2}\Big|_{T=T\inf N2} = \frac{c}{T_{\inf N2}} \quad (31)$$

C. Comparison with bandgap reference characteristics with no error sources

As it is seen from the above obtained inflection points, curvature and V_{ref} magnitude information, it is worthy making a comparison between ideal characteristics and the ones with error sources V_{os} (T) and temperature-dependent R_0-R_2. The results are listed in Table I. The effects of the temperature dependence of R_0-R_2 and the temperature dependence of V_{os} on the inflection point, the curvature, and the value of the reference output are demonstrated.

TABLE I. COMPARISION BETWEEN DIFFERENT CONDITIONS

Conditions	Characteristics Comparison		
	Inflection point (T_{inf})	***Curvature around T_{inf}***	***Vref(T_{inf})***
Ideal Transfer Curve	$\exp(-b/c-1) = T_{infI}$	c/T_{infI}	$a+bT_{infI}+cT_{infI}\ln(T_{infI})$
Curve with $V_{os}(T)$	$T_{infI} \cdot \exp(-d/c \cdot TCV_{os})$	$c/[T_{infI} \cdot \exp(-d/c \cdot TCV_{os})]$	$a+bT_{infN2}+cT_{infN2}\ln(T_{infN2})+d \cdot Vos(T_{infN2})$
Curve with $R_0(T)$-$R_2(T)$	$f(T_{infI}, TCR, m) = T_{infN1}$	$(-k/q \cdot TCR)+(-k/q \cdot TCR)^2+c/T_{infN1}$	$a+bT_{infN1}+cT_{infN1}\ln(T_{infN1})$

III. NUMBERICAL EXAMPLES AND SPECTRE SIMULATION

Kujik's circuit in Fig.2 is implemented in AMI0.6u process in Cadence to verify the effectiveness of derivation in previous section. The key values are as follows: V_{dd}=5V, V_{ss}=0V, R_{2N}=R_{1N}=5.95KΩ, R_{0N}=786Ω, A2:A1=8:1, A(s=0)=100dB, $V_{os}(T_0=27C)$=1mV. The effect of temperature dependent resistor $R_0(T)$-$R_2(T)$ on the inflection point T_{inf}, the curvature near T_{inf} and the value of the bandgap output V_{ref} at T_{inf} is simulated, and then compared with the calculated values that can be found from explicit expression in (19), (25) and (26). It is worthy mentioning that in the simulation, T_{inf} and V_{ref} at T_{inf} can be easily determined by measuring V_{ref} peak location in the plot of V_{ref} vs. temperature transfer curve. To get the curvature near T_{inf}, quadratic curve fitting can be first applied to fit the transfer curve near T_{inf} by a parabola, and the curvature is the second order coefficient. From Fig. 3-5, the calculated values from the explicit model are in close agreement with the simulated values. The differences are mainly due to the approximation made during the derivation, the model parameters and the I_C-V_D diode characteristics.

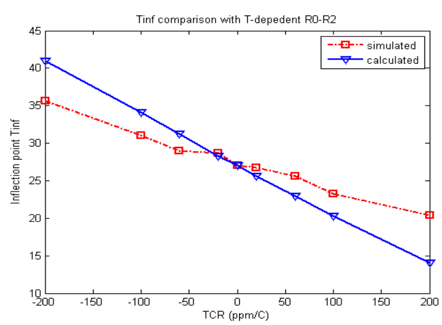

Figure 3. Inflection point vs. temperature coefficient of R_0-R_2 (TCR)

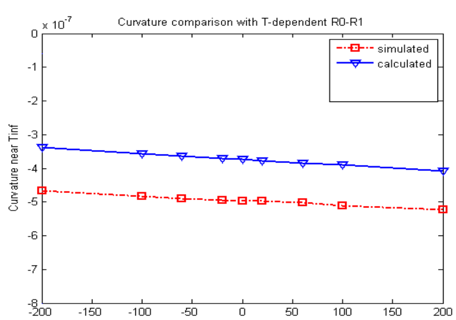

Figure 4. Curvature near T_{inf} vs. TCR

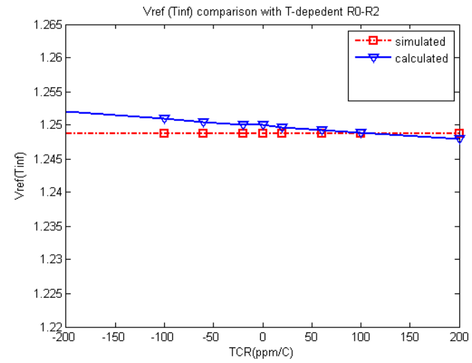

Figure 5. V_{ref} value at T_{inf} vs. TCR

IV. CONCLUSION

A systematical approach is proposed in this paper to analytically determine the effects of the temperature dependence of non-ideal components on the inflection point location, on the curvature of bandgap curve and on the magnitude of the output voltage. The effects of two of the most common non-ideal components, the temperature-dependent gain-determining resistors and the amplifier offset voltage V_{os} (T), on the temperature characteristics of basic bandgap circuits are analyzed. The effectiveness of the derived model is proved by comparing with simulation results using BSIM3v3 model in Spectre. This new approach can allow the circuit designers to have a better understanding of main limitations of the adopted voltage references and to improve the approach to design high precision reference circuits.

REFERENCES

[1] B.-S. Song and P. R. Gray, "A precision curvature-compensated CMOS bandgap reference," *IEEE J. Solid-State Circuits*, vol. SC-18, pp. 634–643, Dec. 1983.

[2] I. Lee, G. Kim, and W. Kim, "Exponential curvature-compensated BiCMOS bandgap references," *IEEE J. Solid-State Circuits*, vol. 29, pp. 1396–1403, Nov. 1994.

[3] X. Dai, D.Chen and R. Geiger, "Explicit characterization of bandgap references," in Proc. ISCAS 2006, 4 pp., May 2006

[4] K. Kuijk, "A Precision Reference Voltage Source", IEEE Journal of Solid State Circuits, Vol. 8, pp. 222-226, June 1973.

[5] Y. P. Tsividis, "Accurate analysis of temperature effects in IC VBE characteristics with application to bandgap reference sources," IEEE J. Solid-State Circuit, vol. SC-15, pp. 1076-1084, Dec. 1980.

[6] G.A. Rincon-Mora, "Voltage References", IEEE Press, John Wiley & Sons, Inc., 2002.

[7] D. Johns and K. Martin, " Analog Integrated Circuit Design", Wiley, New York, 1997

[8] P. Brokaw, " A Simple Three-Terminal IC Bandgap Reference", IEEE Journal of Solid State Circuits, Vol. 9, pp.388-393, Dec.1974