A Faster Method for Accurate Spectral Testing without Requiring Coherent Sampling

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***Abstract*—The Discrete Fourier Transform (DFT) is the ubiquitous method of choice for spectral testing. Nevertheless, the correct application of DFT for slightly distorted sinusoidal signals requires either strict coherent sampling, or careful windowing, or other methods that are not computationally efficient. This paper introduces an improved method for achieving accurate spectral testing for approximate sinusoidal signals without the need for coherent sampling or windowing. Theoretical analysis, simulation and experimental results show that the proposed method is faster than the original method and more robust when the signal frequency is close to Nyquist frequency. Comparative studies demonstrate that the proposed algorithm achieves better spectral testing accuracies than those obtained using the windowing techniques.**

Keywords-Discrete Fourier Transform; coherent sampling; spectral testing

I. INTRODUCTION

Spectral performance of an integrated circuit is of critical concern in many important application areas such as signal processing and communications. It is well known that Discrete Fourier Transform (DFT) or its fast implementation Fast Fourier Transform (FFT) is the most prevalent method for spectral testing. However, when performing FFT for spectral testing of slightly distorted sinusoidal signals, the signal frequency and the sampling clock frequency of the data acquisition system must satisfy very stringent coherency condition. In the FFT algorithm, even the slightest mismatches between the two frequencies could cause a frequency leakage phenomenon in which energy from the fundamental spectral line is spread into neighboring frequencies. This causes the appearance of a "skirt" around the fundamental spectral line.

In order to combat frequency leakage, the IEEE standard [1] as well as the best practice in industry is to require coherent sampling, which means the sampling clock signal should be perfectly synchronized with the signal under test. As a result, an integer multiple of signal periods are captured in a data record M. When this condition is guaranteed, direct use of FFT is permitted and the data analysis is computationally very efficient when M is the power of 2. It requires only $O(M\log_2 M)$ operations. Unfortunately, strictly coherent sampling requires an accurate synthesizer, which requires more design effort and large die area and is therefore unsuitable for on-chip implementation.

An alternative method is to use the windowing technique [2][3] while allowing noncoherent sampling. This technique does not remove the skirting due to noncoherency, rather it merely suppresses the skirting levels at frequencies far away from the fundamental frequency. By doing so it alters the heights of the original spectral lines. Care must be taken in order to correctly recover the spectral lines. Furthermore, due to the fact that the amount of skirt suppression is limited, the windowing technique is not effective in spectral analysis of high purity signals.

Other methods to combat spectral leakage include singular value decomposition [4], 2-D FFT [5], filter banks [6] and removing leakage in frequency domain [7]. These methods are accurate but they are very inefficient in computation.

In order to overcome the shortcomings of the above methods, the concept of fundamental identification and replacement was first introduced in [8]. In this method, the amplitude, frequency and phase of the fundamental harmonic component are estimated first. Then the noncoherent fundamental harmonic component is replaced by a sine component that has the same amplitude and phase but with a slightly modified frequency so that it becomes coherent with the sampling clock. The method does not require coherent sampling or windowing. However, in the cases when the selected data record length M has large prime factors or M itself is a prime number, the computational efficiency of FFT (a component of the algorithm) is poor. Furthermore, the robustness of the method is reduced when the signal frequency is near Nyquist rate and additive noise is relative large.

In this paper an improved fundamental identification and replacement technique is proposed. The new method uses a data record length that has only prime factors of 2 or 3, the smallest prime numbers. This guarantees that the new method is computational efficient with minimal negative impact on spectral analysis. The paper also introduces a new algorithm for counting the signal periods in the data which makes the proposed method more robust when the signal frequency is near Nyquist rate. In Section II, an improved fundamental identification and replacement technique is proposed in detail. The simulation and experimental results are reported in Section III and IV respectively. Section V concludes this paper.

II. THE IMPROVED FUNDAMENTAL IDENTIFICATION AND REPLACEMENT TECHNIQUE

This section we will discuss the improved fundamental identification and replacement method in detail. Denote f_s as the sampling frequency, $T_s=1/f_s$ as the sampling intervals, f_i as the unknown input signal frequency, and M_0 as the nominal data record length. Then $J=M_0f_i/f_s=J_0+\Delta$ will be the number of periods of the input signal in the data record, where J_0 is the integer part of J, Δ is the fractional part of J. J_0 and M_0 are assumed to be co-prime. Δ is unknown, so is J (J_0 could be known).

Suppose the input signal is

$$x(t) = A\sin(2\pi f_i t + \theta) + hd \tag{1}$$

where $A \approx 1$, $\theta \in [0, 2\pi)$, hd is the sum of 2nd and higher harmonic components.

The samples of x(t) at sampling rate f_s are given by:

$$x[k] = A\sin(2\pi f_i \frac{k}{f_s} + \theta) + hd \tag{2}$$

Since $J=M_0f_i/f_s=J_0+\Delta$,

$$x[k] = A\sin(2\pi \frac{J}{M_0}k + \theta) + hd = A\sin(2\pi \frac{J_0+\Delta}{M_0}k + \theta) + hd$$
$$= x_1[k] + x_h[k] \tag{3}$$

In (3), $x_1[k]$ is the fundamental harmonic component of x[k], $x_h[k]$ is the sum of the 2nd and higher harmonic components of x[k]. From [8], we know that as long as Δ is non-zero, which means the data record length is not exactly an integer number of signal periods, the DFT algorithm introduces an error term (skirt term) in the Fourier transform of the fundamental component. This leakage term can be so large that it completely inundates the harmonic distortion components, making it impossible to correctly test the true spectrum of the signal.

In order to estimate and remove the skirt term from the DFT spectrum, the concept of fundamental identification and replacement was first introduced in [8]. In this method, the amplitude, frequency and phase of the fundamental harmonic component are estimated firstly in the time domain. Then the noncoherent fundamental harmonic component is replaced by a sine component that has the same amplitude and phase but with a slightly modified frequency so that it becomes coherent with the sampling clock. After that the standard FFT spectral analysis is done as usual.

Unfortunately, the method in [8] has two shortcomings. The first shortcoming is that when the selected data record length M has large prime factors or M itself is a prime number, the computational efficiency of FFT (a component of the original algorithm) is poor. In order to avoid the cases, we choose the data record length M that has only prime factors of 2 or 3, the two smallest prime numbers. Furthermore, the pairs far away from zero, such as the pairs whose absolute values are larger than $A/\sqrt{2}$, are used to select the most suitable data record length M so that the effects of noise can be reduced greatly. For example, we find the first point $x[k_1]$ whose absolute value is larger than $A/\sqrt{2}$ in the original data record, then compare $x[k_1]$ with $x[M_1+k_1]$, $x[M_2+k_1]$, ……, $x[M_i+k_1]$, ……, $x[M_N+k_1]$, (M_i has only prime factors of 2 or 3, the smallest prime numbers. M_N is close to the length of original data record), to find $x[M_i+k_1]$ that minimizes the following equation

$$(x[M_i+k_1]-x[k_1])^2 + (x[M_i+k_1+1]-x[k_1+1])^2$$
$$+(x[M_i+k_1+2]-x[k_1+2])^2 \quad \text{(i=1, 2, 3, ..., N)} \tag{4}$$

then $x[M_i+k_1]$ most closely matches $x[k_1]$. That is, the sub-sequence from $x[k_1]$ through $x[M_i+k_1]$ most closely matches an integer number of signal periods. Then M_i which has only prime factors of 2 or 3 is the most suitable data record length. When this is guaranteed, the proposed method is computationally more efficient than the original method. After choosing the most suitable data record length M, we still use the positive zero-crossing point as the starting point of the data record. For instance, assuming $x[k_2]$ to be a positive zero-crossing point near $x[k_1]$, then $x[k_2]$ to $x[M+k_2-1]$ is used for spectral analysis. By doing so, θ will be approximately 0 and the errors in estimating θ will have less effects.

The second shortcoming is that the robustness of the original method is reduced when the signal frequency is near Nyquist rate and additive noise is relative large. The reason is that the method in [8] doesn't count the integer cycles of input signal correctly in the cases. Therefore, a new accurate method for counting the integer cycles of input signal is applied in the new method.

The new method for counting the integer cycles J_{int} in the data sequence from $x[k_2]$ to $x[M+k_2-1]$ is introduced as follows (here we call it method I).

a) *Let*
   y[k]=1 if $x[k] \geq A/\sqrt{2}$,
   y[k]=-1 if $x[k] \leq -A/\sqrt{2}$,
   y[k]=0 if $A/\sqrt{2} < x[k] < A/\sqrt{2}$
   then generate a new sequence y[k] (by doing this, the effects of noise are reduced greatly).
b) *In the sequence y[k], if the adjacent elements are the same, choose only one element, then get a new sequence w[k].*
c) *Define the variable c*
   – *If for all k, we have w[k]=w[k+4], then let c=0 indicating* $f_{in} \leq f_s/4$.
   – *If there is at least one k with w[k]≠w[k+4], then let c=1 indicating* $f_{in} > f_s/4$.
d) *Let* $z[k] = |y[k]|$, *then get the new sequence z[k].*
e) *In the sequence z[k], if the adjacent elements are the same, choose only one element, then get a new sequence zz[k].*
f) *Count the sum sum(zz) of the non-zreo elements in sequence zz[k],*
g) *Compute the integer cycles* J_{int}
   J_{int}=floor(sum(zz)/2) if c=0

J_{int}=M/2-floor(sum(zz)/2) if c=1.

The algorithm (in step c) which determines whether the signal frequency is larger than $f_s/4$ can be easily proved by pigeonhole principle [9]. Because of page limitation, we won't discuss it in this paper.

It should be pointed out that this method for counting the integer cycles is not robust when f_{in} is close to $f_s/4$. In this case, the step size of x[k]'s phase is close to π/2. Because of the noise, y[k]'s value is not accurate when x[k] is close to $A/\sqrt{2}$ or $-A/\sqrt{2}$. Therefore, when f_{in} is close to $f_s/4$, we use another method to count the integer cycles of input signal (Here we call it method II). After method I is executed, we get J_{int}, if $20\% < \frac{J_{int}}{M} < 30\%$, we use method II to recount J_{int}. Method II is summarized by the following two steps:

1) *Define the variable R*
   For all i, if $x[i+1] > x[i]$, let R=1, otherwise R=-1, so R reflects the rising and falling trend of sine wave.
2) *The integer number of cycles J_{int} can be counted at every transition from R=-1 to R=1.*

Method II is extremely robust when f_{in} is close to $f_s/4$.

The procedure of the improved fundamental identification and replacement technique can be outlined in following 11 steps.

1) *Capture a sufficient large number of samples,*
2) *Find the first point $x[k_1]$ whose absolute value is larger than $A/\sqrt{2}$,*
3) *choose the most suitable data record length M. compare $x[k_1]$ with $x[M_1+k_1]$, $x[M_2+k_1]$, $x[M_3+k_1]$, ……,$x[M_i+k_1]$, ……, $x[M_N+k_1]$ (M_i has only prime factors of 2 or 3, M_N is close to the length of original data record), to find the point $x[M_i+k_1]$ that most matches $x[k_1]$ in the data sequence. Then M_i is just the most suitable data record length M.*
4) *Find the positive zero-crossing point $x[k_2]$ near $x[k_1]$, use $x[k_2]$ to $x[k_2+M-1]$ as the data record.*
5) *normalize the data record using the power-based normalization, and get the first estimate of the fundamental harmonic magnitude $\widehat{A}_0$.*
6) *count the integer cycles J_{int} of the sequence from $x[k_2]$ to $x[k_2+M-1]$ using method I,*
7) *If $20\% < \frac{J_{int}}{M} < 30\%$, recount J_{int} using method II,*
8) *Compute the fractional cycle*

$$\Delta = \frac{1}{2\pi}[\arcsin(x[k_2]/\widehat{A}_0) - \arcsin(x[k_2+M]/\widehat{A}_0)] \qquad (5)$$

Then the input signal frequency is

$$f_i = f_s \frac{J_{int}+\Delta}{M} \qquad (6)$$

9) *At a subset of data points write*

$$\begin{aligned} x[k] &= A_0\sin(2\pi f_i t_k + \theta) \\ &= A_0\cos(\theta)\sin(2\pi f_i t_k) + A_0\sin(\theta)\cos(2\pi f_i t_k) \end{aligned} \qquad (7)$$

and use least square method to identify $A_0cos(\theta)$ and $A_0sin(\theta)$.

10) *Perform the fundamental component replacement*

$$\begin{aligned} \widehat{x}[k] &= x[k] - A_0\sin(2\pi f_i t_k + \theta) \\ &+ A_0\sin(2\pi f_s J_{int}/M + \theta) \end{aligned} \qquad (8)$$

The new data $\widehat{x}[k]$ is generated by replacing the fundamental component from the original data (which is sampled noncoherently and causes possibly large skirts) with one that is coherent with the sampling clock. This can be achieved by simply subtracting a sine component with the identified parameters and adding a sine component with the same A_0 and θ but with Δ being rounded to zero.

11) *Perform FFT analysis on $\widehat{x}[k]$ as usual.*

The improved method uses the data record length M that has only prime factors of 2 or 3, the two smallest prime numbers. This guarantees that the new method is computational efficient. Furthermore, the new algorithm for counting the integer cycles of input signal makes the improved method more robust and more immune to noise. It works well even when the signal frequency is close to Nyquist rate and the additive noise is relative large.

III. SIMUALTION RESULTS

Simulation study has been conducted in MATLAB in order to verify the performance of the proposed algorithm. During the simulation, ADC (analog-to-digital converter) is modeled as a set of transition levels. Its nonlinearity error is chosen to be a Gaussian random variable with zero mean and standard deviation σ_{DNL}. In this section we present the spectral testing example of 14-bit ADC with σ_{DNL} of 0.008 LSB. Additive noise of input signal is also chosen to be a Gaussian random variable with zero mean and standard deviation of 1 LSB (least significant bit). The signal frequency is set to be close to Nyquist frequency. The data record length is 16384.

For comparison, three different spectral testing methods are simulated first. They are: (1) straightforward application of FFT assuming periodic sampled sequence, (2) the proposed method, (3) perfect coherent sampling. Fig. 1 shows a time domain illustration of the coherent and noncoherent data. Fig. 2 shows the spectrum of straightforward application FFT to the noncoherent data samples. From Fig. 2, we can see that there is a large "skirt" around the fundamental spectral line. Therefore straightforward application of FFT suffered from large errors due to noncoherency. The spectrum of the noncoherent dada samples using the proposed method and the spectrum from perfect coherent sampling data are shown in Fig. 3 and Fig. 4 respectively. From Fig. 3 and Fig. 4 we see that both the spectrums show zero or minimal skirt, which indicates that the proposed method achieves comparable spectral testing accuracy as the perfect coherent sampling method.

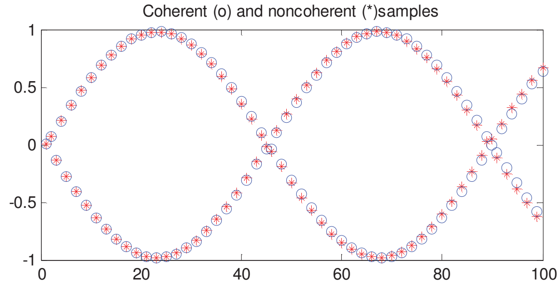

Fig. 1 Data samples from coherent and noncoherent sampling

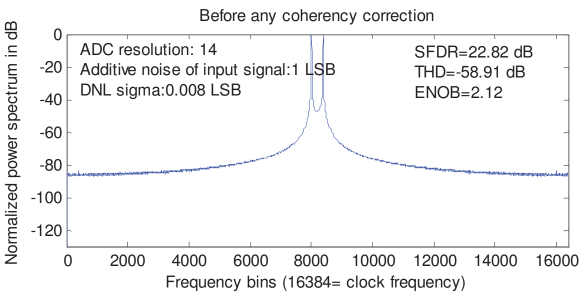

Fig. 2 Straightforward application FFT to the noncoherent data samples

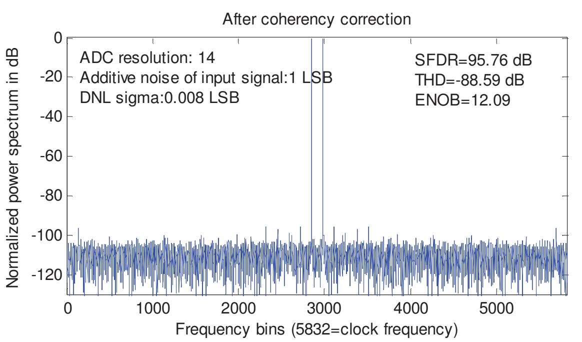

Fig. 3 Spectrum of the noncoherent data samples using the proposed method

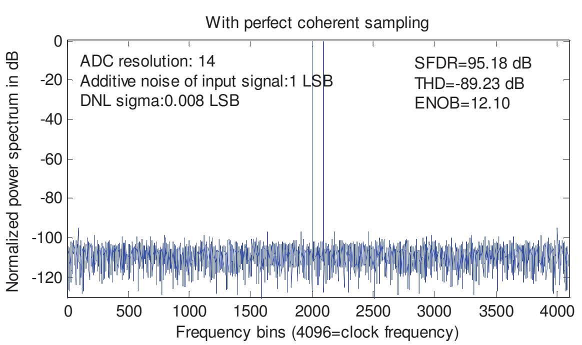

Fig. 4 Spectrum from perfect coherent sampling

We also compare the proposed method with the windowing techniques and the original method [8] using the same noncoherent data record. The corresponding comparative results are summarized in Table I. From Table I we can see that the result of straightforward application of FFT is totally wrong although the computation is most efficient. Most windowing techniques only achieve limited

TABLE I. M, ENOB, AND TIME USING DIFFERENT METHODS

Methods	M	ENOB	Time(ms)
Straightforward FFT	16384	2.12	6.75
Hanning	16384	8.45	7.27
Hamming	16384	5.22	7.68
Blackman	16384	9.96	7.43
Blackmanharris	16384	12.04	7.59
Original method	$12769(113\times113)$	12.07	82.49
Proposed method	$5832(2^3\times3^6)$	12.09	9.44

accuracies although the computation is efficient. Both the proposed method and the original method achieve similar spectral accuracies as the perfect coherent sampling method. But the original method is most computational inefficient because the most suitable data record length of the original method is 12769, which comprises large factor 113. Therefore the computation of FFT is computational inefficient. Fortunately, the proposed method is computationally very efficient with only slightly more computations than FFT.

Therefore, the simulation results show that the proposed method can achieve better spectral testing accuracies than those obtained using the windowing techniques. The computational efficiency of the algorithm is excellent with only minimal addition to the computational complexity of FFT.

IV. EXPERIMENTAL RESULTS

Since the proposed method exhibit excellent spectral performance, we want to validate the algorithm with experimental data. Fig. 5 shows the segment of captured noncoherent data in time domain. The data is the output of a commercial 16-bit SAR (successive approximate register) ADC and the noncoherent data record length is 32767.

To analyze the spectral contents of the signal, one can straightforwardly apply FFT to the raw data. The resultant spectrum is shown in Fig. 6. The clearly visible skirting completely dominates any harmonic distortion components that may be present. Hence no testing information is obtained.

Several different windowing techniques are applied to analyze the spectral contents of the collected signal. Fig.7 shows the spectrum using the hamming window. Compared to the spectrum obtained by performing FFT directly (see Fig. 6), the frequency leakage are combated a little. Nevertheless, the skirt is still large and the accurate spectral information cannot be available. In fact, in terms of windowing techniques, only one of the best windows such as Blackmanharris window can achieve accurate spectral testing result. Fig. 8 shows the spectrum using the Blackmanharris window. From Fig. 8 we can see that the frequency leakage is combated effectively.

Then the proposed method was applied to analyze the spectral contents of the captured signal. The resultant spectrum is shown in Fig. 9. From Fig. 9 we can see that the best data record length that applied in spectral analysis is 7776. We also notice that all skirting effects have been removed and the noise floor has been pushed down to the −127dB level. Rich spectral contents are clearly shown. The corresponding dynamic parameters such as SFDR, THD and ENOB are 106.78 dB, -103.31 dB and 14.99 respectively.

Fig. 10 illustrates the spectral comparison of the Blackmanharris method and the proposed method around the signal bin. The horizontal axis is the frequency normalized versus the clock frequency. The vertical axis is the normalized power in dB. From Fig .10 we see that, for Blackmanharris method, there is still a small skirt around the signal bin. In order to compute the signal power accurately in the spectral testing, we should add the power of the bins around the signal bin. Care must be taken in order to conduct the spectral testing accurately. Fortunately, for the proposed method, the skirt is removed thoroughly. The sole signal bin is enough to compute the signal power accurately. Therefore, the proposed method can be easily applied for the spectral testing.

In fact, the noncoherent data is obtained by discarding the last point of the perfectly coherent sampling data record whose length is 32768. The spectrum of the perfectly coherent sampling data is shown in Fig. 11. The computed SFDR, THD, ENOB are 106.85 dB, -104.56 dB and 14.98 respectively. Comparing the spectral parameters obtained by the proposed method and perfectly sampling method respectively, we can see that they are very close to each other. Furthermore, comparing Fig. 6 with Fig. 11 we can conclude that the coherent sampling condition is very strict and even tiny mismatches (lack of one point) can cause disastrous measurement results.

We also compare the proposed method with the original method using the same noncoherent data record. All the comparative results are summarized in Table II. From Table II we can see that the result of straightforward application of FFT is totally wrong although the computation is most efficient. Most windowing techniques only achieve limited accuracies although the computation is efficient. Both the proposed method and the original method achieve similar spectral accuracies as the perfect coherent sampling method. But the original method is most computational inefficient because the most suitable data record length of the original method is 30947, which comprises a large factor 4421. Therefore the computation of FFT is computational inefficient. Fortunately, the proposed method is computationally very efficient with only slightly more computations than FFT.

Therefore, the experimental results show that the proposed method achieves better spectral testing accuracies than those obtained using the windowing techniques.

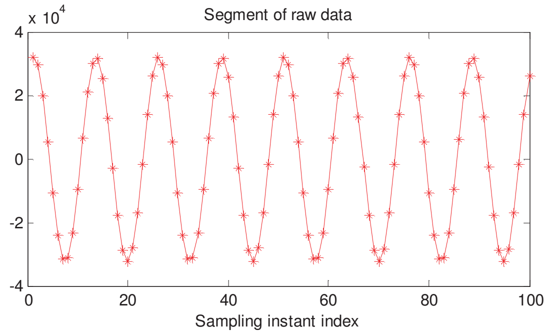

Fig. 5 Time domain data samples from the noncoherent sampling

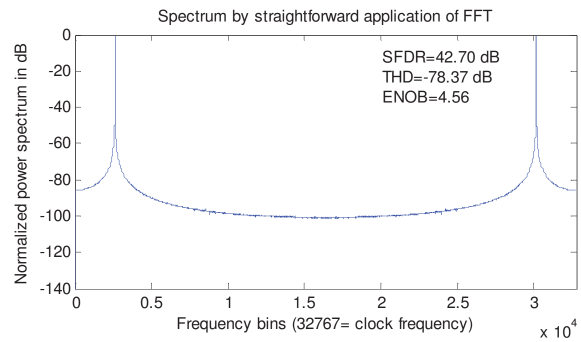

Fig. 6 Straightforward application of FFT to the noncoherent data samples

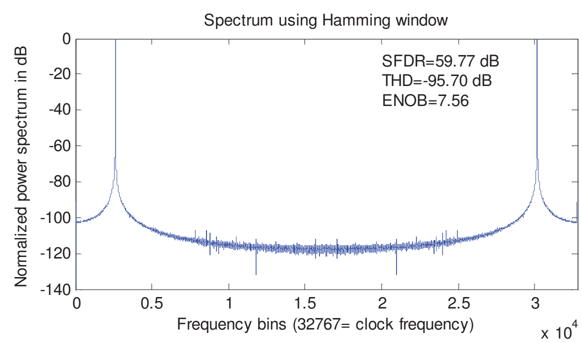

Fig. 7 Spectrum using Hamming window

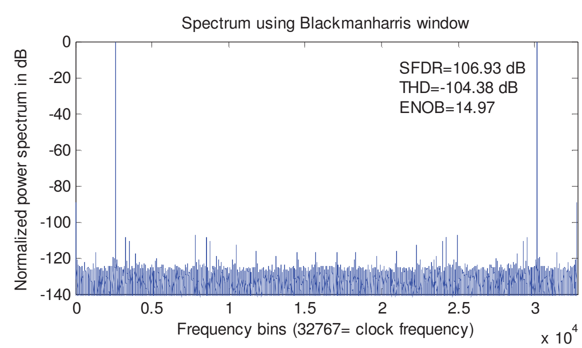

Fig. 8 Spectrum using Blackmanharris window

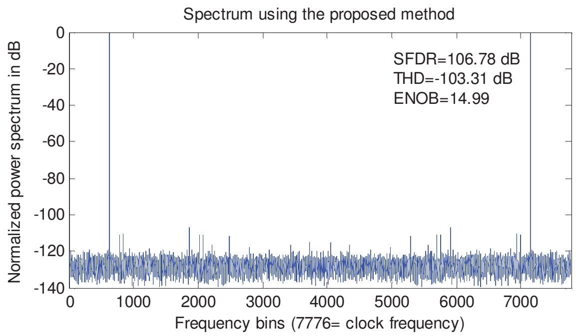

Fig. 9 Spectrum obtained by the proposed method

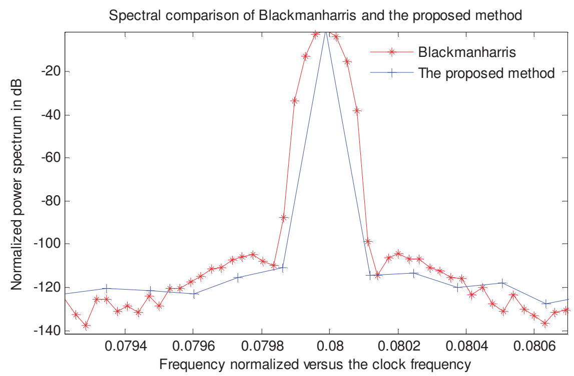

Fig. 10 Spectral comparison of Blackmanharris method and the proposed method around the signal bin

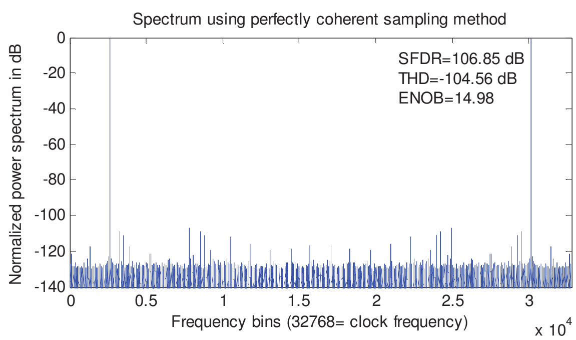

Fig. 11 Spectrum obtained by the perfectly cohrent sampling method

TABLE II. M, ENOB, AND TIME USING DIFFERENT METHODS

Methods	M	ENOB	Time(ms)
Straightforward FFT	32767	4.56	27.32
Hanning	32767	12.76	32.53
Hamming	32767	7.56	33.66
Blackman	32767	13.97	31.82
Blackmanharris	32767	14.97	35.75
Original method	$30947\left(7\times4421\right)$	14.97	423.24
Proposed method	$7776\left(2^5\times3^5\right)$	14.99	51.27
Perfect coherent sampling method	32768	14.98	30.99

V. CONCLUSIONS

An improved method for fast and accurate spectral testing is proposed. The proposed method does not require coherent sampling or the use of windowing. The proposed method uses the data record length that has only prime factors of 2 or 3, the smallest prime numbers. This guarantees that the new method is computational efficient. Therefore, the new method is faster than the original method. This paper also introduces a new algorithm for counting the signal periods in the data which makes the proposed method more robust and more immune to noise. Simulation and experimental results show that the proposed method exhibits fast and accurate performance for spectral testing.

REFERENCES

[1] IEEE Standard for Digitizing Waveform Recorders, IEEE Std. 1057TM-2007, 2008.

[2] P. Carbone, E. Nunzi, and D. Petri, "Windows for ADC Dynamic Testing via Frequency-Domain Analysis", IEEE Trans. Instr. & Meas., vol.50, No.6, pp.1571-1576, 2001.

[3] F.J.Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform", Proc. IEEE, vol.66, No.1, pp.51-83, 1978.

[4] J. Zhang and S.J Ovaska, " ADC Characterization based on singular value decomposition", IEEE Trans. Instr. & Meas., vol.51, No.1, pp.138-143, 2002.

[5] X. Gao, S.J Ovaska and S. Shenghe,et al, " Analysis of second-order harmonic distortion of ADC using bispectrum", IEEE Trans. Instr. & Meas., vol.45, No.1, pp.50-55, 1996.

[6] C. Rebai, D. Dallet and P Marchegay, " Non-coherent Spectral Analysis of ADC using Filter Bank", IEEE Trans. Instr. & Meas., Tech. Conf., pp.183-187, 2002.

[7] F. xu, "Algorithm to Remove Spectral Leakage, Close-in Noise, and Its Application to Converter," Proceedings IEEE Instrumentation and Measurement Technology Conference, pp. 1038-1042, 2006.

[8] Z. Yu, D. Chen, and R. L. Geiger, "A Computationally Efficient Method for Accurate Spectral Testing without Requiring Coherent Sampling ", Proceedings IEEE Int. Test Conference, pp.1398-1407, 2004.

[9] Wojciech A. Trybulec, "Pigeon Hole Principle", Journal of Formalizaed Mathematics, vol.2, pp.1-5, 1990.