

# A Novel Robust and Accurate Spectral Testing Method for Non-coherent Sampling

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**Abstract**— Spectral testing is one of the frequently encountered problems in signal processing and communications. It is a challenging task to obtain coherent sampling for accurate spectral testing. Windowing techniques are widely used to perform spectral testing when the sampling is slightly non-coherent. This paper proposes a new Fundamental Identification and Replacement (FIR) method. The proposed method can estimate the spectral characteristics accurately without requiring coherent sampling. The method is robust to any level of non-coherency, which makes on-chip spectral testing possible. The new method is computationally efficient and is applicable for high resolution spectral testing. Furthermore, the proposed method can perform both single tone signal test and multiple tone signal test accurately. The method gives accurate results even in situations when the windowing techniques cannot give correct results. Simulation results show the robustness and the computational efficiency of the proposed method. The method is also validated with the experimental data.

## I. INTRODUCTION

Spectral testing is a very widely encountered problem. The Discrete Fourier Transform (DFT) or its fast implementation the Fast Fourier Transform (FFT) is usually used to perform spectral testing [1]. To accurately estimate the spectral characteristics of a signal, the IEEE standard 1057 for digitizing waveform recorders [2] and IEEE standard 1241 for Analog to Digital Converter (ADC) testing [3] recommend the input signal to be coherently sampled. Figure 1 shows the setup for traditional spectral performance test of ADCs, in which, a pure sine wave is applied to an ADC under test as stimulus [3]. Two conditions must be satisfied in traditional spectral testing to achieve valid test results. The first is that the sine wave must be pure enough so that its distortion is much lower than ADC under test. The second is that the input signal frequency must be well controlled to achieve coherent sampling.

If coherent sampling is not available, the frequency leakage will be present in the spectrum which gives erroneous test results. This problem due to non-coherent sampling will become more significant when on-chip testing needs to be done. It is because, such systems cannot afford to have highly accurate on-chip signal and clock generators. Furthermore, due to the noise and jitter, it is difficult to obtain coherency if we do not use PLLs and stick to low cost. If LC oscillators or PLLs are used, it is more expensive for on-chip testing. Therefore, there is an urgent demand for a spectral testing method that totally makes the non-coherency a non-issue.

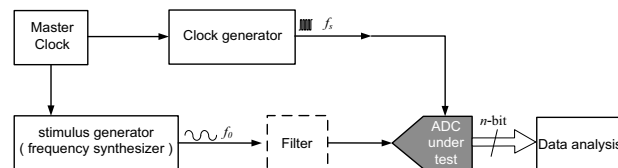


Fig. 1: Spectral test setup [3]

Recently, numerous methods are proposed in the literature to tackle the frequency leakage problem. In [4-7], the fundamental identification and replacement methods were proposed. A singular value decomposition method with a time complexity of  $O(M^3)$  was proposed in [8], a 2-D FFT method with a time complexity of  $O(M^2 \log^2 M)$  was proposed in [9] and a filter bank method that results in an increase in testing circuitry area was proposed in [10]. In [11-12], Interpolated DFT techniques were introduced to suppress the spectral leakage.

Another method that has been in use for a long time is to use windows on non-coherently sampled data [13]-[15]. The windowing technique is widely used in low resolution spectral testing if the sampling is slightly non-coherent. However, for the situations when the sampling is not close to coherent sampling and for high resolution spectral testing, as we point out later in this paper, the windowing technique is not sufficient to provide accurate results.

This paper proposes a new spectral testing method that completely relaxes the condition of coherent sampling and is faster than the state-of-the-art testing methods with non-coherent sampling [4-6]. As a result, the test setup cost can be reduced because low accuracy signal generators and clock generators can be used for spectral testing. As non-coherency is no longer an issue in the proposed method, this method can be applied for on-chip spectral testing. The method is computationally efficient and can be used for high resolution spectral testing.

This paper is arranged as follows. Section II describes the spectral testing and coherency. Section III proposes the new method to accurately estimate spectral characteristics. Simulation results are presented in Section IV. Section V compares the new method with windowing technique and state-of-the-art method. Section VI presents the experimental results and section VII concludes the paper.

## II. SPECTRAL TESTING AND COHERENCY

In this section, we will briefly discuss the spectral testing problem of a slightly distorted cosine signal. The distortion is caused due to the presence of harmonic components at frequencies that are integer multiples of the fundamental. The noise present in the signal is assumed to be a random Gaussian variable.

Let the input signal be

$$x(t) = A_1 \cos(2\pi f_i t + \phi) + \sum_{h=2}^H A_h \cos(2\pi h f_i t + \phi_h) + w(t) \quad (1)$$

where  $A_1, f_i$  and  $\phi$  are the amplitude, frequency and initial phase of the fundamental component of input signal respectively,  $H$  is the total number of harmonics present in the input signal,  $A_h$  and  $\phi_h$  are amplitude and initial phase of the  $h^{\text{th}}$  harmonic respectively where  $A_h \ll A_1$  and  $\phi_h \in [0, 2\pi)$  for all  $2 \leq h \leq H$  and  $w(t)$  is noise at time  $t$ .

Since most of the applications involve signal processing in digital domain, for digital signal processing (DSP), the input signal is sampled at a given clock frequency  $f_{\text{Samp}}$ . Let  $M$  be the total number of sampled data points and  $J$  be the total number of input signal periods present in  $M$  recorded points. The four parameters  $f_{\text{Samp}}, f_i, J$  and  $M$  are related by (2)

$$J = M \frac{f_i}{f_{\text{Samp}}} \quad (2)$$

The sampling is said to be coherent if  $M, f_i$  and  $f_{\text{Samp}}$  are selected such that  $J$  is an integer and  $J$  and  $M$  are co-prime. This condition assures that the power of fundamental and harmonics are each contained in a single bin corresponding to their frequencies in the spectrum as shown in Figure 2.

From equations (1) and (2), the  $M$  sampled data points of input signal are given by (3), where  $n = 0, 1, 2, \dots, M-1$ .

$$x[n] = A_1 \cos\left(\frac{2\pi J}{M} n + \phi\right) + \sum_{h=2}^H A_h \cos\left(\frac{2\pi h J}{M} n + \phi_h\right) + w[n] \quad (3)$$

Spectral testing involves testing the input signal for total harmonic distortion (THD), spurious free dynamic range (SFDR), signal to noise ratio (SNR) and signal to noise and distortion ratio (SNDR), which require the estimation of signal, harmonic and noise power.

The fundamental and harmonic power can be obtained by taking DFT of  $M$  coherently sampled data points. The DFT of  $x[n]$  is given by (4)

$$X_k = \frac{1}{M} \sum_{n=0}^{M-1} x[n] e^{-j \frac{2\pi k n}{M}}, \text{ for } k = 0, 1, \dots, M-1 \quad (4)$$

where  $k$  represents number of the frequency bin.

The power of the  $h^{\text{th}}$  harmonic can be estimated by taking the sum of squares of absolute values of  $X_{h*J}$  and  $X_{M-(h*J)}$ . Since the absolute values of  $X_{h*J}$  and  $X_{M-(h*J)}$  are equal for a cosine wave, the true values of fundamental power and  $h^{\text{th}}$  harmonic power can be calculated by  $P_1$  and  $P_h$  respectively.

$$P_1 = 2 * |X_{J}|^2 = \frac{A_1^2}{2} \quad (5)$$

$$P_h = 2 * |X_{hJ}|^2 = \frac{A_h^2}{2}, \quad 2 \leq h \leq H \quad (6)$$

The true values of total harmonic distortion (THD) and spurious free dynamic range (SFDR) of the input signal are given by (7).

$$THD = \frac{\sum_{h=2}^H |X_{hJ}|^2}{|X_J|^2}, \quad SFDR = \frac{|X_J|^2}{\max(|X_{hJ}|^2)}, \quad h = 2, 3, \dots, H \quad (7)$$

From (7), accurate estimation of THD and SFDR can be obtained if and only if the magnitude of  $X_k$  is calculated accurately for all values of  $k$  corresponding to the fundamental and harmonic frequencies. However the essential requirement for accurate estimation of  $X_k$  is that the signal should be coherently sampled.

If the condition of coherent sampling is not satisfied, i.e., if  $J$  is not an integer, the DFT algorithm produces severe skirting which results in inaccurate values of DFT coefficients,  $X_k$ . This leads to faulty estimation of harmonic and signal power as the power of fundamental and harmonics are no longer contained in a single bin corresponding to their frequencies as shown in Figure 3. The shape of the spectrum is dependent on the amount of non-coherency present in the fundamental [4].

As mentioned earlier, obtaining perfect coherent sampling is a challenging task and is very expensive. In order to reduce the test cost and to facilitate on-chip spectral testing, it is important to develop new spectral testing methods that can give accurate results even when the input is not coherently sampled.

The goal of this paper is to propose a new method which eliminates the requirement of coherent sampling for spectral testing. The functionality of the method is proved by presenting both simulation and experimental results.

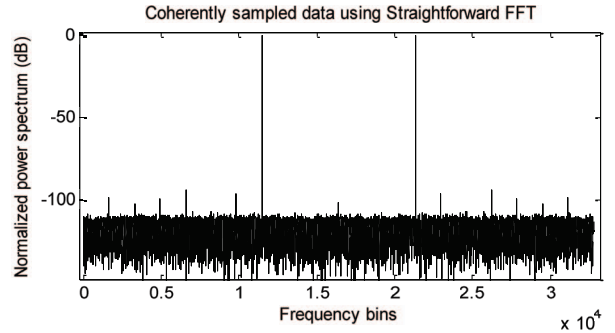


Fig. 2: Power Spectrum of a coherently sampled data that shows no skirting.

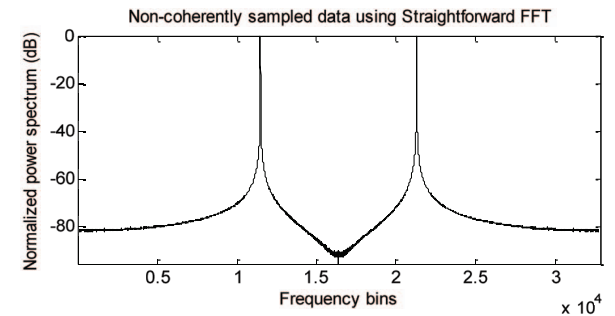


Fig. 3: Power Spectrum of a non-coherently sampled data with skirting.

### III. THE PROPOSED METHOD

This section introduces the contribution of different error sources on harmonic power estimation due to non-coherent sampling. Then, a new method to identify the fundamental is proposed. At last the proposed Algorithms to perform single-tone and multi-tone test are presented.

Consider the case when input is not coherently sampled. As a result,  $J$  in (2) is not an integer. Let  $J = J_{\text{int}} + \delta$ , where  $J_{\text{int}}$  is the integer part closest to  $J$  and  $J > 25$  and  $\delta$  is the fractional part of  $J$  and  $-0.5 < \delta \leq 0.5$ . Let  $M > 1024$  and  $D$  be the minimum number of bins between any two harmonics in the frequency spectrum. The conditions on  $J$  and  $M$  are valid because it is a common practice to select more number of input cycles  $J$ , and collect more data record points  $M$ , to estimate the spectral characteristics of a signal.

#### A. Errors due to non-coherent sampling:

In this section, two theorems are introduced, which discusses the contribution of different error sources on harmonic power estimation when an input is not coherently sampled. Due to space constraints, the proof of the theorems is not given.

*Theorem 1:* For given  $H, M, J, A_1, \phi, \phi_h$  and  $A_h$  as mentioned above, if the input is not coherently sampled, i.e.,  $\delta \neq 0$ , the error in estimating power of the  $q^{\text{th}}$  harmonic using DFT is mainly due to the non-coherency present in fundamental component.

*Theorem 2:* For given  $H, M, J, A_1, \phi, \phi_h$  and  $A_h$  as mentioned above, if the input is not coherently sampled, i.e.,  $\delta \neq 0$ , the error in estimating the  $q^{\text{th}}$  harmonic power due to non-coherency present in other harmonics is negligible compared to the error due to non-coherency in  $q^{\text{th}}$  harmonic itself, provided the non-coherent fundamental is identified and removed.

Figure 4 and Figure 5 illustrate the two theorems. Figure 4 is the spectrum of a non-coherently sampled data. There is severe skirting present in the spectrum as expected. Figure 5 shows the spectrum of the same data after removing the non-coherent fundamental component. Compared to Figure 4, we see that Figure 5 does not have skirting, which explains Theorem 1.

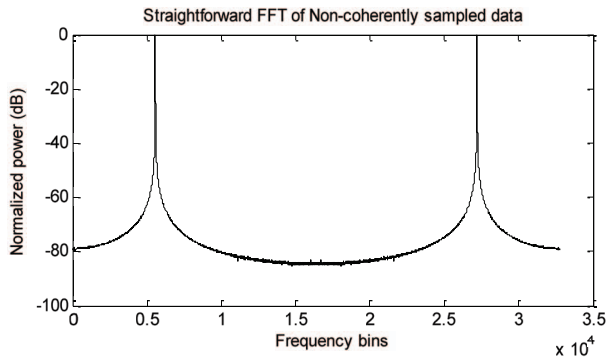


Fig. 4: Spectrum of a direct FFT (Straightforward FFT) of Non-coherently sampled data showing severe skirting due to non-coherency in the fundamental component. The harmonics are not visible.

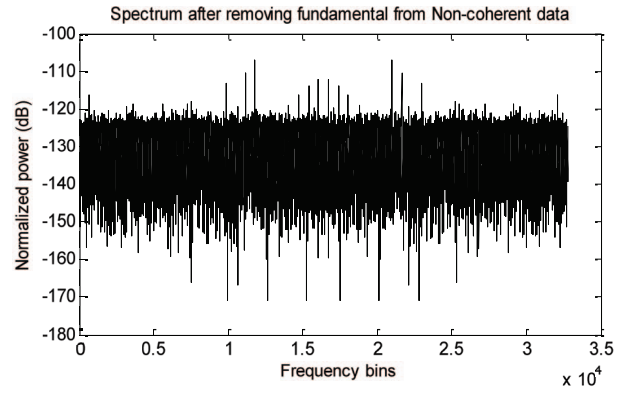


Fig. 5: Spectrum of the non-coherently sampled data after the fundamental frequency is identified and removed. The harmonics are clearly visible.

From the above two theorems, it can be stated that, in order to estimate the spectral characteristics accurately when an input is not coherently sampled, the non-coherent fundamental needs to be estimated accurately and replaced with a cosine component that has the same amplitude and initial phase but a slightly modified frequency so that it becomes coherent with the sampling clock. The harmonics need not be replaced as Theorem 2 states that their effect on estimating the harmonic power is negligible.

#### B. Fundamental Identification

Several fundamental identification methods were proposed in literature [16-17]. In this section, a new method to identify the fundamental from a fixed data record length is proposed. The fundamental frequency is estimated using the frequency domain data, i.e., the DFT.

Let  $x(t)$  be the time domain representation of a pure input signal.

$$\begin{aligned} x(t) &= A \cos(2\pi f_i t + \phi) \\ &= a \cos(2\pi f_i t) + b \sin(2\pi f_i t) \end{aligned} \quad -- (8)$$

where  $A, f_i$  and  $\phi$  are the amplitude, frequency and initial phase of the fundamental frequency respectively,  $a = A \cos(\phi)$  and  $b = -A \sin(\phi)$ ,

Let  $x[n]$  be the analog interpretation of the  $M$  digitized output data points of  $x(t)$ , where  $n = 0, 1, 2, \dots, M-1$ .

$$\begin{aligned} x[n] &= A \cos\left(\frac{2\pi J}{M}n + \phi\right) + \sum_{h=2}^H A_h \cos\left(\frac{2\pi h J}{M}n + \phi_h\right) + w(n) \\ x[n] &= a \cos\left(\frac{2\pi J}{M}n\right) + b \sin\left(\frac{2\pi J}{M}n\right) + HD + w(n) \end{aligned} \quad -- (9)$$

where  $HD$  represents the harmonic components present in the digitized signal.  $HD$  contains the harmonic information of both the input signal and the digitizer. If the digitizer is the device under test, the input signal should be more pure than the digitizer and vice-versa.

The time domain data acquired from the sampler is converted to frequency domain by taking the DFT of the data [17]. Using the DFT coefficients, the fundamental can be accurately identified. To identify the fundamental, the parameters that need to be estimated are  $J, \mathbf{a}$  and  $\mathbf{b}$  in (9).

a) *Estimate J*

As mentioned in section III,  $J = J_{\text{int}} + \delta$ , and  $-0.5 < \delta \leq 0.5$ . To estimate J, it is required to estimate  $J_{\text{int}}$  and  $\delta$ .

$J_{\text{int}}$  is estimated by considering the frequency bin in the half spectrum that contains the maximum power excluding the DC component.

$$\hat{J}_{\text{int}} = \arg \max_{1 \leq k \leq (M/2)} |X_k| \quad - (10)$$

The value of  $\delta$  can be estimated by taking a three-point calibration method using the DFT coefficients. The three bins considered are the fundamental bin and the two bins adjacent to the fundamental which are given by  $X_{J_{\text{int}}}$ ,  $X_{J_{\text{int}}+1}$ ,  $X_{J_{\text{int}}-1}$  respectively. The expression to estimate  $\delta$  can be given by (11), where  $\text{imag}(W)$  represents the imaginary part of  $W$ .

$$\hat{\delta} = -\frac{M}{2\pi} \text{imag} \left\{ \ln \left\{ \frac{\frac{X_{J_{\text{int}}} X_{J_{\text{int}}}}{X_{J_{\text{int}}+1} X_{J_{\text{int}}-1}}}{\frac{X_{J_{\text{int}}} X_{J_{\text{int}}}}{X_{J_{\text{int}}+1} X_{J_{\text{int}}-1}} e^{-j2\pi/M} + e^{j2\pi/M}} \right\} \right\} \quad - (11)$$

The estimated value of J is given by

$$\hat{J} = \hat{J}_{\text{int}} + \hat{\delta} \quad - (12)$$

b) *Estimate a and b.*

Now that J is estimated, **a** and **b** can be estimated using Least Squares method. Multiply equation (9) with  $\cos\left(\frac{2\pi Jn}{M}\right)$  and add all the M points. Neglecting all the other terms except the cosine squared term gives

$$\sum_{n=0}^{M-1} x[n]^* \cos\left(\frac{2\pi Jn}{M}\right) \approx \sum_{n=0}^{M-1} \left\{ a^* \cos^2\left(\frac{2\pi Jn}{M}\right) \right\} \quad - (13)$$

From (13), **a** can be estimated as

$$\hat{a} = \frac{\sum_{n=0}^{M-1} x[n]^* \cos\left(\frac{2\pi Jn}{M}\right)}{\sum_{n=0}^{M-1} \left\{ \cos^2\left(\frac{2\pi Jn}{M}\right) \right\}} \quad - (14)$$

Similarly **b** can be estimated by (15)

$$\hat{b} = \frac{\sum_{n=0}^{M-1} x[n]^* \sin\left(\frac{2\pi Jn}{M}\right)}{\sum_{n=0}^{M-1} \left\{ \sin^2\left(\frac{2\pi Jn}{M}\right) \right\}} \quad - (15)$$

c) *Estimate the non-coherent Fundamental.*

After estimating J, **a** and **b**, the non-coherent fundamental component can be identified. Let  $xnc[n]$  be the estimated fundamental component of the input signal  $x[n]$  that was not sampled coherently.

$$xnc[n] = \hat{a} \cos\left(\frac{2\pi Jn}{M}\right) + \hat{b} \sin\left(\frac{2\pi Jn}{M}\right) \quad - (16)$$

So, the main source of error in estimating the harmonic power when an input is not coherently sampled is estimated.

C. *Algorithm for Proposed Method*

The algorithms to accurately obtain the spectral characteristics of a sampled data using the proposed method for both single-tone and multiple tone testing are given.

a) *Single-tone Test:*

1. Collect M data points from the output of the digitizer, i.e.,  $x[n]$ ,  $n = 0, 1, 2, \dots, M-1$ .
2. Take DFT of the M points to obtain  $X_k$ ,  $k = 0, 1, \dots, M-1$ .
3. Estimate  $J_{\text{int}}$  from the DFT coefficients  $X_k$  using (10).
4. Estimate the value of  $\delta$  using (11).
5. Estimate number of input cycles  $J = J_{\text{int}} + \delta$ .
6. Estimate **a** and **b** using (14) and (15).
7. Construct  $xnc[n]$ , the non-coherent fundamental component in  $x[n]$  using (16).
8. Construct  $xc[n]$ , the coherent fundamental component closest to the actual input signal with  $J_{\text{int}}$  number of cycles.

$$xc[n] = \hat{a} \cos\left(\frac{2\pi J_{\text{int}}n}{M}\right) + \hat{b} \sin\left(\frac{2\pi J_{\text{int}}n}{M}\right) \quad - (17)$$

9. Remove the non-coherent fundamental component from the actual data and replace it with the coherent fundamental component. Let  $\hat{x}[n]$  be the final data,

$$\hat{x}[n] = x[n] - xnc[n] + xc[n] \quad - (18)$$

10. Take DFT of  $\hat{x}[n]$  and perform spectral analysis to accurately estimate the spectral characteristics.

b) *Multi-tone Test.*

Similar to single-tone signal test, we first identify the amplitude, frequency and phase of each non-coherent fundamental component. Once this is done, we replace each non-coherent fundamental component with a cosine component that has the same amplitude and initial phase but a slightly modified frequency so that it becomes coherent with the sampling clock. Let  $m$  be the total number of fundamental tones present in the signal. The suffix  $i$  in algorithm represents the  $i^{\text{th}}$  fundamental that is estimated. Let  $J_{\text{int}_i}$  represent the  $J_{\text{int}}$  of  $i^{\text{th}}$  fundamental in the signal. The procedure is as follows.

1. Collect M data points from the output of the digitizer, i.e.,  $x[n]$ ,  $n = 0, 1, 2, \dots, M-1$ .
2. Let  $i=1$ ,  $x'[n]=x[n]$ , perform DFT of  $x'[n]$  to obtain  $X_k$ .
3. Estimate  $J_{\text{int}_i}$  from the DFT coefficients  $X_k$ .

$$\hat{J}_{\text{int}_i} = \arg \max_{1 \leq k \leq (M/2)} |X_k| \quad --(19)$$

4. Estimate the value of  $\delta_i$  using  $J_{\text{int}_i}$  and equation (11).
5. Estimate number of input signal cycles in M points.

$$\hat{J}_i = \hat{J}_{\text{int}_i} + \hat{\delta}_i$$

6. Estimate  $\mathbf{a}_i$  and  $\mathbf{b}_i$  using equations (14) and (15).
7. Construct  $x_{nc\_i}[n]$ , the  $i^{\text{th}}$  non-coherent fundamental component in  $x[n]$  using equation (16).
8. Construct  $x_{c\_i}[n]$ , the coherent fundamental component closest to the  $i^{\text{th}}$  fundamental in input signal with  $J_{\text{int}_i}$  number of cycles.

$$x_{c\_i}[n] = a_i \cos\left(\frac{2\pi n J_{\text{int}_i}}{M}\right) + b_i \sin\left(\frac{2\pi n J_{\text{int}_i}}{M}\right) \quad --(20)$$

9. Remove the non-coherent fundamental component from the actual data and replace it with the coherent fundamental component.

$$xm[n] = x[n] - x_{nc\_i}[n] + x_{c\_i}[n] \quad --(21)$$

$$x'[n] = xm[n]$$

10. Let  $X_{J_{\text{int}_i}}=0, i=i+1$ , if  $i < m+1$ , return to step 3
11. Take DFT of  $xm[n]$  and perform spectral analysis to accurately estimate the spectral characteristics.

#### IV. SIMULATION RESULTS

In this section, simulation results to verify the functionality of the proposed method are presented. The robustness of the method with different levels of non-coherency is presented.

##### A. Single-tone Results.

A 14-bit ADC was generated using MATLAB with an INL of 0.6 LSB and input additive noise of 1 LSB standard deviation. The total number of recorded points is 32768.

Figure 6 shows the spectrum when a pure input signal is coherently sampled by the 14-bit ADC. As expected there is no skirting. The true values of THD and SFDR are obtained from this spectrum.

Figure 7 shows the spectrum when direct FFT is used on a non-coherently sampled data. A pure input signal is sampled by the 14-bit ADC. The value of  $\delta$  is 0.01 in this case. Severe skirting is obtained near the fundamental when a direct FFT is used to obtain the spectrum.

Figure 8 shows the spectrum when the proposed method is applied to the same non-coherently sampled data used for Figure 7. As seen in the figure, the skirting is completely removed at the fundamental bin.

Table 1 and Table 2 show the estimated values of THD and SFDR using direct FFT method and the proposed method for  $\delta$

equal to 0.01, a very small value and 0.4, a large value respectively. The true values of ADC's THD and SFDR obtained by coherent sampling are also given in the table. It is shown that the proposed method accurately estimates the values of THD and SFDR for both small and large values of  $\delta$ .

Furthermore, the proposed method works for any arbitrary non-coherency present in the fundamental component. Figure 9 and Figure 10 demonstrate the capability of the proposed method in relaxing the condition of coherency. The simulation involved 10000 runs with randomly selected values of  $\delta$  in the whole range of  $\delta$ . The maximum error in estimating both the THD and SFDR is 1.5 dB. Hence, the condition of coherency is completely eliminated using the proposed method.

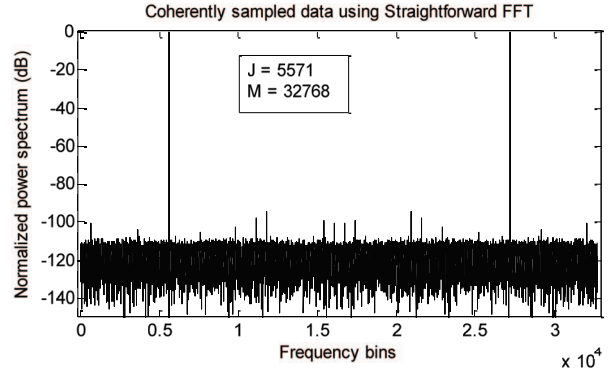


Fig. 6: Spectrum of a coherently sampled 14-bit ADC using direct FFT. The input is a pure sine wave signal.

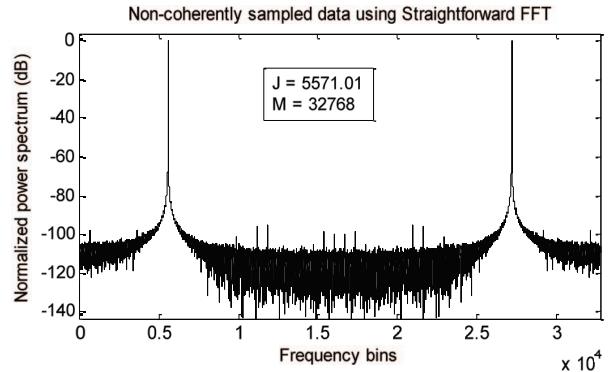


Fig. 7: Spectrum of a non-coherently sampled 14-bit ADC using direct FFT. The input signal is a pure sine wave.

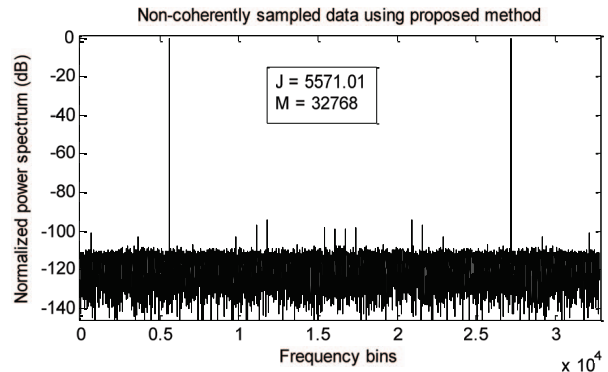


Fig. 8: Spectrum of a non-coherently sampled 14-bit ADC after using proposed Method. The input is a pure sine wave signal.



Table 1: Estimated values of THD and SFDR when an input is not coherently sampled and non-coherency in the input,  $\delta = 0.01$ .

Method	THD (dB)	SFDR (dB)
Direct FFT	-82.9	89.1
Proposed	-88.9	94.3
<b>True values</b>	<b>-89.2</b>	<b>94.5</b>

Table 2: Estimated values of THD and SFDR when an input is not coherently sampled and non-coherency in the input,  $\delta = 0.4$ .

Method	THD (dB)	SFDR (dB)
Direct FFT	-54.6	59.4
Proposed	-89.2	94.2
<b>True values</b>	<b>-89.2</b>	<b>94.5</b>

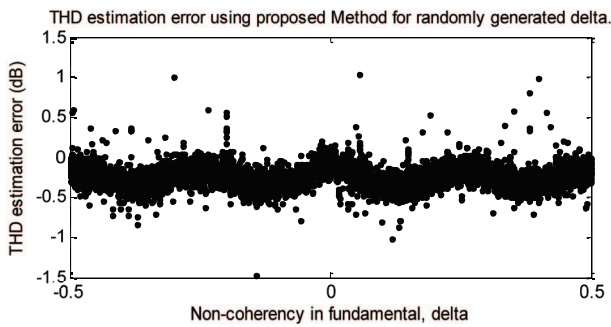


Fig. 9: Error in estimating the THD over the whole range of non-coherency in the fundamental,  $\delta$ .

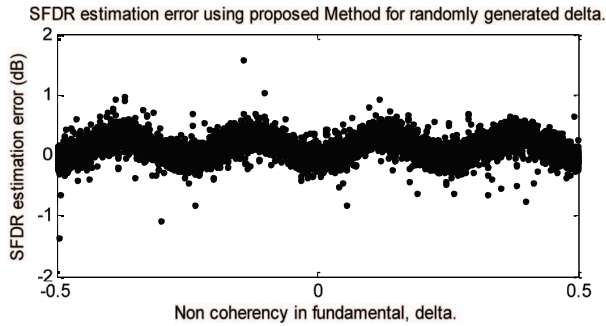


Fig. 10: Error in estimating the SFDR over the whole range of non-coherency in the fundamental,  $\delta$ .

### B. Multi-tone testing:

The proposed method can also be used for multi-tone testing. In this test, a signal with two fundamental components is considered. The harmonics of each fundamental and their inter-modulation components are also present in the signal. For the input signal that is tested here, the ADC used is an ideal 14-bit digitizer. The true values of both the fundamental tones are at  $0.152 * f_{\text{Samp}}$  and  $0.187 * f_{\text{Samp}}$ . In terms of J and M,  $J_{\text{int}_1} = 9989.2857$ ,  $J_{\text{int}_2} = 12266.1985$  and  $M = 65536$ . As seen, since  $J_{\text{int}_1}$  and  $J_{\text{int}_2}$  are not integers the input signal is not coherently sampled. As a result, taking the direct DFT of the sampled data gives severe skirting as shown in Figure 11.

Using the proposed algorithm for multi-tone testing, the skirting is completely removed and the spectral characteristics

of all the harmonics and inter-modulated components can be accurately estimated. Figure 12 shows the result of multi-tone testing using proposed method.

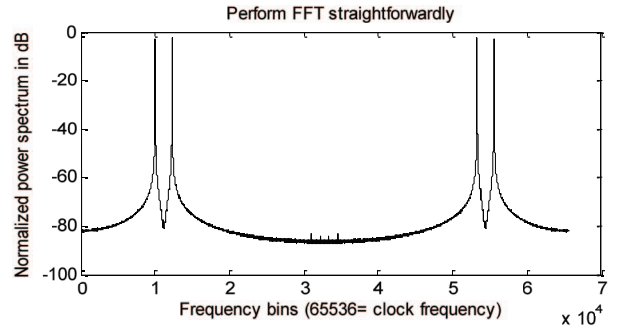


Fig. 11: Spectrum of a non-coherently sampled two-tone input signal with distortion and inter-modulation components using direct FFT method.

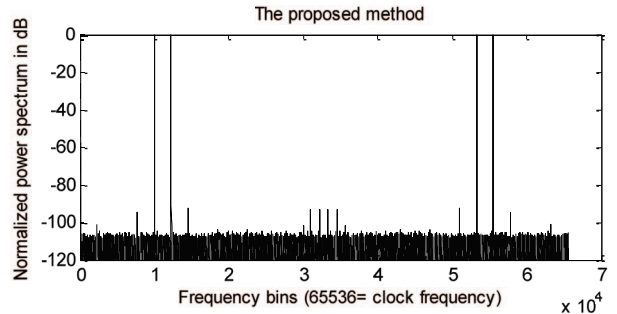


Fig. 12: Spectrum of a non-coherently sampled two-tone input signal with distortion and inter-modulation components using proposed method.

## V. COMPARISON WITH WINDOWING TECHNIQUE AND STATE-OF-THE ART METHODS

In this section, the proposed method is compared with the widely used windowing method and the state-of-the-art fundamental identification and replacement method [4].

### A. Windowing techniques.

The windowing techniques are widely used to obtain accurate spectral characteristics when the input is not coherently sampled. A number of methods were proposed in the literature to perform spectral testing using windows. The windowing techniques are successfully used when the sampling is slightly non-coherent. However, there are some situations when the windowing technique gives inaccurate results.

Now-a-days, the windows cannot be used to test high resolution ADCs as the spectral floor of the windows cannot be lower than the ADC noise floor. In future, due to the advent of on-chip testing, the amount of non-coherency will no longer be small. If windowing technique is used when the non-coherency is large, i.e., when  $\delta$  is large, the actual bin that contains the harmonic power gets shifted from the estimated bin. As a result, the actual harmonic bin falls into the side lobe of the window, which results in giving erroneous results. Also, as the main lobe still has skirting present, if there is any harmonic component that falls in the main lobe, it is not possible to estimate the power of that harmonic accurately. The different situations when the windowing techniques do not work are described below.

As the resolution of ADCs increase, the accuracy of the spectral characteristics calculated using windowing techniques decrease. It is because of the fact that the side lobe spectral floor of the windows that are present today can no longer be lower than the noise floor of the high resolution ADCs. However, the presented method can be used even in such cases to estimate the spectral characteristics accurately. Figure 13 shows the spectrum of a 16-bit ADC when the input is non-coherently sampled. It can be seen that the noise floor obtained from the proposed method is lower than the spectral floor obtained using the Blackman-Harris window. Hence, the proposed method works well for high resolution spectral testing.

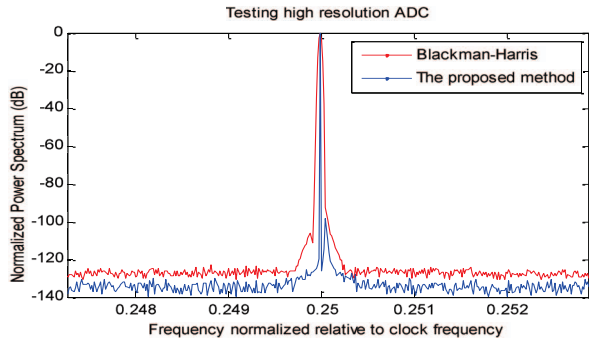


Fig. 13: Spectrum showing the effectiveness of proposed method over windowing method when testing high resolution ADCs. The spectral floor of the window selected is not lower than the ADC noise floor.

If the side lobe of a window is large (i.e., higher than the noise floor of the ADC), the windowing method will not give accurate spectral results. If the side lobe spectral floor of a window is clearly less than the noise floor of the ADC, the windowing technique on non-coherently sampled data can work if the value of  $\delta$  is very small. To achieve such small  $\delta$ , high performance signal and clock generators are required. In order to reduce the test cost, the method should be able to work for large values of  $\delta$ . However, for large values of  $\delta$ , the windowing techniques might give wrong results in estimating the harmonic power. Specifically, when  $\delta$  is large the exact location of the higher order harmonic bins is incorrectly calculated. If such higher order harmonics contain significant power, the windowing technique would result in erroneous results. Such situation will happen in a well designed ADC where in, the power of harmonic components is evenly distributed among many different harmonics This is explained below.

When windowing techniques are used, from the spectrum, the information of only  $J_{int}$  is known. The value of  $\delta$  is unknown. As a result, the bin that corresponds to the  $q^{th}$  harmonic power is estimated as  $q*J_{int}$ . But, the true bin that corresponds to the  $q^{th}$  harmonic power is  $round(q*J)$ . The estimated bin and the actual bin would be the same only if (22) is satisfied. But, as the value of  $\delta$  increases, (22) is not satisfied. This results in a shift of actual bin containing the  $q^{th}$  harmonic from the estimated  $q^{th}$  harmonic bin which leads to errors in estimating harmonic power.

$$round(q*J) = q*J_{int}, \text{ until } |round(q*\delta)| < 0.5 \quad -- (22).$$

To estimate the power of  $q^{th}$  harmonic, it is a common practice to add the power of a span of bins on either side of the estimated  $q^{th}$  harmonic bin. Let  $F$  be the number of bins considered on either side of the estimated harmonic bin so that the total number of bins considered to estimate the  $q^{th}$  harmonic power are  $(2*F+1)$ . For medium values of  $\delta$ , considering a span of bins would help capture the actual harmonic bin. If (23) is satisfied, the error in estimating the harmonic power is decreased as the main bin containing the maximum amount of harmonic power is included.

$$|mod(round(q*J),M) - mod(q*J_{int},M)| < F \quad -- (23).$$

But for large  $\delta$ , the shift in the actual harmonic bin might be more than  $F$  bins from the estimated harmonic bin. As a result, the actual bin that contains the harmonic power is not considered when using windowing technique. For large values of  $\delta$ , the difference between the estimated  $q^{th}$  harmonic bin and the actual  $q^{th}$  harmonic bin increases with increase in the value of  $q$ .

If equation (23) is not satisfied, the windowing method can not estimate the  $q^{th}$  harmonic power accurately.

The phenomenon can be explained by the following example. Consider  $J = 206.45$  and  $M = 4096$ . The value of  $J_{int}$  obtained using Blackman-Harris window is 206. As the value of  $\delta$  is not known when using windowing technique, the estimated values of index containing the second, third and fourth harmonic bins are given as 412, 618 and 824 respectively. But, the actual bins containing second, third and fourth harmonics are 413, 619 and 826 respectively as shown in Figure 14. So, we see that the actual second, third and fourth harmonic bins are shifted by 1, 1 and 2 bins respectively from the estimated bins. In the figure there is an offset of 1 as the frequency bins start from 1 rather than 0. If we consider  $F = 1$ , estimating second and third harmonics would result in less errors as the actual bin is considered in the span. However, the error in estimating the fourth harmonic power is very large as the actual bin containing the fourth harmonic power is not considered in the span. Hence, it is shown that if  $\delta$  is large, the harmonic power estimated by the method of windowing gives inaccurate results. Here  $F = 1$  is considered as an example to explain.

However, this problem is not observed in the proposed method as the value of  $\delta$  is estimated and hence, the estimated  $q^{th}$  harmonic bin is equal to the actual  $q^{th}$  harmonic bin.

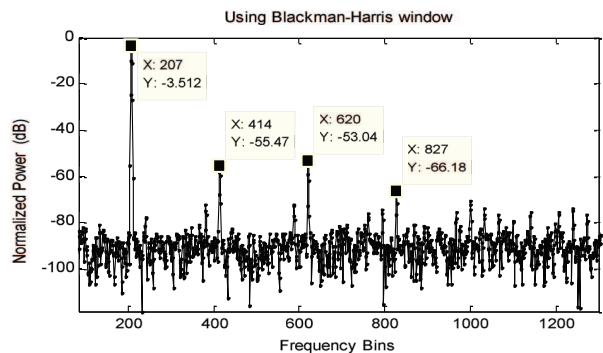


Fig. 14: Spectrum showing the shift in the actual harmonic bins due to large non-coherency present in the fundamental,  $\delta$ . Here  $\delta = 0.45$ .

To improve accuracy using the windowing techniques, one method that can be used is to consider the number of periods of input signal  $J$ , such that the estimated bins containing the harmonic components of the signal are equal to the bins containing the notches (if the windowing results in notches) of the window. The bins containing the notches of the window are noted from the spectrum plot of the window. But, this becomes difficult to realize in practical cases when the input signal is not coherently sampled and the non-coherency ( $\delta$ ) is large. If  $\delta$  is not known, it is not possible to place the harmonic bins on the bins containing the notches of window. This is because the estimated harmonic bins are not equal to the actual harmonic bins. This results in harmonic power estimation errors. However, the proposed method does not have the above mentioned problem and it provides accurate spectral results.

The windowing techniques do not remove skirting completely. The power of the fundamental is distributed in the main lobe of the window selected. There is a possibility of a situation when the  $q^{\text{th}}$  harmonic bin gets folded and falls inside the main lobe. The condition for this case to happen is

$$|\text{mod}(\text{round}(q \cdot J), M) - J| < ((W_m - 1)/2), \quad -- (24).$$

where  $W_m$  is the total width of main-lobe. The function 'mod(A,B)' gives the remainder of A/B.

As the harmonic component is inside the main lobe, it is not possible to accurately obtain the power of such harmonic using windows method. However, the presented method works well even in such cases. An example of such case is given below with simulation result.

Consider  $M = 65536$  and  $J = 16382.936$ .  $J$  is selected randomly from the condition in (24). Let  $W_m = 39$  which is the main lobe width. In this case, the distance between the third harmonic bin and the fundamental bin comes out to be 4 bins using (24). Since  $4 < 19$ , windows method fails to accurately estimate the spectral characteristics. Figure 15 shows the spectrum of the above non-coherent data obtained with Blackman-Harris window and the proposed method. The third harmonic is clearly visible in the spectrum when the proposed method is used. Table 3 shows that the THD and SFDR values are accurately estimated by the proposed method, but the Blackman-Harris window cannot estimate accurately.

Table 3: Estimated values of THD and SFDR when a harmonic bin is inside the main lobe of the window.

Method	THD (dB)	SFDR (dB)
Blackman-Harris	-109	109.8
Proposed	-97.6	98.5
<b>True Values</b>	<b>-97.3</b>	<b>97.8</b>

### B. Issues with Best Data record length Method

The method proposed in [4] accurately estimates the spectral characteristics except in some cases as described in [4]. A best data record length,  $M_{\text{best}}$  is selected such that  $(M/2) < M_{\text{best}} < M$  and  $\delta$  is very small. However, this method has a disadvantage that the calculation time is very large. This is because of the fact that the FFT algorithm is optimal only if the data record length is a power of 2.

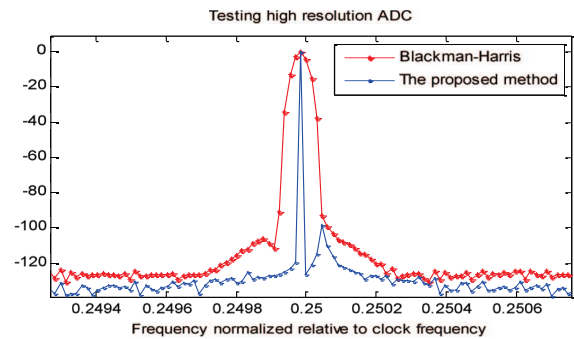


Fig. 15: Spectrum showing the inability of the windowing method in estimating the harmonic power when the harmonic bin falls inside the main lobe. However, the proposed method accurately estimates the harmonic power.

Since the algorithm selects  $M_{\text{best}}$ ,  $M_{\text{best}}$  could be any number, not just a power of 2. As a result, the worst case calculation time is of the order of  $M_{\text{best}}^2$  when  $M_{\text{best}}$  is a prime number. So, the time complexity involved in [4] is large.

In the proposed method,  $M$  can be selected by the user, which can be selected to be a power of 2. So, the calculation time is very small compared to that of the best data record length method. The time complexity involved in this method is of the order of  $2 \cdot M \cdot \log(M)$ . For the same recorded data of size  $M = 4096$ , the time consumed by both the presented method and the best data record length method are given in Table 4. It is seen that the proposed method is about 4 times faster than the state-of-the-art best data record length [4] method. This shows that the presented method is computationally efficient.

Table 4: Table showing that the proposed method is faster than the state-of-the-art Best data record length method [4].

	Proposed Method	Best data record Method
<b>Time (ms)</b>	2	8.2

## VI. EXPERIMENTAL RESULTS

### A. Experimental Results.

In this section, the proposed method is tested using experimental data from a 13-bit ADC. The full scale of the ADC is 0.5V with an LSB of  $61.035 \mu\text{V}$ . The data was collected in the industry. The setup was trying the best to achieve coherent sampling, but apparently the coherency was not sufficient. The exact values of signal frequency and clock frequency are irrelevant in this setup as the ratio of signal to clock frequencies equal the ratio of  $J$  to  $M$  (From (2)). A total of  $2^{19}$  samples are recorded. The reason for considering huge data record is described below.

The clock signal and the input sine wave are generated from a frequency synthesizer which is driven by a master clock. The synthesizer is supposed to guarantee coherency. But, due to the presence of noise and jitter in high frequency Voltage Controlled Oscillator (VCO), the sine wave and clock signal have relative jitter. To reduce the jitter, the Phase Locked Loop (PLL) bandwidth is reduced to have high loop gain. So, the data acquisition time needs to be large, at least several times over the PLL bandwidth. This leads to the large data record length.



Figure 16 shows the spectrum of output data when direct FFT method is used. It is seen that the skirting in the spectrum is very large in spite of taking such a huge data set. Hence the direct FFT method cannot be used to estimate the spectral characteristics of the ADC.

There are simple windows present such as Hamming window, which works well for some applications. In Figure 17, spectrum of the whole data processed with Hamming window is shown. Though the skirting in this spectrum is reduced compared to the skirting in spectrum using direct FFT, the skirting is not low enough to perform spectral testing. Hence, this window cannot be used for spectral testing in this case.

There is another class of windows that are complicated such as Blackman-Harris window. Figure 18 shows the spectrum of the total output data processed with Blackman-Harris window. In the spectrum, most of the leakage has been suppressed, but there is still a small skirting around the bin containing the fundamental. This is shown in Figure 19 by taking a closer look at the fundamental.

Figure 20 shows the spectrum of total output data processed with the proposed method. Comparing Figure 20 with Figures 16 and 17, it can be said that this method gives better spectral results than using the direct FFT method and the Hamming window method. Comparing Figure 20 and Figure 18, it is seen that the spectrum of the data using proposed method closely matches the spectrum of the data using Blackman-Harris (B-H) window. Looking closer at the bins surrounding the fundamental, it is seen from Figure 21 that skirting is eliminated in the proposed method. From Figure 21 and Figure 19, it can be said that the spectrum of the experimental data obtained by using the proposed method is cleaner than that obtained by using Blackman-Harris window.

The estimated THD and SFDR values of the ADC using the direct FFT method, the B-H window and the proposed method are given in Table 5 below.

Using the proposed method, the estimated value of the total number of cycles that are sampled in  $2^{19}$  points, i.e.,  $J$ , is calculated. It was found that  $J = 176.509$ . As  $J$  is not an integer, the severe skirting observed in Figure 16 is explained. This shows that in spite of taking huge effort in collecting the data for coherent sampling, the data obtained might not be coherent. Figures 20 and 21 show that the proposed method works well for non-coherently sampled experimental data.

Blackman-Harris window is widely used and is one of the best windows available today to perform low resolution ADC testing when the sampling is not coherent. The spectrum as well as the spectral parameter values (from Table 5) of the data using proposed method closely matches with that of the data using Blackman-Harris window. Also, the spectrum of the data using the proposed method is cleaner than that of the data using Blackman-Harris window at the fundamental bin. From the details mentioned above, it can be said that the proposed method gives accurate spectral testing results even when the input is not coherently sampled.

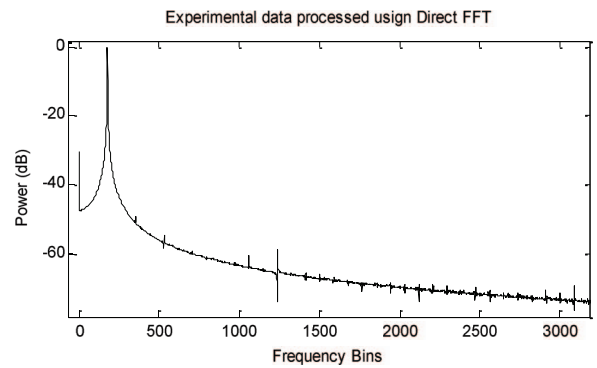


Fig. 16: Partial spectrum of the experimental data using direct FFT method.

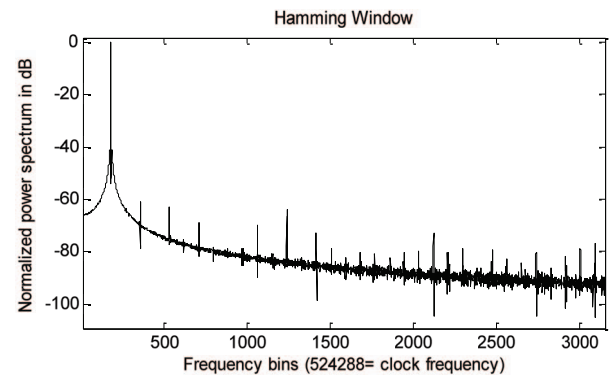


Fig. 17: Partial spectrum of the experimental data using Hamming window.

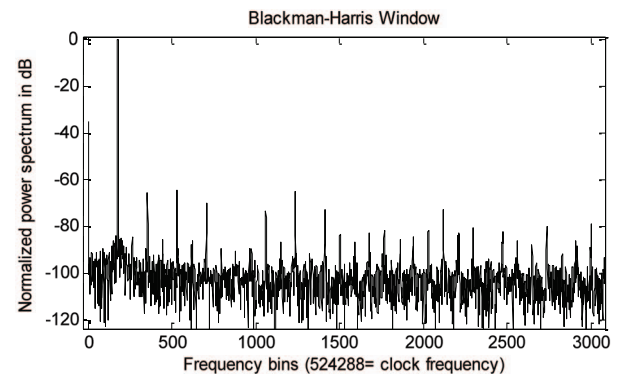


Fig. 18: Partial Spectrum of the experimental data using Blackman-Harris window

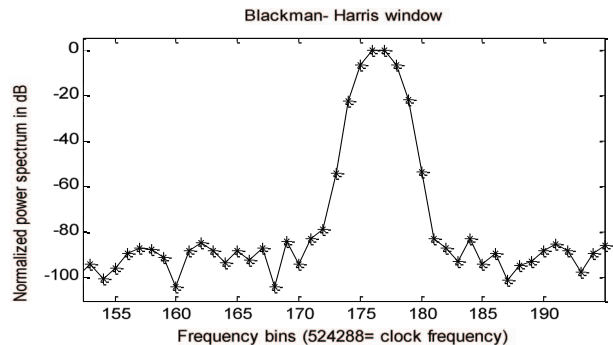


Fig. 19: Zoomed in version of Fig.18 to show skirting.

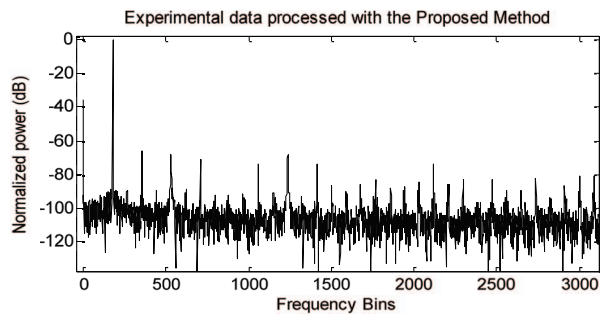


Fig. 20: Partial Spectrum of the experimental data using the proposed method

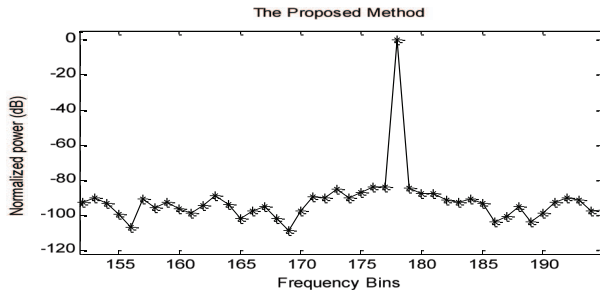


Fig. 21: Zoomed in version of Fig. 20 to show that skirting is completely eliminated

Table 5: Estimated THD and SFDR values of a real ADC using direct FFT method and proposed method.

	Proposed Method	Direct FFT	B-H Window
THD(dB)	-59.5	-44.9	-60
SFDR(dB)	64.9	47.2	64.6
SNR(dB)	51.4	12	51.3

Hence, it is shown that with the industry data of a 13-bit ADC, the spectrum obtained from using the proposed method is cleaner than that obtained from using the Blackman-Harris window. So, this validates that the presented method gives accurate results when estimating the spectral characteristics of a non-coherently sampled data.

## VII. CONCLUSION

A new spectral testing method was proposed. The proposed method eliminates the requirement of coherent sampling in spectral testing, which makes on-chip spectral testing possible. Compared to windowing technique, the method removes the consideration of window selection and also has better accuracy and wider applicability (for all values of  $\delta$ ). The proposed method is slightly slower than the straight forward FFT method (with coherent sampling) and windowing technique but is highly faster than previous methods with non-coherent sampling[4-5]. The straightforward FFT method is applicable to any situation (provided we have coherent sampling). However, the proposed method is limited to situations where the fundamental power is dominant in the total spectrum. When distortion powers become comparable to the fundamental power, the method may not work. However, for all mixed signal testing environment, the conditions stated above are satisfied.

Simulation results show that the proposed method is robust to any level of non-coherent sampling and is suitable for high resolution spectral testing. The method is computationally efficient and can perform both single tone and multiple tone spectral testing accurately. It is also shown that the proposed method gives accurate results even in the situations where windowing techniques cannot provide correct results. The method was validated using experimental results. Compared to windowing techniques, the proposed method achieved a cleaner spectrum.

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