

Linearity testing of ADCs using low linearity stimulus and Kalman Filtering

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Abstract—Traditional linearity testing of ADCs involves using a spectrally pure or a highly linear stimulus, along with a large number of samples per code to average out the effects of noise. Test equipments need to house expensive instruments to provide the highly linear stimulus. The large number of samples required for the procedure results in long test times. These two factors are prime contributors to the test cost. In this paper, algorithms which use low linearity stimuli and a Kalman Filter to reduce both the hardware resources and the test time for the test procedure have been proposed. Simulations results for a 14-bit ADC show that a 7-bit linear stimulus with one sample per code can be used to measure the INL of the ADC with a maximum estimation error of 1 LSB.

Keywords: Analog to Digital Converters (ADC), Code Density Testing, Samples per code, INL (Integral Non Linearity), DNL (Differential Non Linearity), Kalman Filtering.

I. INTRODUCTION

Static performance testing of Analog to Digital Converters is one of the most challenging tasks in mixed signal circuit testing. Code density testing [1]-[2] is the standard method for measuring the static performance parameters, INL and DNL of the ADC under test. Conventional code density testing involves spectrally pure or highly linear stimulus that are typically 3 or more bits more linear than the ADC under test. Large numbers of samples are collected to average out noise for better measurement precision and accuracy. During production test, code density testing is carried out using very expensive Automated Test Equipments (ATE). Considering the fact that ADCs are high volume products, any small saving on hardware resources and/or test time for a single ADC will have a huge impact on the entire test procedure cost. Recently introduced methods [3]-[4] for linearity testing of ADCs dramatically relax the linearity requirements on signal generators. It has been shown in [5] that with the knowledge of input noise and error sources in the circuit, Kalman Filter can be used along with code density testing to obtain better estimates of INL of ADCs. In the current work, two methods that use two low linearity stimuli, SEIR and the algorithm in [4] (denoted as KHK algorithm), are used with a Kalman filter to obtain accurate estimates of INL with small number of samples. The computational complexity of these two new methods is not of concern given the available computation power in today's computers. The fewer number

of samples required by the proposed algorithms to achieve a fixed accuracy, directly results in the reduction of test time, as it reduces the data acquisition time. The rest of the paper is organized as follows. Section II reviews the SEIR algorithm, the KHK algorithm and Kalman Filter for the two algorithms. Section III discusses Simulation results and Section IV, the conclusion.

II. INL OPTIMIZATION USING KALMAN FILTER

A. Kalman Filter

The Kalman Filter is an efficient recursive estimator based on linear dynamical systems. The essence of the filter is to obtain estimates of the state variables from noisy measurement data. The state variables and the measurement variables are represented by the following dynamical system of equations (discrete case):

$$\tilde{x}[k] = F[k] \cdot \tilde{x}[k-1] + G \cdot u[k] + w[k] \quad (1)$$

$$\dot{x}[k] = H[k] \cdot \tilde{x}[k] + v[k] \quad (2)$$

where, $\tilde{x}[k]$ = state variable/process, $\dot{x}[k]$ = measurement variable/process, $\tilde{u}[k]$ is a known vector. $w[k]$ is the process noise and $v[k]$ is the measurement noise. $w[k]$ and $v[k]$ are assumed to be white Gaussian with zero mean, and are not correlated with each other.

Kalman Filter Algorithm:

In the Kalman Filter algorithm, the estimate of the state from previous time step and current measurement are used to calculate the current state based on the model specified by (1) and (2). The Filter in the proposed algorithm is used to obtain optimal estimates of INL[k] of the ADC from noisy INL[k] measurements.

After initializing the state and error covariance matrix $\tilde{x}[0]$ and $P_M[0]$ respectively, the below equations are executed from $k=0$ to M variables of interest.

$$\text{Step 1: } K_p[k] = P_M[k] \cdot H[k]^T (H[k] \cdot P_M[k] \cdot H[k]^T + R)^{-1}$$

$$\text{Step 2: } \tilde{x}[k] = \tilde{x}[k] + G \cdot \tilde{u}[k] + K_p[k] (\dot{x}[k] - H[k] \cdot \tilde{x}[k])$$

$$\text{Step 3: } P[k] = (I - K_p[k] \cdot H[k]) P_M[k]$$

$$\text{Step 4: } \tilde{x}[k] = \tilde{x}[k] \cdot F[k]$$

$$\text{Step 5: } P_M[k+1] = F[k] \cdot P[k] \cdot F[k]^T + Q$$

$K_p[k]$ is the Kalman gain, R is the measurement noise covariance matrix, Q the process noise covariance matrix and $P_M[k]$ is the prediction error variance. The ADC linearity test problem is formulated in such a way that the above equations can be directly used to characterize the static parameters of the

ADC. For a detailed description of the Kalman filter refer to [6].

B. Stimulus for the SEIR and KHK Algorithm:

The SEIR algorithm and KHK algorithm use two stimuli, R_1 and R_2 , which is the shifted from R_1 by a constant offset, α . Two sets of histogram counts, C_1 and C_2 , are obtained from R_1 and R_2 respectively. For a detailed working of the algorithms refer to [3] and [4]. Simple signal generators described in [7] can be used to generate the stimuli.

C. Kalman Filter for SEIR (denoted as SEIR-KF)

The SEIR algorithm uses the histogram data, C_1 and C_2 , to identify the non-linearities of the stimulus. This information is then used to calculate $\tilde{INL}[k]$ s. The equation used to estimate $\tilde{INL}[k]$ is:

$$\begin{aligned} \tilde{INL}[k] = & \frac{(N-2)}{2} (t_1[k] + t_2[k]) \\ & + \sum_{j=1}^M \frac{a_j}{2} (F_j(t_1[k]) + F_j(t_2[k])) - k - \frac{\alpha}{2} \end{aligned} \quad (3)$$

, where $k=1, 2, \dots, N-3$

In the above expression, $N = 2^n$, where n is the resolution of the ADC, a_j is the co-efficient of the basis function $F_j(t)$, $t_1[k]$ or $t_2[k]$ is a measure of the transition time at the transition level k of the ADC, obtained from the histogram data as follows:

$$t_1[k] = T_{C1} \sum_{i=1}^k C_1[i], \text{ where } T_{C1} = \left(\sum_{i=1}^{N-2} C_1[i] \right)^{-1} \quad (4)$$

$$t_2[k] = T_{C2} \sum_{i=1}^k C_2[i], \text{ where } T_{C2} = \left(\sum_{i=1}^{N-2} C_2[i] \right)^{-1} \quad (5)$$

Changing the index of (3) from k to $k-1$,

$$\begin{aligned} \tilde{INL}[k-1] = & \frac{(N-2)}{2} (t_1[k-1] + t_2[k-1]) \\ & + \sum_{j=1}^M \frac{a_j}{2} (F_j(t_1[k-1]) + F_j(t_2[k-1])) - k + 1 - \frac{\alpha}{2} \end{aligned} \quad (6)$$

, where $k=2, 3, \dots, N-3$

Subtracting (4) from (3) we get,

$$\begin{aligned} \tilde{INL}[k] = & \tilde{INL}[k-1] \\ & + \frac{(N-2)}{2} (t_1[k] - t_1[k-1] + t_2[k] - t_2[k-1]) \\ & + \sum_{j=1}^M \frac{a_j}{2} \{F_j(t_1[k]) + F_j(t_2[k]) \\ & - F_j(t_1[k-1]) - F_j(t_2[k-1])\} - 1 \end{aligned} \quad (7)$$

The $DNL[k]$ in LSBs can be expressed in terms of transition times, given by (4) and (5) as,

$$DNL[k] = (N-2) (t[k] - t[k-1]) - 1 \quad (8)$$

Substituting (8) in (7), we have,

$$\begin{aligned} \tilde{INL}[k] = & \tilde{INL}[k-1] + DNL[k] \\ & + \sum_{j=1}^M \frac{a_j}{2} \{F_j(t_1[k]) + F_j(t_2[k]) \\ & - F_j(t_1[k-1]) - F_j(t_2[k-1])\} \end{aligned} \quad (9)$$

Expression (9) is the state expression for the Kalman Filter to be used with the SEIR algorithm. $\tilde{INL}[k]$ is viewed as the state variable and $DNL[k]$ is modeled as the state process noise. $DNL[k]$ is approximated as white Gaussian random variable with zero mean and variance Q .

$$E(DNL[k].DNL[l]) \approx Q.\delta_{kl}, \quad (10)$$

where $E()$ is the expectation operator, δ_{kl} is the dirac delta function, $\delta_{kl}=1$ only when $l=k$ and $\delta_{kl}=0$ elsewhere.

The $\tilde{INL}[k]$ estimates obtained from the SEIR process serve as noisy observations from which optimal estimates are obtained using the Kalman Filter. Thus the process measurement equation is:

$$\tilde{INL}[k] = \tilde{INL}[k] + v[k] \quad (11)$$

$v[k]$ is the measurement noise that is approximated as white Gaussian with zero mean and variance R .

$$E(v[k].v[l]) \approx R.\delta_{kl} \quad (12)$$

In a flash type ADC structure, mismatch among components and comparator errors result in $DNL[k]$. This can be characterized for a family of ADCs. The distribution of $DNL[k]$ s for such ADCs remains fairly constant. Q is thus characterized. The variance of $v[k]$ is not known before hand. Its value can be approximated by characterizing the noise in the test environment.

Comparing (9) and (11) with (1) and (2), we get the following matrices for the Kalman Filter:

$$F = [1], H = [1], Q = [\sigma_{DNL} \cdot \sigma_{DNL}], R = [\sigma_{NOISE} \cdot \sigma_{NOISE}],$$

$$G = [a_1 \ a_2 \ \dots \ a_M],$$

$$u[k] = [F_1(t_1[k]) + F_1(t_2[k]) - F_1(t_1[k-1]) - F_1(t_2[k-1]);$$

$$F_2(t_1[k]) + F_2(t_2[k]) - F_2(t_1[k-1]) - F_2(t_2[k-1]);$$

⋮

$$F_M(t_1[k]) + F_M(t_2[k]) - F_M(t_1[k-1]) - F_M(t_2[k-1])]$$

The flowchart in Figure [1] summarizes the procedure using Kalman Filter with SEIR.

D. Kalman Filter for KHK Algorithm

The KHK algorithm characterizes the stimulus by evaluating the slope of the stimulus at every code index using histogram data. This information is used to estimate the voltage offset, α , followed by INL estimation.

The KHK algorithm can be described in the following steps:

Step 1: The average of histogram data obtained from

the two ramps is calculated.

$$C[i] = \frac{C_1[i] + C_2[i]}{2}, \quad i = 1, 2, \dots, N-2 \quad (13)$$

Step 2: The cumulative difference of histogram data from the two ramps is calculated.

$$D[i] = \sum_{j=0}^i (C_2[i] - C_1[i]) \quad (14)$$

Step 3: The shift voltage, α , is estimated.

$$\hat{\alpha} = \frac{N-2}{\sum_{i=1}^{N-2} D[i]} \quad (15)$$

Step 4: The code widths are then estimated using:

$$cw[i] = \hat{\alpha} \frac{C[i]}{D[i]} \quad (16)$$

From the code widths, the INL[k] s and the DNL[k] s can be calculated as shown in [1]. The Kalman Filter model used in this case is a simple one as described in [5]. The INL[k] estimates obtained using the above algorithm serve as noisy observations for the Kalman Filter. Thus have the measurement process equation:

$$\dot{I}NL[k] = \tilde{I}NL[k] + v[k] \quad (17)$$

The state variable $\tilde{I}NL[k]$ is the Kalman filter estimate obtained from the noisy measurements, $\dot{I}NL[k]$. The state process equation is :

$$\tilde{I}NL[k] = \tilde{I}NL[k-1] + DNL[k] \quad (18)$$

The DNL[k] and $v[k]$ are approximated to have a white Gaussian distribution with variances Q and R respectively.

Comparing (17) and (18), with (1) and (2), we get the following matrices for our Kalman Filter:

$$F = [1], H = [1], Q = [\sigma_{DNL} \cdot \sigma_{DNL}], R = [\sigma_{NOISE} \cdot \sigma_{NOISE}]$$

$$G = [\text{zeros}]$$

The block diagram in Figure [1] summarizes the test procedure using Kalman Filter with KHK Algorithm. The models that are described in this section have been developed for a flash type ADC structure. The models can be easily extended to Pipelined ADC structure as shown in [6].

III. SIMULATION RESULTS

Simulation conditions:

The low linearity stimulus used for simulations is 7-bit linear, generated from simulations of signal generator circuit suggested in [7]. The ADC that has been used for simulation is a 14-bit flash type ADC. The INL of the ADCs used for simulations varies from 0.5LSB to 14LSB. The second ramp is shifted from the first ramp by 150 LSBs. For the algorithm, 24 sinusoidal basis functions are used to characterize the non-linearity of the stimulus. The simulations have been carried out for standard deviation of additive input noise, σ_{NOISE} , and

different number of samples per code, N_s . For each pair of parameters, σ_{NOISE} and N_s , the four algorithms are run 1000 times. The mean and the standard deviation of the maximum estimation error are reported in Table 1.

Figure [1] shows a plot of the true INL[k] of the ADC with those estimated using SEIR, and the proposed algorithm, SEIR-KF. The INL[k]s are estimated with $N_s = 8$ samples per code for an input noise and a $\sigma_{NOISE} = 0.5$ LSB. The first plot shows true INL[k] along with those obtained with SEIR and SEIR-KF. The second plot is a zoomed in version of the first plot over a few bins. In the Error plot the improvement in the estimation error for the SEIR-KF is evident. Similar trends can be observed in plots of KHK and KHK-KF algorithms.

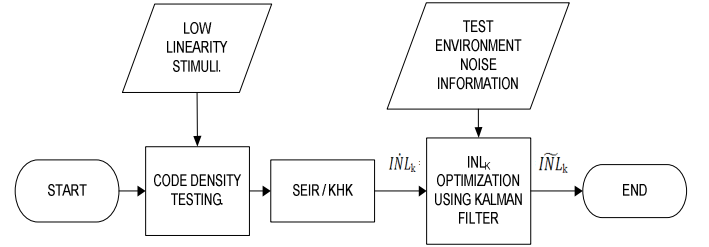


Figure 1: Flowchart: SEIR-KF /KHK-KF

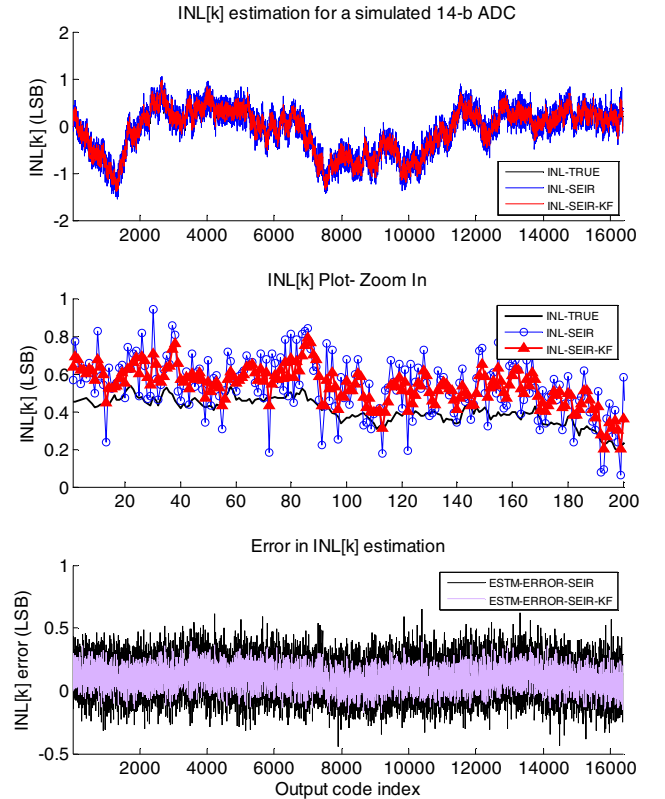


Figure 2: INL and INL estimation error plots for SEIR-KF

Table 1: Simulation Results

		σ_{NOISE}									
		0.25 LSB		0.5 LSB		1 LSB		2 LSB		3 LSB	
Ns	Algorithms	$\mu_{\text{MAX ERR}}$	$\sigma_{\text{MAX ERR}}$	$\mu_{\text{MAX ERR}}$	$\sigma_{\text{MAX ERR}}$	$\mu_{\text{MAX ERR}}$	$\sigma_{\text{MAX ERR}}$	$\mu_{\text{MAX ERR}}$	$\sigma_{\text{MAX ERR}}$	$\mu_{\text{MAX ERR}}$	$\sigma_{\text{MAX ERR}}$
8	SEIR	0.4916	0.0624	0.709	0.09049	1.042	0.143	1.5998	0.2428	2.1067	0.376
	SEIR-KF	0.3621	0.0606	0.477	0.08728	0.698	0.137	1.1127	0.239	1.4924	0.367
	KHK	0.4955	0.0632	0.714	0.0911	1.055	0.1433	1.622	0.244	2.146	0.375
	KHK-KF	0.3637	0.0611	0.495	0.0876	0.768	0.1397	1.147	0.2422	1.5972	0.3717
4	SEIR	0.691	0.086	0.991	0.1267	1.487	0.208	2.288	0.376	3.007	0.513
	SEIR-KF	0.4649	0.084	0.6267	0.1237	0.923	0.199	1.504	0.239	2.091	0.509
	KHK	0.696	0.0866	1	0.129	1.505	0.209	2.322	0.383	3.069	0.517
	KHK-KF	0.4645	0.0849	0.6256	0.1243	0.967	0.204	1.604	0.373	2.178	0.512
2	SEIR	0.9634	0.12	1.3972	0.1787	2.088	0.288	3.229	0.536	4.1965	0.745
	SEIR-KF	0.5945	0.116	0.8104	0.1724	1.247	0.276	2.133	0.531	2.89	0.741
	KHK	0.9725	0.1216	1.411	0.179	2.1231	0.2959	3.288	0.5296	4.288	0.746
	KHK-KF	0.6078	0.169	0.828	0.177	1.2616	0.286	2.06	0.536	2.976	0.746
1	SEIR	1.3895	0.196	1.947	0.2535	2.93	0.4018	4.49	0.6998	5.791	1.097
	SEIR-KF	0.879	0.1929	1.072	0.2438	1.652	0.397	2.75	0.687	3.911	1.074
	KHK	1.4262	0.209	1.985	0.2673	3.011	0.466	4.6077	0.7289	6.163	1.099
	KHK-KF	0.9187	0.2056	1.1205	0.2438	1.7821	0.4218	3.042	0.718	4.254	1.0838

Table 1 summarizes the results of the simulations carried out in MATLAB. From the table it can be concluded that using a Kalman filter improves the performance by at least 0.13LSB (For $N_s=8$, $\sigma_{\text{NOISE}} = 0.25\text{LSB}$ with SEIR-KF). For the case of $\sigma_{\text{NOISE}} = 3\text{LSB}$ and $N_s=1$ sample per code, the improvement in the performance is as high as 1.9 LSBs (With SEIR-KF).

The maximum error in INL estimation that is acceptable during testing is often dictated by the target application of the ADC. For example, consider a case where the maximum acceptable estimation error is 1LSB and the noise in the tester setup has a standard deviation of around 0.5LSB. From the table, the SEIR algorithm and KHK algorithm would require at least $N_s=4$ to achieve the level of performance. With the proposed approach, $N_s=1$ can be used to achieve the same level of accuracy that SEIR algorithm or KHK algorithm achieves with $N_s=4$. The trade off here is a slight increase in computational complexity. But this is negligible given the computation power of today's PCs. Thus the data acquisition time and hence the test time is scaled by 4. This method also employs a low linearity stimulus, thus obviating the need for a spectrally pure or a highly linear stimulus.

IV. CONCLUSION

Optimizing static performance testing by using Kalman Filter with two low-linearity stimulus based linearity testing algorithms has been discussed. Two simple models for linearity testing of ADCs to be used in Kalman Filter have been developed. By employing the SEIR algorithm

and KHK algorithm with a Kalman Filter it has been shown that test cost can be reduced by reducing both test time and hardware resources.

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