A Faster and Accurate Method for Spectral Testing Applicable to Noncoherent Data

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***Abstract*—The Fast Fourier Transform is the standard approach for spectral testing. However, its correct application to sinusoidal signals requires either strict coherent sampling, or careful windowing, or other methods that are not computationally efficient. This paper introduces an improved method for achieving accurate and robust spectral testing for sinusoidal signals without the need for coherent sampling or windowing. Theoretical analysis, extensive simulation results, and experimental results show that the proposed method is always faster than the original method and robust when the signal frequency is close to Nyquist frequency. Statistical analysis and comparative studies demonstrate that the proposed algorithm achieves almost the same spectral testing accuracies as those obtained under perfect coherent sampling.**

I. INTRODUCTION

Spectral performance of an integrated circuit is of critical concern in many important application areas such as signal processing and communications. It is well known that DFT (Discrete Fourier Transform) or FFT (Fast Fourier Transform) is the most prevalent method for spectral testing. However, when performing FFT for spectral testing of sinusoidal signals, one must make sure that the data record being used in the FFT algorithm represents exactly an integer number of periods of the signal. In other words, the signal frequency and the sampling clock frequency of the data acquisition system must satisfy coherent condition. In FFT algorithm, even the slightest mismatches between the two frequencies will cause the frequency leakage phenomenon in which energy from the fundamental spectral line is spread into neighboring frequencies causing the appearance of a "skirt" around the spectral line.

In order to combat the frequency leakage, the IEEE standard [1] as well as industry best practice is to require coherent sampling, meaning that the sampling clock signal should be perfectly synchronized with the signal under test so that an integer multiple of signal periods are captured in a data record. When this is guaranteed, direct use of FFT is permitted and the data analysis is computationally very efficient. Unfortunately, strict coherent sampling is difficult to maintain, especially for on chip implementation.

Another alternative method is to use the windowing technique [2, 3] while allowing noncoherent sampling. This technique does not remove the skirting due to non-coherency, rather it merely suppresses the skirting levels at frequencies far away from the fundamental frequency. By doing so it alters the heights of the original spectral lines. Care must be taken in order to correctly recover the spectral lines. Another limitation is due to the fact the amount of skirt suppression is limited and hence it is not sufficiently accurate for many high resolution application.

Other methods to combat spectral leakage include singular value decomposition [4], 2-D FFT [5] and filter banks [6]. These methods are accurate but they are computationally very inefficient.

In order to overcome the shortcomings of the above methods, the concept of fundamental identification and replacement was first introduced in [7]. In this method, the amplitude, frequency and phase of the fundamental harmonic component are estimated firstly. Then the noncoherent fundamental harmonic component was replaced by a sine component that has the same amplitude and phase but a slightly modified frequency so that it becomes coherent with the sampling clock. The method did not require coherent sampling and windowing. However, the method is robust only when the signal frequency is not very high. It is vulnerable when the signal frequency is close or above ADC's Nyquist frequency. Furthermore, the method is occasionally not computationally efficient if the data record length M has larger prime factors, especially M is a prime number.

An improved fundamental identification and replacement technique is proposed in this paper. The method can achieve accurate and robust spectral testing for sinusoidal signals without the need for coherent sampling or windowing. The improved method works well when the signal frequency is close to Nyquist frequency. Furthermore, the method is computational more efficient than the original one. In Section II, an improved fundamental identification and replacement technique is proposed. The simulation and experimental results are reported in Section III and IV respectively. Section V presents statistical analysis of extensive simulation results showing that the proposed method achieves spectral testing accuracies comparable to those obtained with perfect coherent sampling in an ideal noise-free environment. Section VI concludes this paper.

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II. THE IMPROVED FUNDAMENTAL IDENTIFICATION AND REPLACEMENT TECHNIQUE

Let f_s be the sampling frequency, $T_s=1/f_s$ the sampling intervals, f_i the unknown input signal frequency, and M_0 the nominal data record length. Then $J=M_0f_i/f_s=J_0+\Delta$ will be the number of periods input signal in the data record, where J_0 is the integer part of J, Δ is the fraction part of J. J_0 and M_0 are assumed to be co-prime. Δ is unknown, so is J (J_0 could be known).

Let the input signal be

$$x(t) = A\sin(2\pi f_i t+\theta)+\sum_{n\geq 2}(b_n \sin 2\pi n f_i t + a_n \cos 2\pi n f_i t) \quad (1)$$

Where $A \approx 1$, $\theta \in [0, 2\pi)$, a_n, b_n for n≥2 are all unknown, but together they satisfy

$$\sum_{n\geq 2}(b_n^2 + a_n^2) \ll 1 \quad (2)$$

$$\left|\sum_{n\geq 2}(b_n \sin 2\pi n f_i t + a_n \cos 2\pi n f_i t)\right| \ll 1 \quad (3)$$

The samples of x(t) at sampling rate f_s are given by:

$$x[k] = A\sin(2\pi f_i \frac{k}{f_s}+\theta) + \sum_{n\geq 2}(b_n \sin 2\pi f_i \frac{k}{f_s} + a_n \cos 2\pi f_i \frac{k}{f_s}) \quad (4)$$

Since $\frac{f_i}{f_s} = \frac{J}{M_0} = \frac{J_0+\Delta}{M_0}$,

$$\begin{aligned} x[k] &= A\sin(2\pi \frac{J}{M_0}k+\theta) + \sum_{n\geq 2}(b_n \sin 2\pi f_i \frac{k}{f_s} + a_n \cos 2\pi f_i \frac{k}{f_s}) \\ &= A\sin(2\pi \frac{J_0+\Delta}{M_0}k+\theta) + \sum_{n\geq 2}(b_n \sin 2\pi f_i \frac{k}{f_s} + a_n \cos 2\pi f_i \frac{k}{f_s}) \\ &= x_1[k] + x_h[k] \end{aligned} \quad (5)$$

In (5), $x_1[k]$ is the fundamental harmonic component of x[k], $x_h[k]$ is the sum of the 2nd and higher harmonic components of x[k]. From [7], we know that as long as Δ is non-zero, which means the data record length is not exactly an integer number of signal periods, the DFT algorithm introduces an error term (skirt term) in the Fourier transform of the fundamental component. This leakage term can be so large that it completely inundates the harmonic distortion components, making it impossible to correctly test the true spectrum of the signal. In order to estimate and remove the skirt term from the DFT spectrum, the concept of fundamental identification and replacement was first introduced in [7]. In this method, the amplitude, frequency and phase of the fundamental harmonic component are estimated firstly. Then the noncoherent fundamental harmonic component was replaced by a sine component that has the same amplitude and phase but a slightly modified frequency so that it becomes coherent with the sampling clock. After that the standard FFT spectral analysis is done as usual.

Unfortunately, the method in [7] has two shortcomings. The first one is that it is occasionally computationally unefficient. The method in [7] chooses the best data record length M by searching samples from M_0 to $2M_0$. As we all know, the algorithm of FFT is efficient and requires O(MlogM) operations if M has small factors, the algorithm of FFT will require $O(M^2)$ operations if M is a prime number. In order to avoiding the unlikely case that M is a prime number, we choose the best data record length M having only prime factors of 2 or 3, the two smallest prime numbers. Furthermore, the pairs far away from zero, such as the pair whose absolute values are larger than $A/\sqrt{2}$, are used to select the best data record length M so that the effects of noise can be reduced greatly. For example, search through M_1, M_2, ……, M_i, …… (M_i's factors comprise only 2 or 3) points, to find the $(1+M_i)$th data points that most closely matches the 1st point in the data sequence. That is, x[1] through $x[M_i+1]$ most closely match an integer number of signal periods. Then M_i is the best data record length. Therefore, the proposed method would be computationally more efficient than the method in [7].

After choosing the best data record length M, we still use the positive zero-crossing point as the starting point of the data record. For instance, in the data record from x[1] to x[M], x[1] is the positive zero-crossing point. By doing so, θ will be approximately 0 and the errors in estimating θ will have less effects.

The second one is that it is vulnerable when the signal frequency is near ADC's Nyquist frequency. The reason may be that the method in [7] doesn't count the integer cycles of input signal correctly when the signal frequency is high. Therefore, a new accurate method for counting the integer cycles of input signal is applied in the improved fundamental identification and replacement technique.

The new method for counting the integer cycles J_{int} in the data sequence from x[1] to x[M] is introduced as follows (here we call method I).

a) Let

y[k]=1 if $x[k] \geq A/\sqrt{2}$,

y[k]=-1 if $x[k] \leq -A/\sqrt{2}$,

y[k]=0 if $A/\sqrt{2} < x[k] < A/\sqrt{2}$

then get a new sequence y[k] (by doing this, the effects of noise can be reduced greatly).

b) In the sequence y[k], if the adjacent elements are the same, choose only one element, then get a new sequence w[k].

c) Define the variable c

- *If for all k, we have w[k]=w[k+4], then let c=0 indicating* $f_{in} \leq f_s/4$.
- *If there is at least one k with w[k]≠w[k+4], then let c=1 indicating* $f_{in} > f_s/4$.

d) Let $z[k] = |y[k]|$, *then get the new sequence z[k].*

e) *In the sequence z[k], if the adjacent elements are the same, choose only one element, then get a new sequence zz[k].*

f) *Count the sum sum(zz) of the non-zreo elements in sequence zz[k],*

g) *Compute the integer cycles J_{int}*
J_{int}=floor(sum(zz)/2) if c=0
J_{int}=M/2-floor(sum(zz)/2) if c=1.

The algorithm (in step c) determining whether the signal frequency is larger than $f_s/4$ or not can be easily verified by pigeonhole principle. Because of page limitation, we won't discuss it in this paper.

It should be pointed out that this method for counting the integer cycles is not robust when f_{in} is close to $f_s/4$. In this case, the step size of x[k]'s phase is close to π/2. Because of the noise, y[k]'s value is not accurate when x[k] is close to $A/\sqrt{2}$ or $-A/\sqrt{2}$. Therefore, when f_{in} is close to $f_s/4$, we use another method to count the integer cycles of input signal (Here we call it method II).

After method I is executed, we get J_{int}, if $20\% < \frac{J_{\text{int}}}{M} < 30\%$, we use method II to recount J_{int}. Because of page limitation, we cannot discuss method II in detail. Method II use the nature of sine wave, rising and falling, to count the integer cycles of input signal. Method II is extremely robust if f_{in} is close to $f_s/4$.

The procedure of the improved fundamental identification and replacement technique can be outlined in following 11 steps.

1) *Capture a sufficient number of samples,*
2) *Find the first point x[k1] whose absolute value is larger than $A/\sqrt{2}$,*
3) *choose the best data record length M. compare $x[k_1]$ with $x[M_1+k_1]$, $x[M_2+k_1]$, $x[M_3+k_1]$, ……, $x[M_i+k_1]$, ……$x[M_N+k_1]$ (M_i has only prime factors of 2 or 3, M_N is close to the length of original data record), to find the point $x[M_i+k_1]$ that most matches $x[k_1]$ in the data sequence. Then M_i is just the best data record length M.*
4) *count the integer cycles J_{int} of the sequence from $x[k_1]$ to $x[k_1+M-1]$ using method I,*
5) *If $20\% < \frac{J_{\text{int}}}{M} < 30\%$, recount J_{int} using method II,*
6) *Find the positive zero-crossing point $x[k_2]$ near $x[k_1]$, use all the data points between $x[k_2]$ and $x[k_2+M-1]$ as the data record.*
7) *normalize the data record using the power-based normalization, and get the first estimate of the fundamental harmonic magnitude $\widehat{A}_0$.*
8) *Compute the fractional cycle*

$$\Delta = \frac{1}{2\pi}[\arcsin(x[k_2]/\widehat{A}_0) - \arcsin(x[k_2+M]/\widehat{A}_0)] \tag{6}$$

Then the input signal frequency is

$$f_{in} = f_s \frac{J_{\text{int}} + \Delta}{M} \tag{7}$$

9) *At a subset of data points write*

$$\begin{aligned} x[k] &= A_0 \sin(2\pi f_{in} t_k + \theta) \\ &= A_0 \cos(\theta)\sin(2\pi f_{in} t_k) + A_0 \sin(\theta)\cos(2\pi f_{in} t_k) \end{aligned} \tag{8}$$

And use least square method to identify $A_0\cos(\theta)$ and $A_0\sin(\theta)$.

10) *Perform the fundamental component replacement*

$$\begin{aligned} \tilde{x}[k] &= x[k] - A_0 \sin(2\pi f_{in} t_k + \theta) \\ &+ A_0 \sin(2\pi f_s J_{\text{int}}/M + \theta) \end{aligned} \tag{9}$$

The new data $\tilde{x}[k]$ is generated by replacing the fundamental component from the original data (which is sampled noncoherently and causes possibly large skirts) with one that is coherent with the sampling clock. This is done by simply subtracting a sine component with the identified parameters and adding a sine component with the same A_0 and θ but with Δ being rounded to zero.

11) *Perform FFT analysis on $\tilde{x}[k]$ as usual.*

The improved method use the data record length M that has only prime factors of 2 or 3, the two smallest prime numbers. This guarantees that the new method is always computational efficient. Furthermore, the new algorithm for counting the integer cycles of input signal makes the improved method more robust and more immune to noise. It works well when the signal frequency is close to Nyquist frequency. Therefore, the improved method is robust and immune to noise.

III. SIMUALTION RESULT

Extensive simulation study and experimental study have been conducted in order to verify the performance of the proposed algorithm. During the simulations, ADC is modeled as a set of transition levels. Its nonlinearity error is chosen to be a Gaussian random variable with zero mean and standard deviation σ_{DNL}. In this section we present the spectral testing example of 12-bit ADC with σ_{DNL} of 0.02 LSB. Additive noise of input signal is also chosen to be a Gaussian random variable with zero mean and standard deviation of 0.25 LSB.

For comparison, three different spectral testing methods are simulated. They are: 1) straightforward application of DFT assuming periodic sampled sequence, 2) the proposed method, 3) perfect coherent sampling.

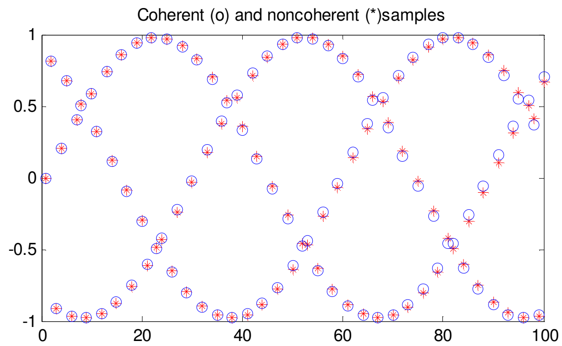

Figure 1 Data samples from coherent and noncoherent sampling

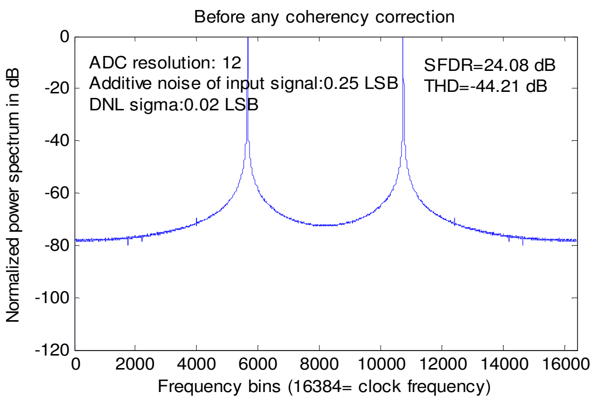

Figure 2 Straightforward application DFT to the noncoherent data samples

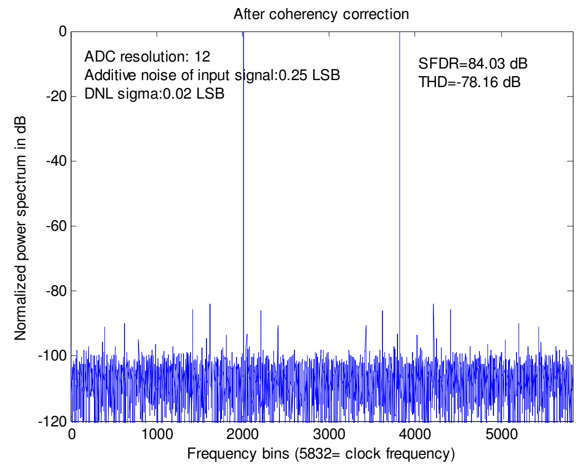

Figure 3 Spectrum of the noncoherent data samples using the proposed method

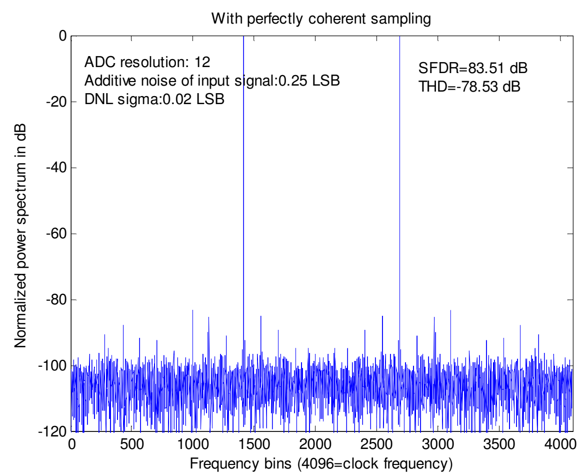

Figure 4 Spectrum from perfect coherent sampling

A time domain illustration of the coherent and noncoherent data is shown in Fig.1. Fig.2 shows the spectrum of straightforward application DFT to the noncoherent data samples. From Fig.2, we can see that there are large “skirt” around the spectral line. So straightforward application of DFT suffered from large errors due to non-coherency. The spectrum of the noncoherent dada samples using the proposed method and the spectrum from perfect coherent sampling data are shown in Fig.3 and Fig.4 respectively. From Fig.3 and Fig.4 we see that both spectrums show zero or minimal skirts.

Therefore, the simulation results show that the proposed method can achieve spectral testing accuracies similar to those obtained with perfect coherent sampling.

IV. Experimental Result

Since the proposed method exhibited excellent spectral performance, we want to validate the algorithms with experimental data. Fig.5 shows the segment of captured noncoherent data in time domain. The data is collected in an industry setting and the original data record length is 524288.

To analyze the spectral contents of the signal, one can straight forwardly apply DFT to the raw data. The resultant spectrum is shown in Fig.6. A small zoomed-in piece is shown on Fig.7. From Fig.7 we can see there are a large “skirt” around the spectral line. So the spectral leakage is serious. Fig. 8 shows the spectrum of the noncoherent data samples using the proposed method. A small zoomed-in piece is shown on Fig.9. From Fig.9, we can see that there are no “skirt” in the spectrum.

TABLE I. *ENOB, Time And M Using Different Methods*

Methods	ENOB	Time (s)	M
Original method	8.13	177.67	383169 =3*337*379
Proposed method	8.15	2.21	279936 $=2^7*3^7$

In order to test the performance of the proposed method, we also conduct the spectral analysis using the original method in [7]. Table I summarizes the ENOB, run time, and the best data record length M of both methods. From Table I we can see that the spectral testing accuracies of both methods are similar. However, the proposed method is significantly faster than the original method. The reason is that the best data record length M of original method has large prime factors, 337 and 339.

Therefore, the experimental results show that the proposed method is faster and robust.

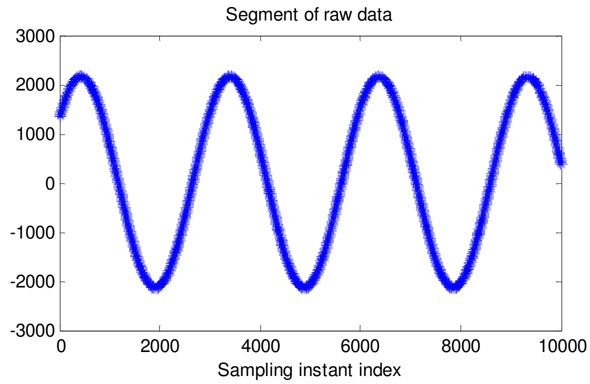

Figure 5 Raw data of noncoherent samples

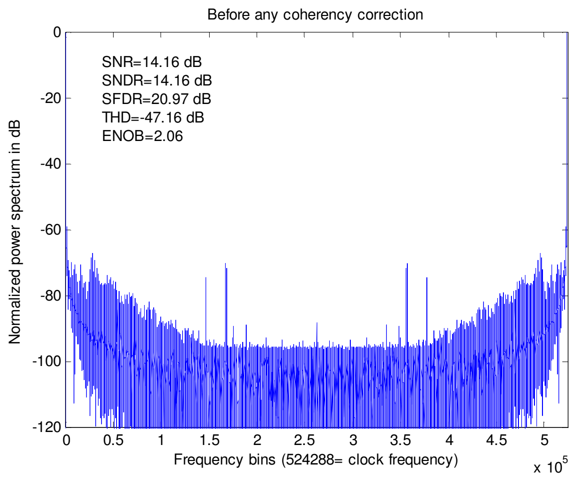

Figure 6 Spectrum by standard application of DFT

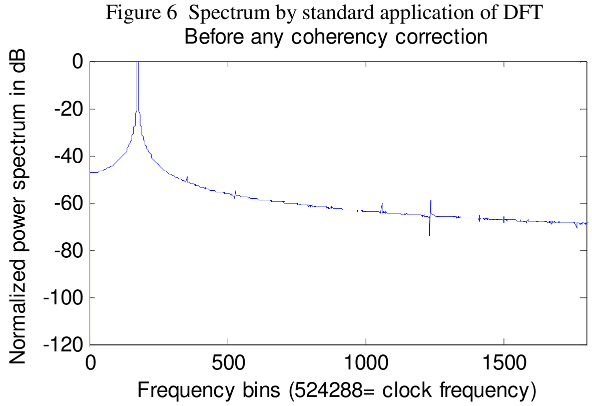

Figure 7 A zoomed-in piece showing the skirting

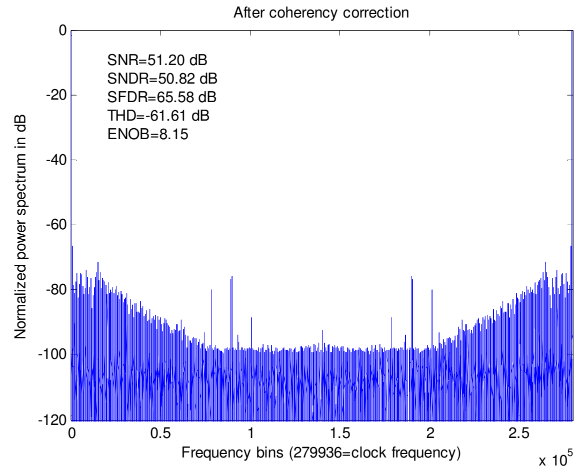

Figure 8 Spectrum of the noncoherent data samples using the proposed method

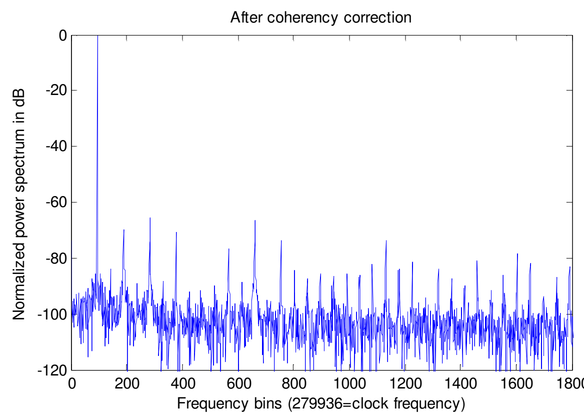

Figure 9 A zoomed-in piece showing no skirting

V. Statistical Performance Study

Statistical performance study is conducted in order to verify the performance of the proposed method. The simulation environment in Matlab is set up so that many parameters are randomly generated. The input signal frequency is generated by selecting a random ratio of f_{in} to f_s. The distorted sine wave signal is generated by adding random amount of harmonic distortion components to a pure sine wave. Additive measurement noise which is chosen to be Gaussian random variable is introduced at the input node of ADC with a standard deviation of around 0.25 LSB.

During the simulations, we avoid using the certain signal and clock frequency combinations if the ratio f_{in} / f_s can be reduced to a rational number with small integers. Furthermore, we also avoid using extremely low frequencies. The reason has been explained in [7].

Simulation of 1000 cases was conducted. Fig.10 and Fig.11 illustrate the signal SFDR testing errors and THD testing errors using the proposed method respectively. Notice that in all 1000 runs, the SFDR testing errors and THD testing errors are within ±3.4 dB and ±0.6 dB respectively. We also notice that SFDR testing errors and THD testing errors are small when the signal frequency is close to Nyquist frequency. Therefore the proposed method is demonstrated to be robust and immune to noise. Table II summarizes the statistics of these comparative study results.

Fig.12 shows the time in 100 runs using the proposed method and the original method in [7] respectively. From Fig.12 we can see that the proposed method is always faster and sometimes significantly faster than the original method.

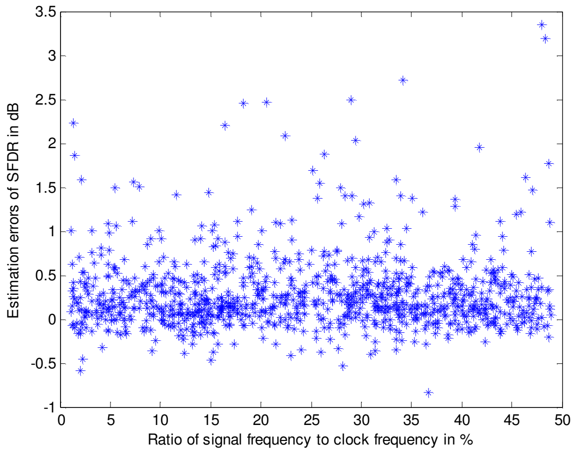

Figure 10 SFDR testing errors in 1000 runs using the proposed method, vs f_{in}/f_s*100

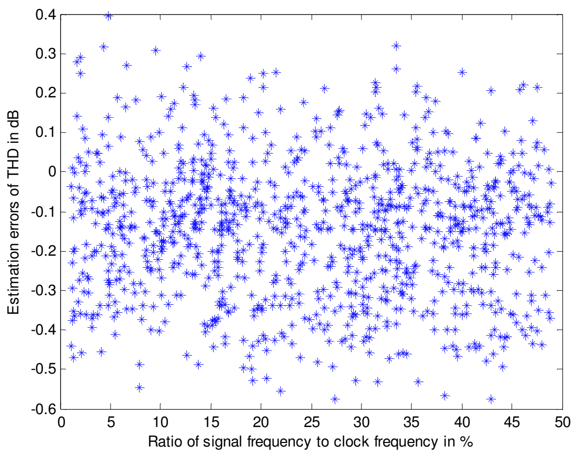

Figure 11 THD testing errors in 1000 runs using the proposed method, vs f_{in}/f_s*100

TABLE II. *SFDR AND THD TESTING ERRORS IN 1000 RUNS USING PROPOSED METHOD*

	SFDR errors (dB)	THD errors (dB)
max	3.35	0.40
min	-0.83	-0.58
mean	0.29	-0.15
std	0.42	0.17

VI. CONCLUSIONS

An improved method for faster and accurate spectral testing is proposed that does not require coherent sampling or the using of windowing. The new method uses the best data record length that has only prime factors of 2 or 3, the smallest prime numbers. This guarantees that the new method is always computational efficient. Because of this, the new method is always faster and sometimes it can be significantly faster than the original method. The paper also introduces a new algorithm for counting the signal periods in the data which make the proposed method is more robust and more immune to noise. Simulation and experimental results, statistical analysis validate this proposed method.

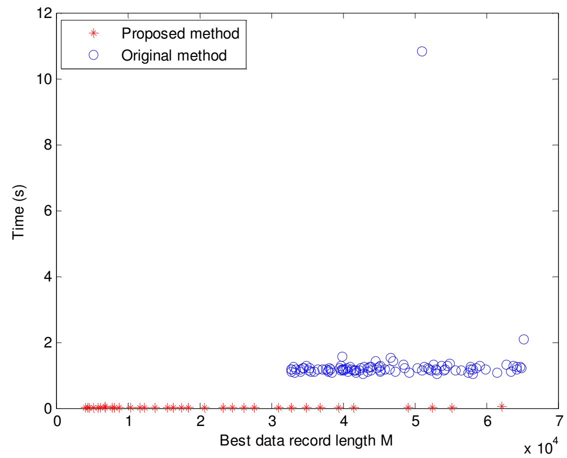

Figure 12 Time in 100 runs using the proposed method and original method respectively, vs the best data record M

REFERENCES

[1] IEEE Standard for Digitizing Waveform Recorders, IEEE Std. 1057TM-2007, April 2008.

[2] P. Carbone, E. Nunzi, and D. Petri, "Windows for ADC Dynamic Testing via Frequency-Domain Analysis", IEEE Trans. Instr. & Meas., vol.50, No.6, pp.1571-1576, 2001.

[3] F.J.Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform", Proc. IEEE, vol.66, No.1, pp.51-83, 1978.

[4] J. Zhang and S.J Ovaska, " ADC Characterization based on singular value decomposition", IEEE Trans. Instr. & Meas., vol.51, No.1, pp.138-143, 2002.

[5] X. Gao, S.J Ovaska and S. Shenghe,et al, " Analysis of second-order harmonic distortion of ADC using bispectrum", IEEE Trans. Instr. & Meas., vol.45, No.1, pp.50-55, 1996.

[6] C. Rebai, D. Dallet and P Marchegay, " Non-coherent Spectral Analysis of ADC using Filter Bank", IEEE Trans. Instr. & Meas., Tech. Conf., pp.183-187, 2002.

[7] Z. Yu, D. Chen, and R. L. Geiger, "A Computationally Efficient Method for Accurate Spectral Testing without Requiring Coherent Sampling ", Proceedings IEEE Int. Test Conference, pp.1398-1407, 2004.

[8] S.M.Kay, Fundamental of Statistical Signal Processing: Estimation Theory, Prentice-Hall, 1993.