A 2-FFT Method for On-chip Spectral Testing without Requiring Coherency.

Siva Sudani, Degang Chen, Randy Geiger
Department of Electrical and Computer Engineering
Iowa State University
Ames, USA
siva@iastate.edu, djchen@iastate.edu, rlgeiger@iastate.edu

Abstract—**Most of the existing methods that test spectral characteristics require stringent coherent sampling. Maintaining coherent sampling is the major bottleneck in spectral testing. We propose a new method for spectral testing that completely relaxes the condition of coherent sampling. This method uses the frequency domain data to identify the non-coherent fundamental. The method is faster than other state of-the art algorithms that do not require coherency. Because of the relaxation in coherency requirement, this method is suitable for on-chip testing. Simulation results are presented to show the effectiveness of this method in the absence of coherency. A maximum error of 1dB and 1.5dB were obtained while estimating THD and SFDR respectively. The proposed 2-FFT method is successfully verified using measurement data.**

Keywords- coherent sampling, digitizer, FFT, on-chip test, SoC.

I. Introduction

The latest developments in circuit design have resulted in integrating several circuit blocks on a single chip, such as System on Chip (SoC). Such systems include a number of analog and digital functional blocks embedded in a large system. All the blocks in such a system require to be tested for their performance. However, it is difficult to obtain access to the internal nodes of each block for testing. Therefore, there is a strong need for on-chip self-testing capability of such embedded systems.

Most existing methods that test the spectral characteristics require coherent sampling [1]. The data record length should contain exactly an integer number of periods of the input signal. This requirement is stringent because, any tiny errors in the number of periods of input signal considered will result in wrong spectral results. Due to inherent random mismatches present in the semiconductor devices and noise, perfect coherent sampling cannot be guaranteed without using expensive frequency synthesizers. But, embedded systems cannot afford such expensive synthesizers as they consume large area. So, there is a high demand to develop new testing methods that do not require coherent sampling.

Recently, in literature there has been study on methods to relax the condition of coherency. One method is the fundamental identification, removal and replacement method [2] which selects the best data record length and processes the final data to obtain accurate testing results. Other methods include the use of windows [3], the singular value decomposition method [4] which involves a time complexity of O (M^3), the 2-D FFT method [5] which requires O ($M^2 \log^2 M$) operations, the filter banks method [6] whose area linearly increases with the number of harmonics to be measured, the resampling technique [7] which uses an interpolator and a decimator to change the sampling frequency that further increases the area. These methods are accurate but they are inefficient in terms of either computational time or device area.

This paper introduces a new method that eliminates the requirement of coherency for spectral testing. The other advantage includes faster computation with no additional area over head. This paper is arranged as follows. Section II explains the method to identify the non-coherent fundamental component and presents the proposed method. Two theorems are stated that form the basis for proposed method. Section III provides simulation results that show the effectiveness of this new method. Section IV presents measurement results using the proposed method. Section V concludes the discussion.

II. The 2-FFT Method

Let f_{Sig} be the frequency of input signal, f_{Samp} be the clock frequency, M be the total number of data points recorded to measure the spectral characteristics and J be the total number of periods of the input signal in the recorded data. The four parameters are related by the equation.

$$J = M * \frac{f_{Sig}}{f_{Samp}} \tag{1}$$

The sampling is said to be coherent if J is an integer and non-coherent if J is not an integer.

Let x(t) be the time domain representation of the analog signal. The signal is ideally a pure sine wave.

$$\begin{aligned} x(t) &= A\cos\left(2*\pi*f_{Sig}*t+\phi\right) \\ &= a*\cos\left(2*\pi*f_{Sig}*t\right)+b*\sin\left(2*\pi*f_{Sig}*t\right) \end{aligned} \tag{2}$$

where, A is the amplitude of the sine wave, Φ is the phase. $\boldsymbol{a}$ = A*cos(Φ) and $\boldsymbol{b}$ = -A*sin(Φ).

Let x[n] be the analog interpretation of the digital output obtained from the digitizer. x[n] can be represented by the following relation.

$$x[n] = A\cos\left(\frac{2*\pi*J*n}{M}+f\right) + h.o.h + noise + jitter$$
$$= a\cos\left(\frac{2*\pi*J*n}{M}\right) + b\sin\left(\frac{2*\pi*J*n}{M}\right) + h.o.h + noise + jitter \quad (3)$$

for n = 0,1,2,….,M-1. M is usually selected to be a power of 2 for faster processing of the FFT algorithm. 'h.o.h' represents the higher order harmonic components in the output data. These higher order harmonic terms include the harmonics information of both the input signal and the digitizer. If the spectral characteristics of the digitizer such as an ADC need to be measured, the input source should be more pure than the ADC and vice-versa.

If the input signal of a digitizer is not coherently sampled, the DFT of the digitized data results in large spectral measurement errors. However, using the proposed method, accurate spectral characteristics can be obtained even if the input is not coherently sampled. The motivation for the proposed method is obtained from the two theorems that are stated in the following section. The proof of the theorems is not presented due to space constraints.

A. Foundation for the Proposed Method

In this section, two theorems are proposed that could serve as a theoretical foundation for the proposed method.

Consider the case when the input is not coherently sampled. As a result, J in equation (2) is not an integer. Let $J = J_{int} + \delta$, where J_{int} is the integer part closest to J and δ is the fractional part of J such that $-0.5 < \delta \le 0.5$. Let J > max(25, 0.01*M), 0.01 < (J/M) < 0.49 and M > 1024. Let H be the total number of harmonics present in the digitized signal, A_h and φ_h be the amplitude and initial phase of h[th] harmonic respectively such that $A_h << A$ and $\varphi_h \in [0,2\pi)$ for all $2 \le h \le H$.

The above set of J and M is valid because it is a common practice to select more number of input cycles, J and collect more data record points, M to estimate the spectral characteristics of a signal.

If Discrete Fourier Transform (DFT) is applied on a non-coherently sampled data, severe skirting is observed in the frequency spectrum as shown in Figure 1. This skirting would affect the accurate estimation of harmonic power and signal power. It is because the bin corresponding to the particular frequency of interest not only contains the power of that frequency but also contains a fraction of the power of other frequencies (due to non-coherent sampling). The following theorems look at the major sources of error in estimating the harmonics when the input is not coherently sampled.

Theorem1: For H, M, J, A, φ, φ_h and A_h as mentioned above, if the input is not coherently sampled, i.e., $\delta \neq 0$, the error in estimating the power of q[th] harmonic using DFT is mainly dominated by the non-coherency present in the fundamental component.

Theorem 2: For H, M, J, A, φ, φ_h and A_h as mentioned above, if the input is not coherently sampled, i.e., $\delta \neq 0$, the error in estimating the power of q[th] harmonic from DFT due to non-coherency in other harmonic components is negligible provided the non-coherent fundamental is removed and the harmonic frequency bins are separated by a minimum of 20 bins.

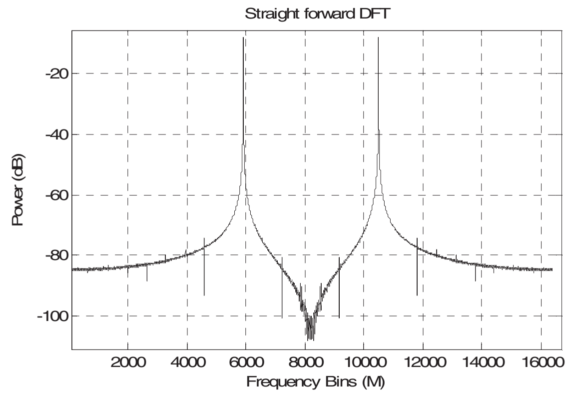

Fig. 1: Spectrum of a non-coherently sampled data showing the skirting.

So, if the non-coherent fundamental can be identified, removed and replaced by a coherent fundamental, the errors in estimating the harmonic power will be considerably reduced.

B. Identifying the non-coherent fundamental.

To identify the non-coherent fundamental from the output data of the digitizer, it is required to estimate the exact value of J, **a** and **b** in equation (3).

For a non-coherently sampled input signal, J is not an integer. In this method, J is estimated using the frequency domain data, X_k, which is obtained after taking the Discrete Fourier Transform (DFT) of x[n].

The index containing the maximum absolute value of the first (M/2+1) Fourier coefficients X_k (setting the DC component to zero), equals $\widehat{J}_{int}$.

To estimate the value of δ, a three point calibration is done using the Fourier coefficients. From [8], for M > 1024, J > max(25 and 0.01*M), the Fourier coefficient X_k can be given as shown below.

$$X_k = \frac{A}{2}\frac{\sin(\pi(J-k))}{\sin\left(\frac{\pi(J-k)}{M}\right)}e^{i(a(J-k)+\varphi)}, \quad a = \frac{\pi*(M-1)}{M} \quad (4)$$

From (4), using the values of X_{Jint}, X_{Jint+1} and X_{Jint-1}, the value of δ can be estimated by equation (5).

$$\hat{\delta} = -\frac{M}{2\pi}\, imag\left(\ln\left\{\frac{\frac{X_{J_{int}}}{X_{J_{int}+1}} - \frac{X_{J_{int}}}{X_{J_{int}-1}}}{\frac{X_{J_{int}}}{X_{J_{int}+1}} - \frac{X_{J_{int}}}{X_{J_{int}-1}} - e^{-j2\pi/M} + e^{j2\pi/M}}\right\}\right) \quad \text{-- (5)}$$

From $\widehat{J}_{int}$ and $\hat{\delta}$, estimate $\widehat{J}$ using $\widehat{J} = \widehat{J}_{int} + \hat{\delta}$.

Now that $\widehat{J}$ is known, **a** and **b** can be estimated using Least Squares Method. Multiplying equation (3) with $\cos\left(\frac{2*\pi*\widehat{J}*n}{M}\right)$ and adding all the M points gives us,

$$\sum_{n=0}^{M-1} x[n]*\cos\left(\frac{2*\pi*\hat{J}*n}{M}\right) = \sum_{n=0}^{M-1}\left\{a*\cos^2\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\} + \sum_{n=0}^{M-1}\left\{b*\sin\left(\frac{2*\pi*\hat{J}*n}{M}\right)*\cos\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\} + \sum_{n=0}^{M-1}\left\{(h.o.h.+noise+jitter)*\cos\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\} \quad (6)$$

Assuming the terms containing sine and cosine product, higher order harmonics, noise and jitter are negligible compared to that of the cosine squared term in (6), we have

$$\sum_{n=0}^{M-1} x[n]*\cos\left(\frac{2*\pi*\hat{J}*n}{M}\right) \approx \sum_{n=0}^{M-1}\left\{a*\cos^2\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\} \quad (7)$$

Similarly, multiplying equation (3) with $\sin\left(\frac{2*\pi*\hat{J}*n}{M}\right)$ and adding all the M points gives us

$$\sum_{n=0}^{M-1} x[n]*\sin\left(\frac{2*\pi*\hat{J}*n}{M}\right) \approx \sum_{n=0}^{M-1}\left\{b*\sin^2\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\} \quad (8)$$

From (7) and (8), **a** and **b** can be estimated as

$$\hat{a} \simeq \frac{\sum_{n=0}^{M-1} x[n]*\cos\left(\frac{2*\pi*\hat{J}*n}{M}\right)}{\sum_{n=0}^{M-1}\left\{\cos^2\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\}} \quad (9)$$

$$\hat{b} \simeq \frac{\sum_{n=0}^{M-1} x[n]*\sin\left(\frac{2*\pi*\hat{J}*n}{M}\right)}{\sum_{n=0}^{M-1}\left\{\sin^2\left(\frac{2*\pi*\hat{J}*n}{M}\right)\right\}} \quad (10)$$

After estimating J, **a** and **b**, the non-coherent fundamental component can be identified. Let xnc[n] be the fundamental component of the actual non-coherent input signal x[n].

$$xnc[n] \approx \hat{a} * \cos\left(\frac{2*\pi*\hat{J}*n}{M}\right) + \hat{b} * \sin\left(\frac{2*\pi*\hat{J}*n}{M}\right) \quad (11)$$

C. The 2-FFT Algorithm.

In this section, the steps required to be followed to obtain accurate spectral characteristics from the output data using the newly proposed 2-FFT method are given.

1. Obtain M recorded data points from the output of the digitizer, i.e., x[n], n = 0, 1, 2, …., M-1.
2. Take DFT of the M points. Let X_k be the DFT coefficients calculated. k = 0,1,2,….,M-1

$$X_k = \sum_{n=0}^{M-1} x[n]*e^{-j\frac{2*\pi*k*n}{M}} \quad (12)$$

3. Estimate J_{int} from the Fourier coefficients X_k using equation (13).

$$\hat{J}_{int} = \arg\max_{1 \le k \le \frac{M}{2}} |X_k| \quad (13)$$

4. Estimate the value of δ using equation (5).
5. Estimate number of input cycles $\hat{J} = \hat{J}_{int} + \hat{\delta}$.
6. Estimate **a** and **b** using equations (9) and (10).
7. Reconstruct xnc[n], the non-coherent fundamental component in x[n] using equation (11).
8. Reconstruct xc[n], the coherent fundamental component closest to the actual input signal corresponding to $\hat{J}_{int}$ number of cycles.

$$xc[n] = \hat{a} * \cos\left(\frac{2*\pi*\hat{J}_{int}*n}{M}\right) + \hat{b} * \sin\left(\frac{2*\pi*\hat{J}_{int}*n}{M}\right) \quad (14)$$

9. Remove the non-coherent fundamental component from the actual data and replace it with the coherent fundamental component. Let $\hat{x}(n)$ be the final data,

$$\hat{x}[n] = x[n] - xnc[n] + xc[n] \quad (15)$$

10. Take DFT of $\hat{x}[n]$ and perform spectral analysis to accurately estimate the spectral characteristics.

D. Spectral Analysis.

To calculate the signal power, the power of the bins adjacent to the actual bin corresponding to the fundamental component should also be added to reduce estimation errors. Care should be taken while calculating the harmonic power. To calculate the q^{th} harmonic power, the main bin that should be considered is "round (q*$\hat{J}$)" not "(q*J_{int})". Also the power of the bins surrounding the main harmonic bin needs to be added to estimate the harmonic power so that the errors due to non-coherency in harmonics can be reduced.

III. SIMULATION RESULTS

For the simulation results, an ADC was generated using MATLAB with a resolution of 12 bits and DNL standard deviation of 0.02 LSB and noise at 0.5 LSB level.

The THD and SFDR values of the ADC were recorded by sending in a pure sine wave with integer number of samples which is equivalent to using the direct DFT method with coherent sampling (standard testing).

Next, a pure sine wave with non-integer number of cycles of the input signal was fed to the same ADC. This corresponds to the practical case where the input is not sampled coherently. The use of direct DFT method on such data resulted in large estimation errors. However, the proposed method produced accurate values of THD and SFDR.

When the ADC is fed with a coherently sampled input, a neat spectrum without any leakage in power is observed as shown in Fig. 2.

However, if the input is not coherently sampled and a direct DFT is performed, leakage in the power of fundamental is observed in the output spectrum as shown in Fig 3.

Using the proposed method after obtaining output of the ADC, the skirting due to fundamental component is completely removed as shown in Fig 4. Power in all the three figures (Fig. 2-4) is normalized to the fundamental power.

Table 1 shows the estimated values of THD and SFDR of ADC using both the direct DFT method and the proposed 2-FFT method. It is observed that the estimation errors using proposed method are very less.

To show the complete relaxation of the condition of coherency using the proposed method, 10000 runs were run on randomly generated δ values ranging from -0.5 to 0.5 (the whole range of δ) for a particular ADC and J_{int}. The THD and SFDR of the ADC obtained by coherent sampling are -73.04dB and 75.72dB respectively. The errors obtained in estimating the THD and SFDR values of the ADC are shown in Fig.5 and Fig.6 respectively. Table 2 presents the statistical results of proposed method which show its effectiveness.

Another advantage that the 2-FFT method has compared to the state of the art 'best data record length' method [2] is the calculation speed. In the proposed method, M can be chosen to be a power of 2 to obtain faster and optimal performance of the Fourier transform. The time complexity is of the order of M*log(M). But, in the best data record length method [2], M_{best} is not a power of 2 and hence the calculation time is large. The worst case time complexity of the method in [2] is of the order of M_{best}^2, where M_{best} is the best data record length. Table 3 shows the calculation time taken for both the methods to estimate the spectral characteristics of an ADC using non-coherent sampling in MATLAB.

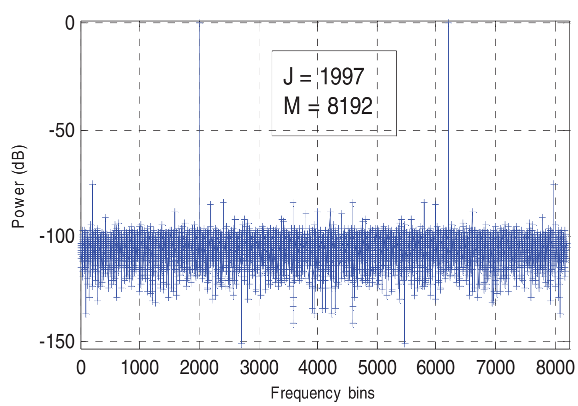

Fig.2: Spectrum of the ADC output when coherently sampled using the direct DFT Method. (J/M = 1997/8192)

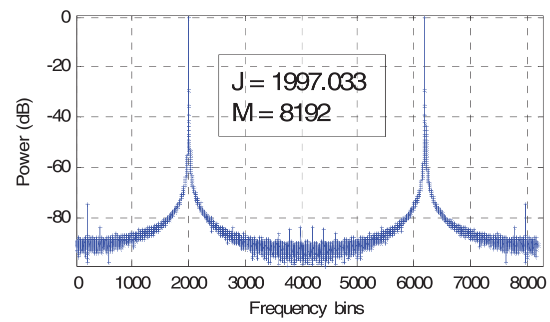

Fig.3: Spectrum of the ADC output when sampled non-coherently using the Direct DFT method. (J/M = 1997.033/8192)

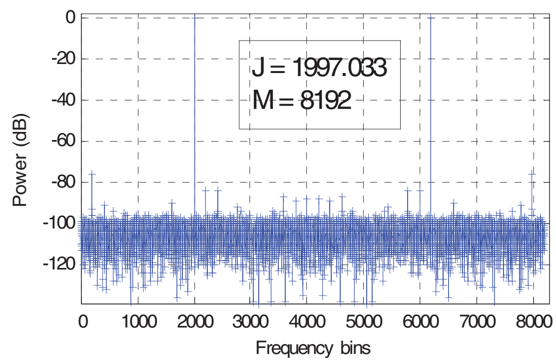

Fig.4: Spectrum of the ADC output when sampled non-coherently using the 2-FFT Method. (J/M = 1997.033/8192)

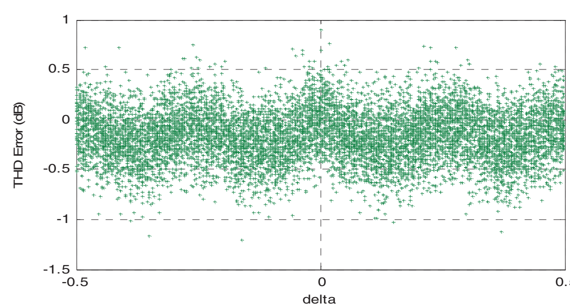

Fig. 5: THD estimation error plot for varying delta using 2-FFT method. δ varies from -0.5 to 0.5

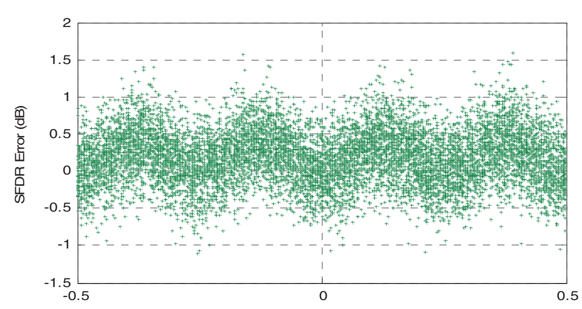

Fig. 6: SFDR estimation error plot for varying delta using 2-FFT method. δ varies from -0.5 to 0.5

TABLE 1: ESTIMATEDVALUES OF THD AND SFDR

Method	THD (dB)	SFDR (dB)
2-FFT	-73.05	75.67
Direct DFT	-63.05	68.35
Ideal Values	**-73.04**	**75.72**

TABLE 2: STATISTICAL RESULTS OF THD AND SFDR ESTIMATION FOR 10000 RUNS

Method	Mean THD (dB)	Mean SFDR (dB)	σ_{THD} (dB)	σ_{SFDR} (dB)
2-FFT	-73.17	75.89	0.19	0.27

TABLE 3: COMPARISON OF CALCULATION TIME

Method	Time (s)
2-FFT Method	0.0175
Best Data Record Method [2]	0.1562

IV. MEASUREMENT RESULTS

In this section, six different Analog to Digital Converters (ADCs) with different speeds, resolutions and architectures are considered to perform spectral analysis. The input to each ADC is coherently sampled and the spectral characteristics obtained from coherent sampling are considered as the reference (or true) values. Later, the input signal to the ADCs is sampled non-coherently and the proposed method is used to estimate the spectral characteristics of each ADC.

Figure 7 shows the spectrum of a coherently sampled 14-bit pipelined ADC. There is no skirting present in the spectrum. The values of second and third harmonic power with respect to signal power (HD2 and HD3), signal to noise ratio (SNR), Total Harmonic Distortion (THD) and Spurious Free Dynamic Range (SFDR) are noted and are considered as reference (or true) values.

The input is then non-coherently sampled by the same ADC. Figure 8 shows the spectrum if a straight forward DFT is performed on the non-coherently sampled data. As expected, skirting is observed in the spectrum. Figure 9 shows the spectrum of the same non-coherently sampled data after using the proposed method. It can be seen that the skirting is completely eliminated in Figure 9. So, the proposed method can be used to estimate the spectral characteristics accurately even when the input is not coherently sampled.

The spectral characteristics of all the six ADCs are estimated and presented in Table 4. The table contains the results obtained by performing both coherent and non-coherent sampling on the ADCs. The TRUE method represents the results obtained by coherent sampling. The proposed method represents the results obtained by performing non-coherent sampling and using the proposed method to estimate the spectral characteristics of each ADC.

The first three rows show the results of three different 9-bit ADCs operating at different speeds. From the results, it can be mentioned that the proposed method is independent of the speed of ADC under test. The ADCs considered in rows four, five and six have different resolutions and architectures. The spectral characteristics of each ADC were accurately estimated by the proposed method. This shows the independence of the proposed method to various architectures and resolutions.

From Table 4, it can be said that the proposed method accurately estimates the spectral characteristics of any ADC irrespective of the architecture, speed and resolution.

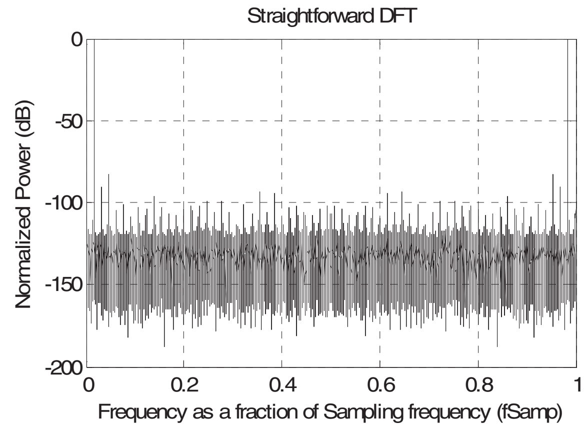

Fig 7: Spectrum of a 14 bit pipelined ADC when coherently sampled using the straight forward DFT method.

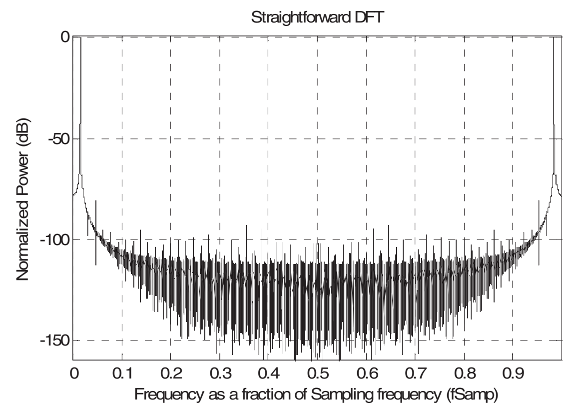

Fig 8: Spectrum of a 14 bit pipelined ADC when non-coherently sampled using the straight forward DFT method.

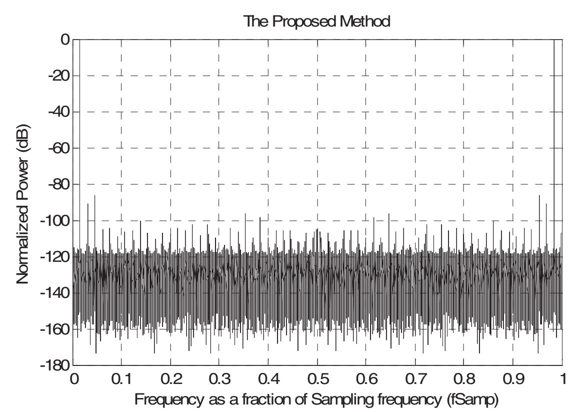

Fig 9: Spectrum of a 14 bit pipelined ADC when non-coherently sampled using the proposed 2-FFT method.

TABLE 4: SPECTRAL RESULTS OF DIFFERENT ADCS USING PROPOSED 2-FFT METHOD (USING NON-COHERENTLY SAMPLED DATA) COMPARED WITH TRUE RESULTS (USING COHERENTLY SAMPLED DATA)

ADC	Method	HD2	HD3	SNR	THD	SFDR
9-bit, 400 MSPS	TRUE	-78.2	-60	49.8	-56.2	60
	Proposed	-77.2	-60.6	49.6	-56.2	60.6
9-bit 400 MSPS	TRUE	-75.7	-56.7	49.6	-54.9	56.7
	Proposed	-75.3	-56.9	49.3	-54.5	56.9
9-bit 800 MSPS	TRUE	-78.6	-58.2	43.8	-55.9	58.2
	Proposed	-78.9	-58.6	43.9	-55.9	58.6
12-bit pipeline	TRUE	-92.6	-94.3	71.9	-85.1	89.6
	Proposed	-92.8	-93.8	72	-85.1	89.8
14-bit Pipeline	TRUE	-90.8	-82.4	72.3	-81.4	82.4
	Proposed	-90.8	-82.7	72.3	-81.7	82.7
16-bit SAR	TRUE	-121	-107	92.2	-104	106.8
	Proposed	-119	-107	92.2	-103	107

V. CONCLUSION

A new method to perform spectral testing using frequency domain data to identify the non-coherent fundamental was proposed. Simulation results were presented that illustrates the capability of the 2-FFT method to test for spectral characteristics even when the input is not coherently sampled. The worst case estimation errors of THD and SFDR values were 1dB and 1.5dB respectively. So, the major bottleneck of maintaining coherency for spectral testing is relaxed. The 2-FFT method is faster than the state of-the art method by about 10 times. The method achieves the above performance at no additional cost in terms of area. Also, measurement results were provided that shows the effectiveness of the proposed 2-FFT method for ADCs with different speed, resolution and architecture. From the advantages mentioned above, this 2-FFT method can be used in embedded signal processing for on-chip spectral testing where coherency is very challenging to obtain.

REFERENCES

[1] M. Burns and G.W.Roberts, *An Introduction to Mixed-Signal IC Test and Measurement.* Oxford University Press, New York, USA, 2000.

[2] Z. Yu, D. Chen, and R. Geiger, “A computationally efficient method for accurate spectral testing without requiring coherent sampling,” in Proc ITC, 2004, pp. 1398–1407.

[3] P. Carbone, E. Nunzi, D. Petri, “Windows for ADC Dynamic Testing via Frequency-Domain Analysis,” IEEE Trans. Instr. & Meas., 50(6), pp 1571-1576, 2001.

[4] J.Q. Zhang and S.J. Oyaska, “ ADC Characterization based on singular value decomposition”, Trans. Instr. & Meas., 51(1), 2002.

[5] X.M. Gao, S.J. Ovaska, S. Shenghe, Y.C. Jenq, “Analysis of second-order harmonic distortion of ADC using bispectrum”, IEEE Trans. Instr. & Meas., 45(1), pp. 50-55, 1996.

[6] C. Rebai, D. Dallet, P. Marchegay, “Non-coheret Spectral Analysis of ADC Using Filter Bank”, IEEE Instr. & Meas. Tech. Conf., pp. 183-187, 2002.

[7] Y.Y. Liu, L. Zhu, C. Wang, B. Zhao, “Non-coherent spectral analysis of ADC using Resampling Method”, ICTD 2009.

[8] V. K. Jain, W. L. Collins Jr., and D. C. Davis, “High-accuracy analog measurements via interpolated-FFT,” IEEE Transactions on Instrumentation and Measurement, vol. IM-28, No. 2, pp. 113–122, June 1979.