ADC Spectral Performance Measurement Uncertainty in DFT Method

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***Abstract* — Spectral performances are important specifications need to be measured for many analog and RF circuits. Measurement errors of these performances are random variables related to additive noise which is normal distributed. Few research results on measurement error can be found in literature. Therefore accuracy or error of measurement is usually estimated by experience. This paper presents rigorous analyses of measurement error of THD and SFDR in DFT testing. Analyses are validated by Matlab simulations and provide useful guidance on how to select number of samples to achieve certain accuracy and confidence level in real testing.**

I. INTRODUCTION

Spectral performances are very important for many analog and mixed signal (AMS) circuit blocks such as LNA, Mixer, ADC, and etc. Among them, THD and SFDR are very most important parameters need to be measured in ADC production test [1]. Discrete Fourier transform (DFT) method is usually used to test spectral performance such as THD and SFDR. After fabrication, spectral performances of any circuit are fixed. Target of measurement is to find true values of these performances. However, due to additive noise, measurement result is different from the true value. The difference can be reduced by increasing number of samples in the testing [2].

Measurement uncertainty in ADC testing due to noise has been investigated and many papers are published. The uncertainty in linearity test is analyzed in [3]. Spectral performance test uncertainty is also investigated in [4] emphasizing noise and SNR. However, test accuracy of THD and SFDR is usually claimed by experience [5] without concrete foundation. Test uncertainty of spectral performance including THD and SFDR still needs to be investigated more detailedly. Rigorous and easy-to-use analysis of the uncertainty can be useful research results.

In this paper, test uncertainty in harmonic power in spectrum test is rigorously analyzed. Statistical behaviors of SFDR and THD test error are investigated. Based on the statistical behavior, a guidance of how to select number of samples for certain accuracy is provided. The rest of this paper is organized as following. In Section II, measurement errors in SFDR and THD are statistically analyzed. In Section III, simulation results are firstly provided as validation of theoretical analyses. And then more calculation is done to provide concise guidance for testing.

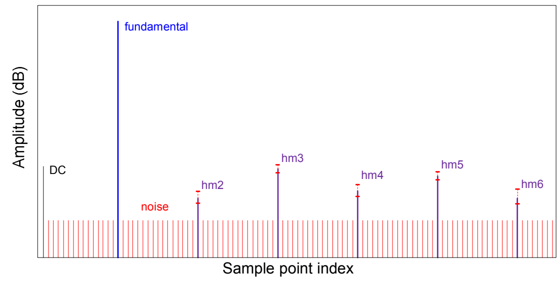

Fig.1. Spectrum of output signal

II. ANALYSIS OF ERROR IN DFT SPECTRAL TESTING

A. Error in harmonic power

The acquired signal in time domain can be expressed as

$$x(n) = s(n) + \delta(n) \tag{1}$$

The acquired signal contains true signal $s(n)$ and Gaussian additive noise $\delta(n)$. The noise is the input referred noise which contains both quantization and device noise. Spectrum of the signal is obtained by applying DFT to the data as (2) and Fig.1 shows. M is total number of samples used in the DFT testing. Statistic properties of this harmonic are examined in this section to show how additive noise in time domain cause error in SFDR test.

$$\begin{aligned} X'(k) &= \frac{1}{M}\sum_{n=0}^{M-1} x(n)\cdot e^{-j\cdot\frac{2\pi}{M}\cdot n\cdot k} \qquad k = 0,1,2,\ldots,M-1 \\ &= \frac{1}{M}\sum_{n=0}^{M-1} s(n)\cdot e^{-j\cdot\frac{2\pi}{M}\cdot n\cdot k} + \frac{1}{M}\sum_{n=0}^{M-1} \delta(n)\cdot e^{-j\cdot\frac{2\pi}{M}\cdot n\cdot k} \end{aligned} \tag{2}$$

Since Fourier transform is linear, the i^{th} harmonic can be expressed as the summation of true value and error term. From the second term of (2), we can further write the error term into (3).

$$\varepsilon_k = \frac{1}{M}\sum_{n=0}^{M-1} \delta(n)\cos\left(\frac{2kn\pi}{M}\right) + \frac{1}{M} j\sum_{n=0}^{M-1} \delta(n)\sin\left(\frac{2kn\pi}{M}\right) \tag{3}$$

From (3), we can find the following two properties of ε_k by simple calculation. The first one is that the expected values of real and imaginary part of ε_k are zero. The second property is that the expected value of square of ε_k magnitude is half of noise floor, which agrees with the fact that energy in time equals to that in frequency domain. σ^2 is the variance of input referred noise.

$$\begin{cases} E\left(\varepsilon_k\right) = 0 + j0 \\ E\left(\left|\varepsilon_k\right|^2\right) = \dfrac{\sigma^2}{M} \end{cases} \tag{4}$$

Spectrum obtained by DFT is symmetrical. Suppose the k^{th} and M-k bin in the spectrum capture the i^{th} harmonic component. So the i^{th} harmonic power is calculated from two bins as shown in (5). Since M-k bin is the conjugate of k^{th} bin, the power can be expressed as two times of the power of k^{th} bin. Power of harmonic bins is widely used as the harmonic distortion power by practitioners, however in order to achieve an unbiased estimate, the noise power per bin should be subtracted from it.

$$\begin{aligned} \hat{P}_{hi} &= \left|X'_k\right|^2 + \left|X'_{M-k}\right|^2 - \frac{2\sigma^2}{M} \\ &= 2\left|X_k + \varepsilon_k\right|^2 - \frac{2\sigma^2}{M} \end{aligned} \tag{5}$$

The true harmonic distortion power is $2|X_k|^2$. So the error in the i^{th} order harmonic distortion power caused by noise is shown in (6).

$$\begin{aligned} P_{hi\_err} &= \hat{P}_{hi} - 2\left|X_k\right|^2 \\ &= 2\left(c+a\right)^2 + 2\left(d+b\right)^2 - 2\left(a^2+b^2\right) - \frac{2\sigma^2}{M} \end{aligned} \tag{6}$$

in which, a and b are real and imaginary part of X_k, and c and d is the real and imaginary part of ε_k. For a certain ADC, the value of a and b are fixed, which means a and b is in (6) can be treated as constants. From (3), c and d have normal distribution and their mean is 0, and variance is $\sigma^2/2M$. Since cosine and sine function are orthogonal, c and d are independent to each other. Therefore, the expected value of P_{hi_err} is 0.

We are interested in the probability density function (pdf) and variance value of the error. Define another random variable as

$$Y = \left[\frac{\left(c+a\right)^2}{\sigma^2/2M} + \frac{\left(d+b\right)^2}{\sigma^2/2M}\right] \tag{7}$$

in which, $\left(c+a\right)$ and $\left(d+b\right)$ are independent and normal distributed. Y follows noncentral chi square distribution.

$$Y \sim \chi_k^2\left(\lambda\right) \tag{8}$$

In this case, k equals 2 and λ is

$$\lambda = \frac{a^2+b^2}{\sigma^2/2M} \tag{9}$$

Now (6) can be written as

$$P_{hi\_err} = \frac{\sigma^2}{M}\left(Y - \lambda - k\right) \tag{10}$$

The above analysis can be applied to any harmonic distortion and non-harmonic spurious signal. Suppose the ith harmonic power is the largest spurious signal, the tested SFDR value is

$$SFDR = 10\log\left(\frac{P_s}{P_{hi}}\frac{1}{\left(1+\dfrac{P_{hi\_err}}{P_{hi}}\right)}\right) \tag{11}$$

The error of SFDR in dB is

$$E_{SFDR} = -10\log\left(1+\frac{P_{hi\_err}}{P_{hi}}\right) \tag{12}$$

E_{SFDR} is a random variable related to Y. From (10) and (12), mean of it is 0, and the standard diviation is

$$\sigma\left(E_{SFDR}\right) \simeq \frac{10}{\ln\left(10\right)} \cdot 2\sqrt{\left(1+\frac{\sigma^2}{M \cdot P_{hk}}\right)\frac{\sigma^2}{M \cdot P_{hk}}} \tag{13}$$

It can be also expressed in terms of SFDR and SNR as following

$$\sigma\left(E_{SFDR}\right) \simeq \frac{20}{\ln\left(10\right)} \cdot \sqrt{\left(1+\frac{10^{\left(\frac{SFDR-SNR}{10}\right)}}{M}\right)\frac{10^{\left(\frac{SFDR-SNR}{10}\right)}}{M}} \tag{14}$$

(14) can be used as a guidance in measurement to select number of samples for DFT to achieve certain accuracy in SFDR measurement. However E_{SFDR} no longer follows a well known named distribution such as normal distribution. It is necessary to obtain the pdf of E_{SFDR} which is written in (15)

$$f\left(u,2,\lambda\right) = \frac{1}{2}e^{-\frac{y+\lambda}{2}} I_0\left(\sqrt{\lambda y}\right) \cdot \frac{\lambda \ln\left(10\right)}{10} \cdot 10^{-\frac{u}{10}} \tag{15}$$

Where $I_0(z)$ is the modified Bessel function of the first kind, u represents E_{SFDR}, and y is the function of u

$$y = k + \lambda \cdot 10^{-\frac{u}{10}} \tag{16}$$

B. *Error in total harmoinc distortion power*

The error in total harmonic distortion (THD) can be anaylized in similar way to that of SFDR. The unbiased tested total harmonic distortion power in DFT method is

$$\begin{aligned} \hat{P}_{thd} &= \sum_{k=2}^{h}\left(\left|X'_k\right|^2 + \left|X'_{M-k}\right|^2 - \frac{2\sigma^2}{M}\right) \\ &= 2\sum_{k=2}^{h}\left|X_k + \varepsilon_k\right|^2 - 2\left(h-1\right)\frac{\sigma^2}{M} \end{aligned} \tag{17}$$

in which h harmonics are taken into consideration. The true totla harmonic distortion power is the summation of all harmonic power. So the error in the tested value caused by input noise is

$$P_{thd\_err} = \hat{P}_{thd} - 2\sum_{k=2}^{h}\left|X_k\right|^2 = 2\sum_{k=2}^{h}\left(c_k + a_k\right)^2 + 2\sum_{k=2}^{h}\left(d_k + b_k\right)^2 - 2\sum_{k=2}^{h}\left(a_k^2 + b_k^2\right) - 2\left(h-1\right)\frac{\sigma^2}{M} \quad (18)$$

in which, a_k and b_k are real and imaginary part of X_k, and c_k and d_k is the real and imaginary part of ε_k. For a certain ADC, the value of a_k and b_k are fixed and can be treated as constants in (18). c_k and d_k are normal distributed and independent. Their mean is 0, and variance is $\sigma^2/2M$. Therefore, the expected value of P_{thd_err} is 0, which shows the unbiasness.

Define another random variable as following

$$W = \frac{\sum_{k=2}^{h}\left(c_k + a_k\right)^2}{\sigma^2/2M} + \frac{\sum_{k=2}^{h}\left(d_k + b_k\right)^2}{\sigma^2/2M} \quad (19)$$

in which, $(c_k + a_k)$ and $(d_k + b_k)$ are independent and normal distributed. W follows noncentral chi square distribution.

$$W \sim \chi_k^2\left(\lambda\right) \quad (20)$$

In this case, k equals $2(h\text{-}1)$ and λ is

$$\lambda = \frac{\sum_{i=2}^{h}\left(a_i^2 + b_i^2\right)}{\sigma^2/2M} \quad (21)$$

Now (18) can be written as

$$P_{thd\_err} = \frac{\sigma^2}{M}\left(W - \lambda - k\right) \quad (22)$$

In spectral performance testing, the THD is calculated as

$$THD = 10\log\left(\frac{P_{thd}}{P_s}\cdot\left(1 + \frac{P_{thd\_err}}{P_{thd}}\right)\right) \quad (23)$$

And the error of tested THD in dB is

$$E_{THD} = 10\log\left(1 + \frac{P_{thd\_err}}{P_{thd}}\right) \quad (24)$$

E_{THD} is a random variable related to W. From (22) and (24), mean of it is 0, and the standard diviation can be expressed in terms of SNR and THD.

$$\sigma\left(E_{THD}\right) \simeq \frac{20}{\ln(10)}\cdot\sqrt{\left(1 + \left(h-1\right)\frac{10^{\left(\frac{|THD|-SNR}{10}\right)}}{M}\right)\frac{10^{\left(\frac{|THD|-SNR}{10}\right)}}{M}} \quad (25)$$

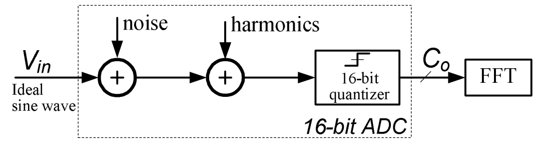

Fig.2. Simulation setup

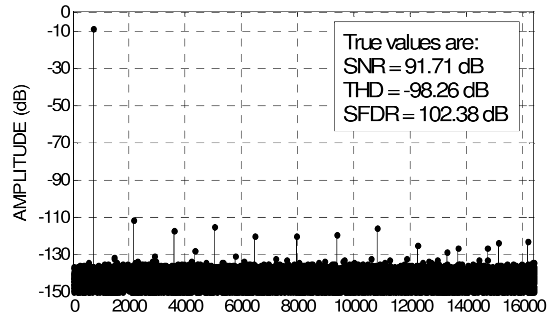

Fig.3. Spectral performance of the ADC

Again, providing standard deviation of E_{THD} (14) can guide the selection of number of samples for DFT to achieve certain accuracy in THD measurement. It is necessary to obtain the pdf of E_{THD} which is written in (26)

$$f\left(v, 2\left(h-1\right), \lambda\right) = \frac{1}{2}e^{-\frac{w+\lambda}{2}} I_0\left(\sqrt{\lambda w}\right)\cdot\frac{\lambda\ln(10)}{10}\cdot 10^{-\frac{v}{10}} \quad (26)$$

Where $I_0(z)$ is the modified Bessel function of the first kind, u represents E_{THD}, and y is the function of v

$$w = k + \lambda\cdot 10^{-\frac{v}{10}} \quad (27)$$

III. Simulation Results

Simulation is firstly done to validate correctness of above analysis. The analysis applies to DFT testing of any type of circuit such as ADC, LNA, Mixer, and etc. An ideal 16-bit ADC is modeled in MATLAB as a set of transition levels. As shown in Fig.2, input signal used for spectral testing is summation of an ideal sine wave, additive noise, and 40 harmonic components. Amplitudes of all harmonic components are randomly generated and scaled to certain levels and shape look similar to what most ADCs have. The additive noise standard deviation is about half LSB and is also randomly generated. ADC with noise and distortion is modeled by components in the dashed block. By generating harmonic distortion and noise intentionally, the true values of SNR, THD, and SFDR are know, which helps validation of the method. From output code of the ADC, DFT method tests SFDR and THD. Fig.3 shows the spectrum of output of the ADC when 2^{15} points are used in DFT.

Test the same ADC 10000 times with different random additive noise. Fig.4 shows the distribution of testing error of 2^{15} point DFT testing, from which we know standard deviations of tested SFDR and THD error are 0.165 dB and 0.102 dB respectively. Put the conditions in Fig.3 into (14)

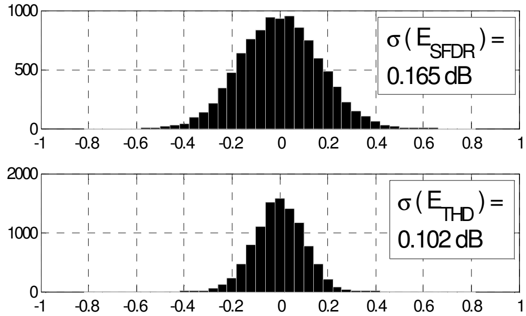

Fig.4. test error of spectral performance in FFT

Table.1. Error of SFDR in FFT testing

# of pts	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
Calculation	1.323	0.930	0.656	0.463	0.327	0.231	0.164
Simulation	1.381	0.964	0.663	0.470	0.330	0.231	0.165
Δ	0.042	0.034	0.007	0.007	0.003	0	0

Table.2. Error of THD in FFT testing

# of pts	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}
Calculation	0.943	0.623	0.424	0.294	0.206	0.145	0.102
Simulation	0.915	0.608	0.419	0.294	0.207	0.144	0.102
Δ	0.028	0.015	0.005	0	0.001	0.001	0

and (25), we can calculate the standard deviations of SFDR and THD test error are 0.164 dB and 0.102 dB respectively.

When smaller number of samples M is used in FFT test, the error will increase. Do the same simulation and calculation for different M. Table.1 and 2 show error standard deviations in simulation and calculation when number of samples change from 2^9 to 2^{15}. From these two tables, the analysis results match simulation very well.

As mentioned in section II, how to select number of samples to obtain certain testing accuracy with a certain confidence level is more useful in testing. Rewrite (9) and (21) as

$$\lambda_s = M \cdot 10^{0.1(SNR-SFDR)} \quad (28)$$

$$\lambda_d = M \cdot 10^{0.1(SNR-|THD|)} \quad (29)$$

Base on equation (15) and (26), required λ_s and λ_d values corresponding to a certain accuracy and confidence level can be calculated. Table.3 and Table.4 shows some calculation results. For example, if we want to test SFDR value so that 99.9% test results has less than 0.1dB error, the required λ_s is 81800.

More detailedly, take ADC test as an example. The nominal values of SNR, THD, and SFDR of the ADC are 92dB, -98dB, and 102dB. SFDR test requirement is 99.9% tests has less than 1dB error. And THD test requirement is 99.9% tests has less than 0.5dB error. So from Table.3 we know λ_s needs to be larger than 850. Regarding the nominal performance, from (28) we can decide that more than 8500 samples are needed in the measurement. Accuracy of THD test needs to be considered at the same time. From Table.4

Table.3. Required λ_s for certain confidence and SFDR error bound

Conf. level / Bound	70%	80%	90%	99%	99.9%
0.1	8108	12394	20415	50100	81800
0.5	327	499	821	2018	3300
1.0	83.3	127	209	515	850
1.5	38.2	57.8	95	237	395
2.0	22.4	33.7	55.2	140	234
3.0	11.1	16.5	27	70	117.7

Table.4. Required λ_d for certain confidence and THD error bound

Conf. level / Bound	70%	80%	90%	99%	99.9%
0.1	8492	12793	20836	50563	83000
0.5	593	790	1143	2409	3750
1.0	262	325	432	790	1156
1.5	174	208	264	445	622
2.0	132.3	155.4	192.5	306.6	417
3.0	94.1	107.9	129.5	193.2	251.9

we know needs to be larger than 3750. From (29) we can decide that more than 14930 samples are needed in the measurement. Based on the calculation, 16384 samples will be acquired for spectral performance test.

IV. CONCLUSION

Spectral performance testing is important for many circuits. Spectrum based method is usually used to test spectral performance such as THD and SFDR. However, few references on measurement error of THD and SFDR can be found in literature. This paper rigorously analyzes statistical behavior of THD and SFDR test error in DFT method. Analyses are validated by simulation. Analysis results are useful guidance on how to select number of samples to achieve certain accuracy and confidence level in real testing.

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